Chapter 1

Solutions to Selected Exercises

Section 1.1

2. $\{2, 4\}$ 3. $\{7, 10\}$ 5. $\{2, 3, 5, 6, 8, 9\}$ 6. $\{1, 3, 5, 7, 9, 10\}$

8. A 9. $\emptyset$ 11. B 12. $\{1, 4\}$ 14. $\{1\}$

15. $\{2, 3, 4, 5, 6, 7, 8, 9, 10\}$ 18. 1 19. 3

22. We find that $B = \{2, 3\}$. Since A and B have the same elements, they are equal.

23. Let $x \in A$. Then $x = 1, 2, 3$. If $x = 1$, since $1 \in \mathbf{Z}^+$ and $1^2 < 10$, then $x \in B$. If $x = 2$, since $2 \in \mathbf{Z}^+$ and $2^2 < 10$, then $x \in B$. If $x = 3$, since $3 \in \mathbf{Z}^+$ and $3^2 < 10$, then $x \in B$. Thus if $x \in A$, then $x \in B$.

    Now suppose that $x \in B$. Then $x \in \mathbf{Z}^+$ and $x^2 < 10$. If $x \geq 4$, then $x^2 > 10$ and, for these values of x, $x \notin B$. Therefore $x = 1, 2, 3$. For each of these values, $x^2 < 10$ and x is indeed in B. Also, for each of the values $x = 1, 2, 3$, $x \in A$. Thus if $x \in B$, then $x \in A$. Therefore $A = B$.

26. Since $(-1)^3 - 2(-1)^2 - (-1) + 2 = 0$, $-1 \in B$. Since $-1 \notin A$, $A \neq B$.

27. Since $3^2 - 1 > 3$, $3 \notin B$. Since $3 \in A$, $A \neq B$. 30. Equal 31. Not equal

34. Let $x \in A$. Then $x = 1, 2$. If $x = 1$,

    $$x^3 - 6x^2 + 11x = 1^3 - 6 \cdot 1^2 + 11 \cdot 1 = 6.$$

    Thus $x \in B$. If $x = 2$,

    $$x^3 - 6x^2 + 11x = 2^3 - 6 \cdot 2^2 + 11 \cdot 2 = 6.$$

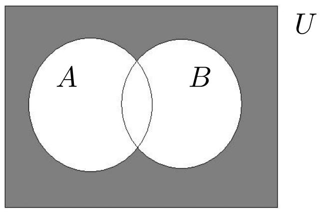
    Again $x \in B$. Therefore $A \subseteq B$.

35. Let $x \in A$. Then $x = (1, 1)$ or $x = (1, 2)$. In either case, $x \in B$. Therefore $A \subseteq B$.

38. Since $(-1)^3 - 2(-1)^2 - (-1) + 2 = 0$, $-1 \in A$. However, $-1 \notin B$. Therefore A is not a subset of B.

39. Consider 4, which is in A. If $4 \in B$, then $4 \in A$ and $4 + m = 8$ for some $m \in C$. However, the only value of m for which $4 + m = 8$ is $m = 4$ and $4 \notin C$. Therefore $4 \notin B$. Since $4 \in A$ and $4 \notin B$, A is not a subset of B.

42.



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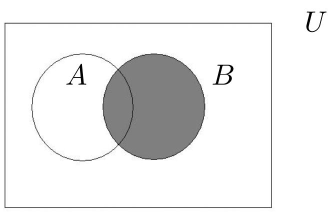

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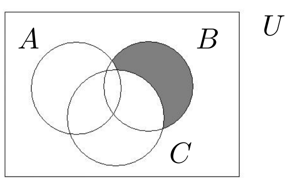

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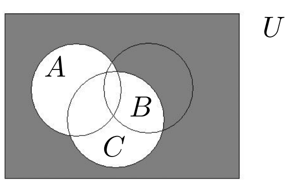
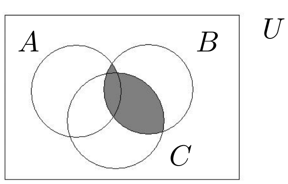
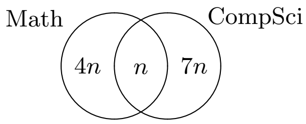


48.



