Chapter 1

1. (a) Conservation of momentum gives $p_{x,\text{initial}} = p_{x,\text{final}}$, or

$$m_{\text{H}} v_{\text{H,initial}} + m_{\text{He}} v_{\text{He,initial}} = m_{\text{H}} v_{\text{H,final}} + m_{\text{He}} v_{\text{He,final}}$$

Solving for $v_{\text{He,final}}$ with $v_{\text{He,initial}} = 0$, we obtain

$$\begin{aligned} v_{\text{He,final}} &= \frac{m_{\text{H}}(v_{\text{H,initial}} - v_{\text{H,final}})}{m_{\text{He}}} \\ &= \frac{(1.674\times10^{-27}\ \text{kg})[1.1250\times10^{7}\ \text{m/s} - (-6.724\times10^{6}\ \text{m/s})]}{6.646\times10^{-27}\ \text{kg}} = 4.527\times10^{6}\ \text{m/s} \end{aligned}$$

(b) Kinetic energy is the only form of energy we need to consider in this elastic collision. Conservation of energy then gives $K_{\text{initial}} = K_{\text{final}}$, or

$$\tfrac{1}{2} m_{\text{H}} v_{\text{H,initial}}^2 + \tfrac{1}{2} m_{\text{He}} v_{\text{He,initial}}^2 = \tfrac{1}{2} m_{\text{H}} v_{\text{H,final}}^2 + \tfrac{1}{2} m_{\text{He}} v_{\text{He,final}}^2$$

Solving for $v_{\text{He,final}}$ with $v_{\text{He,initial}} = 0$, we obtain

$$\begin{aligned} v_{\text{He,final}} &= \sqrt{\frac{m_{\text{H}}(v_{\text{H,initial}}^2 - v_{\text{H,final}}^2)}{m_{\text{He}}}} \\ &= \sqrt{\frac{(1.674\times10^{-27}\ \text{kg})[(1.1250\times10^{7}\ \text{m/s})^2 - (-6.724\times10^{6}\ \text{m/s})^2]}{6.646\times10^{-27}\ \text{kg}}} = 4.527\times10^{6}\ \text{m/s} \end{aligned}$$

2. (a) Let the helium initially move in the *x* direction. Then conservation of momentum gives:

$$p_{x,\text{initial}} = p_{x,\text{final}}: \qquad m_{\text{He}} v_{\text{He,initial}} = m_{\text{He}} v_{\text{He,final}} \cos\theta_{\text{He}} + m_{\text{O}} v_{\text{O,final}} \cos\theta_{\text{O}}$$

$$p_{y,\text{initial}} = p_{y,\text{final}}: \qquad 0 = m_{\text{He}} v_{\text{He,final}} \sin\theta_{\text{He}} + m_{\text{O}} v_{\text{O,final}} \sin\theta_{\text{O}}$$

From the second equation,

$$v_{\text{O,final}} = -\frac{m_{\text{He}} v_{\text{He,final}} \sin\theta_{\text{He}}}{m_{\text{O}} \sin\theta_{\text{O}}} = -\frac{(6.6465\times10^{-27}\ \text{kg})(6.636\times10^{6}\ \text{m/s})(\sin 84.7°)}{(2.6560\times10^{-26}\ \text{kg})[\sin(-40.4°)]} = 2.551\times10^{6}\ \text{m/s}$$

(b) From the first momentum equation,

$$v_{\text{He,initial}} = \frac{m_{\text{He}} v_{\text{He,final}} \cos\theta_{\text{He}} + m_{\text{O}} v_{\text{O,final}} \cos\theta_{\text{O}}}{m_{\text{He}}}$$

$$= \frac{(6.6465\times10^{-27}\text{ kg})(6.636\times10^{6}\text{ m/s})(\cos 84.7°) + (2.6560\times10^{-26}\text{ kg})(2.551\times10^{6}\text{ m/s})[\cos(-40.4°)]}{6.6465\times10^{-27}\text{ kg}}$$

$$= 8.376\times10^{6}\text{ m/s}$$

3. (a) Using conservation of momentum for this one-dimensional situation, we have $p_{x,\text{initial}} = p_{x,\text{final}}$, or

$$m_{\text{He}} v_{\text{He}} + m_{\text{N}} v_{\text{N}} = m_{\text{D}} v_{\text{D}} + m_{\text{O}} v_{\text{O}}$$

