GRAPH THEORY: Modeling, Applications, and Algorithms,
by Geir Agnarsson and Raymond Greenlaw
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Selected solutions (or drafts thereof) to the Exercises
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Chapter 1

1.5:
By (a) we have that both $P(1)$ and $P(2)$ are true. If now $r \in \{1, 2\}$ and $P(3k+r)$ is true, then so is $P(3(k+1)+r)$ by (b). By induction we have that $P(n)$ is true for all $n \geq 1$ that are not divisible by 3.

Since n is divisible by 3 if, and only if, $n+3$ is divisible by 3, and neither 1 nor 2 are divisible by 3, then we cannot conclude from (a) and (b) alone that $P(3k)$ is true for any k.

If we have additionally that $P(3)$ is true (which we do not have in this problem) then by (b) we obtain that $P(3k)$ is true for all $k \geq 1$ and hence $P(n)$ is true for all $n \in \mathbb{N}$.

1.6:
Since $Q(1001)$ is true by (a), then so are $Q(998), Q(995), \ldots, Q(5), Q(2)$ by (b). In fact, if $Q(n)$ is true, then so is $Q(n-3\ell)$ for any ℓ such that $n-3\ell$ is positive. By (c) we therefore have that $Q(4)$ is true and so by (b) $Q(1)$ is true.

We proceed by induction on m: Assume that $Q(3k+1)$ and $Q(3k+2)$ are true. By (c) we have $Q(6k+2)$ and $Q(6k+4)$ are true, and hence by (b) $Q(3(k+1)+2)) = Q((6k+2)-3(k-1))$ is true and also $Q(3(k+1)+1) = Q((6k+4)-3k)$. By induction we have that $Q(n)$ is true for any number that is not divisible by 3.

However, 1001 is not divisible by 3. If n is not divisible by 3, then neither is $n-3$. Finally, if n is not divisible by 3, then neither is $2n$. Hence, we cannot conclude from (a), (b) and (c) alone that $Q(n)$ is true for any n that is divisible by 3.

1.9:
The graph in Figure 1.4 has 11 edges that we can label $\{e_1, e_2, \ldots, e_{11}\}$ from left-to-right along a horizontal line just above the A_i-vertices. So here $G = (V, E, \phi)$ can be given by

$$\begin{aligned} V &= \{A_1, A_2, \ldots, A_7\} \cup \{J_1, J_2, \ldots, J_5\}, \\ E &= \{e_1, e_2, \ldots, e_{11}\}, \end{aligned}$$

[1] The book actually appeared in September of 2006.