51. 32 52. 105 54. 51

56. Suppose that n students are taking both a mathematics course and a computer science course Then $4n$ students are taking a mathematics course, but not a computer science course, and $7n$ students are taking a computer science course, but not a mathematics course. The following Venn diagram depicts the situation:



Thus, the total number of students is

$$4n + n + 7n = 12n.$$

The proportion taking a mathematics course is

$$\frac{5n}{12n} = \frac{5}{12},$$

which is greater than one-third.

58. $\{(a,1),(a,2),(b,1),(b,2),(c,1),(c,2)\}$

59. $\{(1,1),(1,2),(2,1),(2,2)\}$ 62. $\{(1,a,a),(2,a,a)\}$

63. $\{(1,1,1),(1,2,1),(2,1,1),(2,2,1),(1,1,2),(1,2,2),(2,1,2),(2,2,2)\}$

66. Vertical lines (parallel) spaced one unit apart extending infinitely to the left and right.

67. Horizontal lines (parallel) spaced one unit apart extending infinitely up and down.

69. Consider all points on a horizonal line one unit apart. Now copy these points by moving the horizonal line n units straight up and straight down for all integer $n > 0$. The set of all points obtained in this way is the set $\mathbf{Z} \times \mathbf{Z}$.

70. Ordinary 3-space

72. Take the lines described to the solution to Exercise 67 and copy them by moving n units out and back for all $n > 0$. The set of all points obtained in this way is the set $\mathbf{R} \times \mathbf{Z} \times \mathbf{Z}$.

74. $\{1,2\}$
$\{1\},\{2\}$

75. $\{a,b,c\}$
$\{a,b\},\{c\}$
$\{a,c\},\{b\}$
$\{b,c\},\{a\}$
$\{a\},\{b\},\{c\}$

78. False   79. True   81. False   82. True

84. $\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}$, $\{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}$. All except $\{a,b,c,d\}$ are proper subsets.

85. $2^{10} = 1024$; $2^{10} - 1 = 1023$ 88. $B \subseteq A$ 89. $A = U$

92. The symmetric difference of two sets consists of the elements in one or the other but not both.

93. $A \triangle A = \emptyset$, $A \triangle \overline{A} = U$, $U \triangle A = \overline{A}$, $\emptyset \triangle A = A$

95. The set of primes

Section 1.2

2. Is a proposition. Negation: $6 + 9 \neq 15$.

3. Not a proposition

5. Is a proposition. Negation: For every positive integer n, $19340 \neq n \cdot 17$.

6. Is a proposition. Negation: Audrey Meadows was not the original "Alice" in the "Honeymooners."

8. Is a proposition. Negation: The line "Play it again, Sam" does not occur in the movie *Casablanca*.

9. Is a proposition. Some even integer greater than 4 is not the sum of two primes.

11. Not a proposition. The statement is neither true nor false.

13. No heads were obtained. 14. No heads or no tails were obtained. 17. True

18. True 20. False 21. False

23.

p	q	$(\neg p \vee \neg q) \vee p$
T	T	T
T	F	T
F	T	T
F	F	T

24.

p	q	$(p \vee q) \wedge \neg p$
T	T	F
T	F	F
F	T	T
F	F	F

26.

p	q	$(p \wedge q) \vee (\neg p \vee q)$
T	T	T
T	F	F
F	T	T
F	F	T

27.

p	q	r	$\neg(p \wedge q) \vee (r \wedge \neg p)$
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

29.

p	q	r	$\neg(p \wedge q) \vee (\neg q \vee r)$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

31. $\neg(p \wedge q)$. True.

32. $p \vee \neg(q \wedge r)$. True.

34. Lee takes computer science and mathematics.

35. Lee takes computer science or mathematics.

37. Lee takes computer science but not mathematics.

38. Lee takes neither computer science nor mathematics.

40. You do not miss the midterm exam and you pass the course.

41. You play football or you miss the midterm exam or you pass the course.

43. Either you play football and you miss the midterm exam or you do not miss the midterm exam and you pass the course.

45. It is not Monday and either it is raining or it is hot.

46. It is not the case that today is Monday or it is raining, and it is hot.

48. Today is Monday and either it is raining or it is hot, and it is hot or either it is raining or today is Monday.

50. $p \wedge q$

51. $p \wedge \neg q$

53. $p \vee q$

54. $(p \vee q) \wedge \neg p$

56. $p \wedge r \wedge q$

57. $(p \vee r) \wedge q$

59. $(q \vee \neg p) \wedge \neg r$

61. $p \wedge \neg r$

62. $p \wedge q \wedge r$

64. $\neg p \wedge \neg q \wedge r$

65. $\neg(p \vee q \vee \neg r)$

66.

p	q	p *exor* q
T	T	F
T	F	T
F	T	T
F	F	F

68. Inclusive-or

69. Inclusive-or

71. Exclusive-or

72. Exclusive-or

76. "lung disease" -cancer

77. "minor league" baseball team illinois -"midwest league"

Section 1.3

2. If Rosa has 160 quarter-hours of credits, then she may graduate.

3. If Fernando buys a computer, then he obtains \$2000.

5. If a person gets that job, then that person knows someone who knows the boss.

6. If you go to the Super Bowl, then you can afford the ticket.

8. If a better car is built, then Buick will build it.

9. If the chairperson gives the lecture, then the audience will go to sleep.

12. Contrapositive of Exercise 2: If Rosa does not graduate, then she does not have 160 quarter-hours of credits.

14. False 15. False 17. False 18. True 20. True

22. Unknown 23. Unknown 25. True 26. Unknown 28. Unknown

29. Unknown 32. True 33. True 35. True 36. False

38. True 39. False 41. $(p \wedge r) \to q$ 42. $\neg((r \wedge \neg q) \to r)$

45. $(\neg p \vee \neg r) \to \neg q$ 46. $r \to q$ 48. $q \to (p \vee r)$ 49. $(q \wedge p) \to \neg r$

51. If it is not raining, then it is hot and today is Monday.

52. If today is not Monday, then either it is raining or it is hot.

54. If today is Monday and either it is raining or it is hot, then either it is hot, it is raining, or today is Monday.

55. If today is Monday or (it is not Monday and it is not the case that (it is raining or it is hot)), then either today is Monday or it is not the case that (it is hot or it is raining).

57. Let p: $4 > 6$ and q: $9 > 12$. Given statement: $p \to q$; true. Converse: $q \to p$; if $9 > 12$, then $4 > 6$; true. Contrapositive: $\neg q \to \neg p$; if $9 \leq 12$, then $4 \leq 6$; true.

58. Let p: $|1| < 3$ and q: $-3 < 1 < 3$. Given statement: $q \to p$; true. Converse: $p \to q$; if $|1| < 3$, then $-3 < 1 < 3$; true. Contrapositive: $\neg p \to \neg q$; if $|1| \geq 3$, then either $-3 \geq 1$ or $1 \geq 3$; true.

61. $P \not\equiv Q$ 62. $P \equiv Q$ 64. $P \not\equiv Q$ 65. $P \equiv Q$ 67. $P \not\equiv Q$

68. $P \not\equiv Q$ 71. Either Dale is not smart or not funny.

72. Shirley will not take the bus and not catch a ride to school.

75. (a) If p and q are both false, $(p\ \mathit{imp2}\ q) \wedge (q\ \mathit{imp2}\ p)$ is false, but $p \leftrightarrow q$ is true.
    (b) Making the suggested change does not alter the last line of the *imp2* table.

76.

p	q	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

Section 1.4

2. Invalid

$$\begin{array}{l} p \to q \\ \neg r \to \neg q \\ \hline \therefore\ r \end{array}$$

3. Valid

$$\begin{array}{l} p \leftrightarrow r \\ r \\ \hline \therefore\ p \end{array}$$

5. Valid

$$\begin{array}{l} p \to (q \vee r) \\ \neg q \wedge \neg r \\ \hline \therefore\ \neg p \end{array}$$

7. If 4 megabytes of memory is better than no memory at all, then either we will buy a new computer or we will buy more memory. If we will buy a new computer, then we will not buy more memory. Therefore if 4 megabytes of memory is better than no memory at all, then we will buy a new computer. Invalid.

8. If 4 megabytes of memory is better than no memory at all, then we will buy a new computer. If we will buy a new computer, then we will buy more memory. Therefore, we will buy more memory. Invalid.

10. If 4 megabytes of memory is better than no memory at all, then we will buy a new computer. If we will buy a new computer, then we will buy more memory. 4 megabytes of memory is better than no memory at all. Therefore we will buy more memory. Valid.

12. Valid 13. Valid 15. Valid

16. Suppose that $p_1, p_2, \ldots, p_n$ are all true. Since the argument $p_1, p_2 \ / \therefore\ p$ is valid, p is true. Since $p, p_3, \ldots, p_n$ are all true and the argument

$$p, p_3, \ldots, p_n \ / \therefore\ c$$

is valid, c is true. Therefore the argument

$$p_1, p_2, \ldots, p_n \ / \therefore\ c$$

is valid.

19. Modus ponens

20. Disjunctive syllogism

22. Let p denote the proposition "there is gas in the car," let q denote the proposition "I go to the store," let r denote the proposition "I get a soda," and let s denote the proposition "the car transmission is defective." Then the hypotheses are:

$$p \to q, \quad q \to r, \quad \neg r.$$

From $p \to q$ and $q \to r$, we may use the hypothetical syllogism to conclude $p \to r$. From $p \to r$ and $\neg r$, we may use modus tollens to conclude $\neg p$. From $\neg p$, we may use addition to conclude $\neg p \vee s$. Since $\neg p \vee s$ represents the proposition "there is not gas in the car or the car transmission is defective," we conclude that the conclusion does follow from the hypotheses.

23. Let p denote the proposition "Jill can sing," let q denote the proposition "Dweezle can play," let r denote the proposition "I'll buy the compact disk," and let s denote the proposition "I'll buy the compact disk player." Then the hypotheses are:

$$(p \vee q) \to r, \quad p, \quad s.$$

From p, we may use addition to conclude $p \vee q$. From $p \vee q$ and $(p \vee q) \to r$, we may use modus ponens to conclude r. From r and s, we may use conjunction to conclude $r \wedge s$. Since $r \wedge s$ represents the proposition "I'll buy the compact disk and the compact disk player," we conclude that the conclusion does follow from the hypotheses.

25. The truth table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

shows that whenever p is true, $p \vee q$ is also true. Therefore addition is a valid argument.

26. The truth table

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

shows that whenever $p \wedge q$ is true, p is also true. Therefore simplification is a valid argument.

28. The truth table

p	q	r	$p \to q$	$q \to r$	$p \to r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

shows that whenever $p \to q$ and $q \to r$ are true, $p \to r$ is also true. Therefore hypothetical syllogism is a valid argument.

29. The truth table

p	q	$p \vee q$	$\neg p$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	T

shows that whenever $p \vee q$ and $\neg p$ are true, q is also true. Therefore disjunctive syllogism is a valid argument.

Section 1.5

2. The statement is a command, not a propositional function.

3. The statement is a command, not a propositional function.

5. The statement is not a propositional function since it has no variables.

6. The statement is a propositional function. The domain of discourse is the set of real numbers.

8. 1 divides 77. True. 9. 3 divides 77. False. 11. For some n, n divides 77. True.

13. False 14. True 16. True 17. True 19. False 20. True

22. $\neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \neg P(4)$ 23. $\neg(P(1) \wedge P(2) \wedge P(3) \wedge P(4))$

25. $\neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4)$ 26. $\neg(P(1) \vee P(2) \vee P(3) \vee P(4))$

29. Some student is taking a math course.

30. Every student is not taking a math course.

32. It is not the case that every student is taking a math course.

33. It is not the case that some student is taking a math course.

36. There is some person such that if the person is a professional athlete, then the person plays soccer. True.

37. Every soccer player is a professional athlete. False.

39. Every person is either a professional athlete or a soccer player. False.

40. Someone is either a professional athlete or a soccer player. True.

42. Someone is a professional athlete and a soccer player. True.

45. $\exists x(P(x) \wedge Q(x))$

46. $\forall x(Q(x) \rightarrow P(x))$

50. True 51. True 53. False 54. True

56. No. The suggested replacement returns false if $\neg P(d_1)$ is true, and true if $\neg P(d_1)$ is false.

58. Literal meaning: Every old thing does not covet a twenty-something. Intended meaning: Some old thing does not covet a twenty-something. Let $P(x)$ denote the statement "x is an old thing" and $Q(x)$ denote the statement "x covets a twenty-something." The intended statement is $\exists x(P(x) \wedge \neg Q(x))$.

59. Literal meaning: Every hospital did not report every month. (Domain of discourse: the 74 hospitals.) Intended meaning (most likely): Some hospital did not report every month. Let $P(x)$ denote the statement "x is a hospital" and $Q(x)$ denote the statement "x reports every month." The intended statement is $\exists x(P(x) \wedge \neg Q(x))$.

61. Literal meaning: Everyone does not have a degree. (Domain of discourse: People in Door County.) Intended meaning: Someone does not have a degree. Let $P(x)$ denote the statement "x has a degree." The intended statement is $\exists x \neg P(x)$.

62. Literal meaning: No lampshade can be cleaned. Intended meaning: Some lampshade cannot be cleaned. Let $P(x)$ denote the statement "x is a lampshade" and $Q(x)$ denote the statement "x can be cleaned." The intended statement is $\exists x(P(x) \wedge \neg Q(x))$.

64. Literal meaning: No person can afford a home. Intended meaning: Some person cannot afford a home. Let $P(x)$ denote the statement "x is a person" and $Q(x)$ denote the statement "x can afford a home." The intended statement is $\exists x(P(x) \wedge \neg Q(x))$.

65. The literal meaning is as Mr. Bush spoke. He probably meant: Someone in this country doesn't agree with the decisions I've made. Let $P(x)$ denote the statement "x agrees with the decisions I've made." Symbolically, the clarified statement is $\exists x \, \neg P(x)$.

69. Let

$$\begin{aligned} p(x)&: \quad x \text{ is good.} \\ q(x)&: \quad x \text{ is too long.} \\ r(x)&: \quad x \text{ is short enough.} \end{aligned}$$

The domain of discourse is the set of movies. The assertions are

$$\begin{array}{l}\forall x(p(x) \to \neg q(x)) \\ \forall x(\neg p(x) \to \neg r(x)) \\ p(\textit{Love Actually}) \\ q(\textit{Love Actually}).\end{array}$$

By universal instantiation,

$$p(\textit{Love Actually}) \to \neg q(\textit{Love Actually}).$$

Since $p(\textit{Love Actually})$ is true, then $\neg q(\textit{Love Actually})$ is also true. But this contradicts, $q(\textit{Love Actually})$.

72. Let $P(x)$ denote the propositional function "x is a member of the Titans," let $Q(x)$ denote the propositional function "x can hit the ball a long way," and let $R(x)$ denote the propositional function "x can make a lot of money." The hypotheses are

$$P(\text{Ken}),\ \ Q(\text{Ken}),\ \ \forall x\ Q(x) \to R(x).$$

By universal instantiation, we have $Q(\text{Ken}) \to R(\text{Ken})$. From $Q(\text{Ken})$ and $Q(\text{Ken}) \to R(\text{Ken})$, we may use modus ponens to conclude $R(\text{Ken})$. From $P(\text{Ken})$ and $R(\text{Ken})$, we may use conjunction to conclude $P(\text{Ken}) \wedge R(\text{Ken})$. By existential generalization, we have $\exists x\ P(x) \wedge R(x)$ or, in words, someone is a member of the Titans and can make a lot of money. We conclude that the conclusion does follow from the hypotheses.

73. Let $P(x)$ denote the propositional function "x is in the discrete mathematics class," let $Q(x)$ denote the propositional function "x loves proofs," and let $R(x)$ denote the propositional function "x has taken calculus." The hypotheses are

$$\forall x\ P(x) \to Q(x),\ \ \exists x\ P(x) \wedge \neg R(x).$$

By existential instantiation, we have $P(d) \wedge \neg R(d)$ for some d in the domain of discourse. From $P(d) \wedge \neg R(d)$, we may use simplification to conclude $P(d)$ and $\neg R(d)$. By universal instantiation, we have $P(d) \to Q(d)$. From $P(d) \to Q(d)$ and $P(d)$, we may use modus ponens to conclude $Q(d)$. From $Q(d)$ and $\neg R(d)$, we may use conjunction to conclude $Q(d) \wedge \neg R(d)$. By existential generalization, we have $\exists\ Q(x) \wedge \neg R(x)$ or, in words, someone who loves proofs has never taken calculus. We conclude that the conclusion does follow from the hypotheses.

75. By definition, the proposition $\exists x \in D\ P(x)$ is true when $P(x)$ is true for some x in the domain of discourse. Taking x equal to a $d \in D$ for which $P(d)$ is true, we find that $P(d)$ is true for some $d \in D$.

76. By definition, the proposition $\exists x \in D\ P(x)$ is true when $P(x)$ is true for some x in the domain of discourse. Since $P(d)$ is true for some $d \in D$, $\exists x \in D\ P(x)$ is true.

Section 1.6

2. Everyone is taller than someone.

3. Someone is taller than everyone.

7. $\forall x \forall y T_1(x,y)$, False; $\forall x \exists y T_1(x,y)$, False; $\exists x \forall y T_1(x,y)$, False; $\exists x \exists y T_1(x,y)$, True.

8. $\forall x \forall y T_1(x,y)$, False; $\forall x \exists y T_1(x,y)$, False; $\exists x \forall y T_1(x,y)$, False; $\exists x \exists y T_1(x,y)$, False.

11. Everyone is taller than or the same height as someone.

12. Someone is taller than or the same height as everyone.

16. $\forall x \forall y T_2(x,y)$, False; $\forall x \exists y T_2(x,y)$, True; $\exists x \forall y T_2(x,y)$, True; $\exists x \exists y T_2(x,y)$, True.

17. $\forall x \forall y T_1(x,y)$, True; $\forall x \exists y T_1(x,y)$, True; $\exists x \forall y T_1(x,y)$, True; $\exists x \exists y T_1(x,y)$, True.

20. For every person, there is a person such that if the persons are distinct, the first is taller than the second.

21. There is a person such that, for every person, if the persons are distinct, the first is taller than the second.

25. $\forall x \forall y T_3(x,y)$, False; $\forall x \exists y T_3(x,y)$, True; $\exists x \forall y T_3(x,y)$, False; $\exists x \exists y T_3(x,y)$, True.

26. $\forall x \forall y T_3(x,y)$, True; $\forall x \exists y T_3(x,y)$, True; $\exists x \forall y T_3(x,y)$, True; $\exists x \exists y T_3(x,y)$, True.

29. $\forall x \forall y L(x,y)$. False.

30. $\exists x \exists y L(x,y)$. True.

34. $\forall x \neg A(x, \text{Profesor Sandwich})$

35. $\forall x \exists y E(x) \rightarrow A(x,y)$

38. True

39. False

43. True

44. False

46. False

47. False

49. False

50. False

52. True

53. True

55. True

56. False

58. True

59. True

61.
```
for i = 1 to n
    if (forall_dj(i))
        return true
return false

forall_dj(i) {
    for j = 1 to n
        if (¬P(d_i, d_j))
            return false
    return true
}
```

62.
```
for i = 1 to n
    for j = 1 to n
        if (P(d_i, d_j))
            return true
return false
```

64. Since the first two quantifiers are universal and the last quantifier is existential, Farley chooses x and y, after which, you choose z. If Farley chooses values that make $x \geq y$, say $x = y = 0$, whatever value you choose for z,
$$(z > x) \wedge (z < y)$$
is false. Since Farley can always win the game, the quantified propositional function is false.

65. Since the first two quantifiers are universal and the last quantifier is existential, Farley chooses x and y, after which, you choose z. Whatever values Farley chooses, you can choose z to be one less than the minimum of x and y; thus making
$$(z < x) \wedge (z < y)$$
true. Since you can always win the game, the quantified propositional function is true.

67. Since the first two quantifiers are universal and the last quantifier is existential, Farley chooses x and y, after which, you choose z. If Farley chooses values such that $x \geq y$, the proposition
$$(x < y) \to ((z > x) \wedge (z < y))$$
is true by default (i.e., it is true regardless of what value you choose for z). If Farley chooses values such that $x < y$, you can choose $z = (x + y)/2$ and again the proposition
$$(x < y) \to ((z > x) \wedge (z < y))$$
is true. Since you can always win the game, the quantified propositional function is true.

69. The proposition must be true. $P(x, y)$ is true for all x and y; therefore, no matter which value for x we choose, the proposition $\forall y P(x, y)$ is true.

70. The proposition must be true. Since $P(x, y)$ is true for all x and y, we may choose *any* values for x and y to make $P(x, y)$ true.

72. The proposition can be false. Let N denote the set of persons James James, Terry James, and Lee James; let the domain of discourse be $N \times N$; and let $P(x, y)$ be the statement "x's first name is the same as y's last name." Then $\exists x \forall y P(x, y)$ is true, but $\forall x \exists y P(x, y)$ is false.

73. The proposition must be true. Since $\exists x \forall y P(x, y)$ is true, there is some value for x for which $\forall y P(x, y)$ is true. Choosing any value for y whatsoever makes $P(x, y)$ true. Therefore $\exists x \exists y P(x, y)$ is true.

75. The proposition can be false. Let $P(x, y)$ be the statement $x > y$ and let the domain of discourse be $\mathbf{Z}^+ \times \mathbf{Z}^+$. Then $\exists x \exists y P(x, y)$ is true, but $\forall x \exists y P(x, y)$ is false.

76. The proposition can be false. Let $P(x, y)$ be the statement $x > y$ and let the domain of discourse be $\mathbf{Z}^+ \times \mathbf{Z}^+$. Then $\exists x \exists y P(x, y)$ is true, but $\exists x \forall y P(x, y)$ is false.

78. The proposition can be true. Let $P(x, y)$ be the statement $x \leq y$ and let the domain of discourse be $\mathbf{Z}^+ \times \mathbf{Z}^+$. Then $\forall x \forall y P(x, y)$ is false, but $\exists x \forall y P(x, y)$ is true.

79. The proposition can be true. Let $P(x, y)$ be the statement $x \leq y$ and let the domain of discourse be $\mathbf{Z}^+ \times \mathbf{Z}^+$. Then $\forall x \forall y P(x, y)$ is false, but $\exists x \exists y P(x, y)$ is true.

81. The proposition can be true. Let N denote the set of persons James James, Terry James, and Lee James; let the domain of discourse be $N \times N$; and let $P(x, y)$ be the statement "x's first name is different from y's last name." Then $\forall x \exists y P(x, y)$ is false, but $\exists x \forall y P(x, y)$ is true.

82. The proposition can be true. Let $P(x, y)$ be the statement $x > y$ and let the domain of discourse be $\mathbf{Z}^+ \times \mathbf{Z}^+$. Then $\forall x \exists y P(x, y)$ is false, but $\exists x \exists y P(x, y)$ is true.

84. The proposition can be true. Let $P(x, y)$ be the statement $x < y$ and let the domain of discourse be $\mathbf{Z}^+ \times \mathbf{Z}^+$. Then $\exists x \forall y P(x, y)$ is false, but $\forall x \exists y P(x, y)$ is true.

85. The proposition can be true. Let $P(x, y)$ be the statement $x \leq y$ and let the domain of discourse be $\mathbf{Z} \times \mathbf{Z}$. Then $\exists x \forall y P(x, y)$ is false, but $\exists x \exists y P(x, y)$ is true.

87. $\forall x\, \exists y\, P(x, y)$ must be false. Since $\exists x\, \exists y\, P(x, y)$ is false, for every x and for every y, $P(x, y)$ is false. Choose $x = x'$ in the domain of discourse. For this choice of x, $P(x, y)$ is false for every y. Therefore $\forall x\, \exists y\, P(x, y)$ is false.

88. $\exists x\, \forall y\, P(x, y)$ must be false. Since $\exists x\, \exists y\, P(x, y)$ is false, for every x and for every y, $P(x, y)$ is false. Choose $y = y'$ in the domain of discourse. Now, for any choice of x, $P(x, y)$ is false for $y = y'$. Therefore $\exists x\, \forall y\, P(x, y)$ is false.

90. Not equivalent. Let $P(x, y)$ be the statement $x > y$ and let the domain of discourse be $\mathbf{Z}^+ \times \mathbf{Z}^+$. Then $\neg(\forall x \exists y P(x, y))$ is true, but $\forall x \neg(\exists y P(x, y))$ is false.

91. Equivalent by De Morgan's law

94. $\exists \varepsilon > 0\, \forall \delta > 0\, \exists x\, ((0 < |x - a| < \delta) \wedge (|f(x) - L| \geq \varepsilon))$

95. $\forall L\, \exists \varepsilon > 0\, \forall \delta > 0\, \exists x\, ((0 < |x - a| < \delta) \wedge (|f(x) - L| \geq \varepsilon))$

96. Literal meaning: No school may be right for every child. Intended meaning: Some school may not be right for some child. Let $P(x, y)$ denote the statement "school x is right for child y." The intended statement is $\exists x \exists y \neg P(x, y)$.

Problem-Solving Corner: Quantifiers

1. The statement of Example 1.6.6 is

$$\forall x \exists y (x + y = 0).$$

As was pointed out in Example 1.6.6, this statement is true. Now

$$\forall x \forall y (x + y = 0)$$

is false; a counterexample is $x = y = 1$. Also

$$\exists x \forall y (x + y = 0)$$

is false since, given any x, if $y = 1 - x$, then $x + y \neq 0$.

2. Yes; the statement $\forall m \exists n (m < n)$ with domain of discourse $\mathbf{Z} \times \mathbf{Z}$ of Example 1.6.1 also solves problems (a) and (b).