Solving for v_{O} with $v_{\text{N}} = 0$, we obtain

$$v_{\text{O}} = \frac{m_{\text{He}} v_{\text{He}} - m_{\text{D}} v_{\text{D}}}{m_{\text{O}}} = \frac{(3.016\text{ u})(6.346\times10^{6}\text{ m/s}) - (2.014\text{ u})(1.531\times10^{7}\text{ m/s})}{15.003\text{ u}} = -7.79\times10^{5}\text{ m/s}$$

(b) The kinetic energies are:

$$K_{\text{initial}} = \tfrac{1}{2} m_{\text{He}} v_{\text{He}}^2 + \tfrac{1}{2} m_{\text{N}} v_{\text{N}}^2 = \tfrac{1}{2}(3.016\text{ u})(1.6605\times10^{-27}\text{ kg/u})(6.346\times10^{6}\text{ m/s})^2 = 1.008\times10^{-13}\text{ J}$$

$$K_{\text{final}} = \tfrac{1}{2} m_{\text{D}} v_{\text{D}}^2 + \tfrac{1}{2} m_{\text{O}} v_{\text{O}}^2 = \tfrac{1}{2}(2.014\text{ u})(1.6605\times10^{-27}\text{ kg/u})(1.531\times10^{7}\text{ m/s})^2$$
$$+ \tfrac{1}{2}(15.003\text{ u})(1.6605\times10^{-27}\text{ kg/u})(7.79\times10^{5}\text{ m/s})^2 = 3.995\times10^{-13}\text{ J}$$

As in Example 1.2, this is also a case in which nuclear energy turns into kinetic energy. The gain in kinetic energy is exactly equal to the loss in nuclear energy.

4. Let the two helium atoms move in opposite directions along the *x* axis with speeds v_1 and v_2. Conservation of momentum along the *x* direction ($p_{x,\text{initial}} = p_{x,\text{final}}$) gives

$$0 = m_1 v_1 - m_2 v_2 \qquad \text{or} \qquad v_1 = v_2$$

The energy released is in the form of the total kinetic energy of the two helium atoms:

$$K_1 + K_2 = 92.2\text{ keV}$$

Because $v_1 = v_2$, it follows that $K_1 = K_2 = 46.1\text{ keV}$, so

$$v = \sqrt{\frac{2K_1}{m_1}} = \sqrt{\frac{2(46.1\times10^{3}\text{ eV})(1.602\times10^{-19}\text{ J/eV})}{(4.00\text{ u})(1.6605\times10^{-27}\text{ kg/u})}} = 1.49\times10^{6}\text{ m/s}$$

$$v_2 = v_1 = 1.49\times10^{6}\text{ m/s}$$

5. (a) The kinetic energy of the electrons is

$$K_i = \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}(9.11\times10^{-31}\ \text{kg})(1.76\times10^6\ \text{m/s}) = 14.11\times10^{-19}\ \text{J}$$

In passing through a potential difference of $\Delta V = V_f - V_i = +4.15$ volts, the potential energy of the electrons changes by

$$\Delta U = q\Delta V = (-1.602\times10^{-19}\ \text{C})(+4.15\ \text{V}) = -6.65\times10^{-19}\ \text{J}$$

Conservation of energy gives $K_i + U_i = K_f + U_f$, so

$$K_f = K_i + (U_i - U_f) = K_i - \Delta U = 14.11\times10^{-19}\ \text{J} + 6.65\times10^{-19}\ \text{J} = 20.76\times10^{-19}\ \text{J}$$

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(20.76\times10^{-19}\ \text{J})}{9.11\times10^{-31}\ \text{kg}}} = 2.13\times10^6\ \text{m/s}$$

(b) In this case $\Delta V = -4.15$ volts, so $\Delta U = +6.65\times10^{-19}$ J and thus

$$K_f = K_i - \Delta U = 14.11\times10^{-19}\ \text{J} - 6.65\times10^{-19}\ \text{J} = 7.46\times10^{-19}\ \text{J}$$

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(7.46\times10^{-19}\ \text{J})}{9.11\times10^{-31}\ \text{kg}}} = 1.28\times10^6\ \text{m/s}$$

6. (a) $\Delta x_A = v\Delta t_A = (0.624)(2.997\times10^8\ \text{m/s})(124\times10^{-9}\ \text{s}) = 23.2\ \text{m}$
(b) $\Delta x_B = v\Delta t_B = (0.624)(2.997\times10^8\ \text{m/s})(159\times10^{-9}\ \text{s}) = 29.7\ \text{m}$

7. With $T = 35°\text{C} = 308\ \text{K}$ and $P = 1.22\ \text{atm} = 1.23\times10^5\ \text{Pa}$,

$$\frac{N}{V} = \frac{P}{kT} = \frac{1.23\times10^5\ \text{Pa}}{(1.38\times10^{-23}\ \text{J/K})(308\ \text{K})} = 2.89\times10^{25}\ \text{atoms/m}^3$$

so the volume available to each atom is $(2.89\times10^{25}/\text{m}^3)^{-1} = 3.46\times10^{-26}\ \text{m}^3$. For a spherical atom, the volume would be

$$\tfrac{4}{3}\pi R^3 = \tfrac{4}{3}\pi(0.710\times10^{-10}\ \text{m})^3 = 1.50\times10^{-30}\ \text{m}^3$$

The fraction is then

$$\frac{1.50\times10^{-30}}{3.46\times10^{-26}} = 4.34\times10^{-5}$$

8. Differentiating $N(E)$ from Equation 1.23, we obtain

$$\frac{dN}{dE}=\frac{2N}{\sqrt{\pi}}\frac{1}{(kT)^{3/2}}\left[\tfrac{1}{2}E^{-1/2}e^{-E/kT}+E^{1/2}\left(-\frac{1}{kT}\right)e^{-E/kT}\right]$$

To find the maximum, we set this function equal to zero:

$$\frac{2N}{\sqrt{\pi}}\frac{1}{(kT)^{3/2}}E^{-1/2}e^{-E/kT}\left(\frac{1}{2}-\frac{E}{kT}\right)=0$$

Solving, we find the maximum occurs at $E=\tfrac{1}{2}kT$. Note that $E = 0$ and $E = \infty$ also satisfy the equation, but these solutions give minima rather than maxima.

9. For this case $kT=(280\text{ K})(8.617\times10^{-5}\text{ eV/K})=0.0241\text{ eV}$. We take dE as the width of the interval (0.0012 eV) and E as its midpoint (0.0306 eV). Then

$$dN=N(E)\,dE=\frac{2N}{\sqrt{\pi}}\frac{1}{(0.0241\text{ eV})^{3/2}}(0.0306\text{ eV})^{1/2}e^{-(0.0306\text{ eV})/(0.0241\text{ eV})}(0.0012\text{ eV})=1.8\times10^{-2}N$$

10. (a) From Eq. 1.33,

$$\Delta E_{\text{int}}=\tfrac{5}{2}nR\,\Delta T=\tfrac{5}{2}(2.37\text{ moles})(8.315\text{ J/mol}\cdot\text{K})(65.2\text{ K})=3.21\times10^{3}\text{ J}$$

(b) From Eq. 1.34,

$$\Delta E_{\text{int}}=\tfrac{7}{2}nR\,\Delta T=\tfrac{7}{2}(2.37\text{ moles})(8.315\text{ J/mol}\cdot\text{K})(65.2\text{ K})=4.50\times10^{3}\text{ J}$$

(c) For both cases, the change in the translational part of the kinetic energy is given by Eq. 1.31:

$$\Delta E_{\text{int}}=\tfrac{3}{2}nR\,\Delta T=\tfrac{5}{2}(2.37\text{ moles})(8.315\text{ J/mol}\cdot\text{K})(65.2\text{ K})=1.93\times10^{3}\text{ J}$$

11. After the collision, m_1 moves with speed v_1' (in the *y* direction) and m_2 with speed v_2' (at an angle θ with the *x* axis). Conservation of energy then gives $E_{\text{initial}} = E_{\text{final}}$:

$$\tfrac{1}{2}m_1v_1^2=\tfrac{1}{2}m_1v_1'^2+\tfrac{1}{2}m_2v_2'^2 \qquad \text{or} \qquad v^2=v_1'^2+3v_2'^2$$

Conservation of momentum gives: