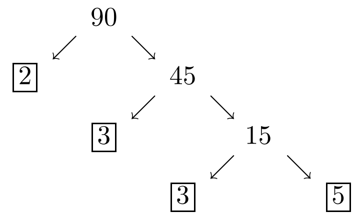
CHAPTER 1 THE REAL NUMBER SYSTEM

1.1 Fractions

1.1 Classroom Examples, Now Try Exercises

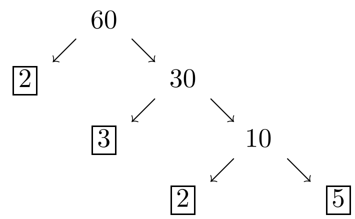
1.



Writing 90 as the product of primes gives us

$$90 = 2 \cdot 3 \cdot 3 \cdot 5.$$

N1.



Writing 60 as the product of primes gives us

$$60 = 2 \cdot 2 \cdot 3 \cdot 5.$$

2. $\frac{12}{20} = \frac{3 \cdot 4}{5 \cdot 4} = \frac{3 \cdot 1}{5 \cdot 1} = \frac{3}{5}$

N2. $\frac{30}{42} = \frac{5 \cdot 6}{7 \cdot 6} = \frac{5 \cdot 1}{7 \cdot 1} = \frac{5}{7}$

3. (a) $\frac{7}{9} \cdot \frac{12}{14} = \frac{7 \cdot 12}{9 \cdot 14}$ *Multiply numerators. Multiply denominators.*

$= \frac{7 \cdot 2 \cdot 2 \cdot 3}{3 \cdot 3 \cdot 2 \cdot 7}$ *Factor.*

$= \frac{2}{3}$ *Write in lowest terms.*

(b) $3\frac{1}{3} \cdot 1\frac{3}{4} = \frac{10}{3} \cdot \frac{7}{4}$ *Change both mixed numbers to improper fractions.*

$= \frac{10 \cdot 7}{3 \cdot 4}$ *Multiply numerators. Multiply denominators.*

$= \frac{2 \cdot 5 \cdot 7}{3 \cdot 2 \cdot 2}$ *Factor.*

$= \frac{35}{6}$, or $5\frac{5}{6}$ *Write as a mixed number.*

N3. (a) $\frac{4}{7} \cdot \frac{5}{8} = \frac{4 \cdot 5}{7 \cdot 8}$ *Multiply numerators. Multiply denominators.*

$= \frac{4 \cdot 5}{7 \cdot 2 \cdot 4}$ *Factor.*

$= \frac{5}{14}$ *Write in lowest terms.*

(b)

$3\frac{2}{5} \cdot 6\frac{2}{3} = \frac{17}{5} \cdot \frac{20}{3}$ *Change both mixed numbers to improper fractions.*

$= \frac{17 \cdot 20}{5 \cdot 3}$ *Multiply numerators. Multiply denominators.*

$= \frac{17 \cdot 5 \cdot 4}{5 \cdot 3}$ *Factor.*

$= \frac{68}{3}$, or $22\frac{2}{3}$ *Write as a mixed number.*

4. (a) $\frac{9}{10} \div \frac{3}{5} = \frac{9}{10} \cdot \frac{5}{3}$ *Multiply by the reciprocal of the second fraction.*

$= \frac{3 \cdot 3 \cdot 5}{2 \cdot 5 \cdot 3}$

$= \frac{3}{2}$, or $1\frac{1}{2}$

(b) $2\frac{3}{4} \div 3\frac{1}{3} = \frac{11}{4} \div \frac{10}{3}$ *Change both mixed numbers to improper fractions.*

$= \frac{11}{4} \cdot \frac{3}{10}$ *Multiply by the reciprocal of the second fraction.*

$= \frac{33}{40}$

N4. (a) $\frac{2}{7} \div \frac{8}{9} = \frac{2}{7} \cdot \frac{9}{8}$ *Multiply by the reciprocal of the second fraction.*

$= \frac{2 \cdot 3 \cdot 3}{7 \cdot 2 \cdot 4}$

$= \frac{9}{28}$

(b) $3\frac{3}{4} \div 4\frac{2}{7} = \frac{15}{4} \div \frac{30}{7}$ *Change both mixed numbers to improper fractions.*

$= \frac{15}{4} \cdot \frac{7}{30}$ *Multiply by the reciprocal of the second fraction.*

$= \frac{15 \cdot 7}{4 \cdot 2 \cdot 15}$

$= \frac{7}{8}$

5. $\frac{1}{9}+\frac{5}{9}=\frac{1+5}{9}$ *Add numerators; denominator does not change.*

$=\frac{6}{9}$

$=\frac{2\cdot 3}{3\cdot 3}$ *Factor.*

$=\frac{2}{3}$

N5. $\frac{1}{8}+\frac{3}{8}=\frac{1+3}{8}$ *Add numerators; denominator does not change.*

$=\frac{4}{8}$

$=\frac{1\cdot 4}{2\cdot 4}$ *Factor.*

$=\frac{1}{2}$

6. **(a)** $\frac{7}{30}+\frac{2}{45}$

Since $30=2\cdot 3\cdot 5$ and $45=3\cdot 3\cdot 5$, the least common denominator must have one factor of 2 (from 30), two factors of 3 (from 45), and one factor of 5 (from either 30 or 45), so it is $2\cdot 3\cdot 3\cdot 5=90$.

Write each fraction with a denominator of 90.

$$\frac{7}{30}=\frac{7\cdot 3}{30\cdot 3}=\frac{21}{90} \quad\text{and}\quad \frac{2}{45}=\frac{2\cdot 2}{45\cdot 2}=\frac{4}{90}$$

Now add.

$$\frac{7}{30}+\frac{2}{45}=\frac{21}{90}+\frac{4}{90}=\frac{21+4}{90}=\frac{25}{90}$$

Write $\frac{25}{90}$ in lowest terms.

$$\frac{25}{90}=\frac{5\cdot 5}{5\cdot 18}=\frac{5}{18}$$

(b) $4\frac{5}{6}+2\frac{1}{3}=\frac{29}{6}+\frac{7}{3}$ *Change both mixed numbers to improper fractions.*

The least common denominator is 6, so write each fraction with a denominator of 6.

$$\frac{29}{6} \quad\text{and}\quad \frac{7}{3}=\frac{7\cdot 2}{3\cdot 2}=\frac{14}{6}$$

Now add.

$$\frac{29}{6}+\frac{7}{3}=\frac{29}{6}+\frac{14}{6}=\frac{29+14}{6}$$

$$=\frac{43}{6},\ \text{or}\ 7\frac{1}{6}$$

N6. **(a)** $\frac{5}{12}+\frac{3}{8}$

Since $12=2\cdot 2\cdot 3$ and $8=2\cdot 2\cdot 2$, the least common denominator must have three factors of 2 (from 8) and one factor of 3 (from 12), so it is $2\cdot 2\cdot 2\cdot 3=24$.

Write each fraction with a denominator of 24.

$$\frac{5}{12}=\frac{5\cdot 2}{12\cdot 2}=\frac{10}{24} \quad\text{and}\quad \frac{3}{8}=\frac{3\cdot 3}{8\cdot 3}=\frac{9}{24}$$

Now add.

$$\frac{5}{12}+\frac{3}{8}=\frac{10}{24}+\frac{9}{24}=\frac{10+9}{24}=\frac{19}{24}$$

(b)

$3\frac{1}{4}+5\frac{5}{8}=\frac{13}{4}+\frac{45}{8}$ *Change both mixed numbers to improper fractions.*

The least common denominator is 8, so write each fraction with a denominator of 8.

$$\frac{45}{8} \quad\text{and}\quad \frac{13}{4}=\frac{13\cdot 2}{4\cdot 2}=\frac{26}{8}$$

Now add.

$$\frac{13}{4}+\frac{45}{8}=\frac{26}{8}+\frac{45}{8}=\frac{26+45}{8}$$

$$=\frac{71}{8},\ \text{or}\ 8\frac{7}{8}$$

7. **(a)** $\frac{3}{10}-\frac{1}{4}$

Since $10=2\cdot 5$ and $4=2\cdot 2$, the least common denominator is $2\cdot 2\cdot 5=20$. Write each fraction with a denominator of 20.

$$\frac{3}{10}=\frac{3\cdot 2}{10\cdot 2}=\frac{6}{20} \quad\text{and}\quad \frac{1}{4}=\frac{1\cdot 5}{4\cdot 5}=\frac{5}{20}$$

Now subtract.

$$\frac{3}{10}-\frac{1}{4}=\frac{6}{20}-\frac{5}{20}=\frac{1}{20}$$

(b) $3\frac{3}{8}-1\frac{1}{2}=\frac{27}{8}-\frac{3}{2}$ *Change each mixed number into an improper fraction.*

The least common denominator is 8. Write each fraction with a denominator of 8. $\frac{27}{8}$ remains unchanged, and

$$\frac{3}{2}=\frac{3\cdot 4}{2\cdot 4}=\frac{12}{8}.$$

Now subtract.

$$\frac{27}{8}-\frac{3}{2}=\frac{27}{8}-\frac{12}{8}=\frac{27-12}{8}=\frac{15}{8},\ \text{or}\ 1\frac{7}{8}$$

N7. **(a)** $\frac{5}{11}-\frac{2}{9}$

Since $11=11$ and $9=3\cdot 3$, the least common denominator is $3\cdot 3\cdot 11=99$. Write each fraction with a denominator of 99.

$$\frac{5}{11}=\frac{5\cdot 9}{11\cdot 9}=\frac{45}{99} \text{ and } \frac{2}{9}=\frac{2\cdot 11}{9\cdot 11}=\frac{22}{99}$$

Now subtract.

$$\frac{5}{11}-\frac{2}{9}=\frac{45}{99}-\frac{22}{99}=\frac{23}{99}$$

(b) $4\frac{1}{3}-2\frac{5}{6}=\frac{13}{3}-\frac{17}{6}$ *Change each mixed number into an improper fraction.*

The least common denominator is 6. Write each fraction with a denominator of 6. $\frac{17}{6}$ remains unchanged, and

$$\frac{13}{3}=\frac{13\cdot 2}{3\cdot 2}=\frac{26}{6}.$$

Now subtract.

$$\frac{13}{3}-\frac{17}{6}=\frac{26}{6}-\frac{17}{6}=\frac{26-17}{6}=\frac{9}{6}$$

Now reduce.

$$\frac{9}{6}=\frac{3\cdot 3}{2\cdot 3}=\frac{3}{2}, \text{ or } 1\frac{1}{2}$$

8. To find out how many gallons of paint Tran should buy, divide the total area to be painted by the area that one gallon of paint covers.

$$\frac{4200}{500}=\frac{42}{5}, \text{ or } 8\frac{2}{5}$$

$8\frac{2}{5}$ gal are needed, so he must buy 9 gal.

N8. To find out how long each piece must be, divide the total length by the number of pieces.

$$10\frac{1}{2}\div 4=\frac{21}{2}\div\frac{4}{1}=\frac{21}{2}\cdot\frac{1}{4}=\frac{21}{8}, \text{ or } 2\frac{5}{8}$$

Each piece should be $2\frac{5}{8}$ feet long.

9. **(a)** In the circle graph, the sector for Europe is the second largest, so Europe had the second largest share of Internet users, $\frac{3}{10}$.

(b) As in Example 9(b),

$$\tfrac{3}{10}(1000)=300 \text{ million.}$$

(c) As in Example 9(c),

$$\frac{3}{10}(970)=\frac{3}{10}\cdot\frac{970}{1}=\frac{2910}{10}=291 \text{ million.}$$

N9. **(a)** In the circle graph, the sector for Other is the smallest, so Other had the least number of Internet users.

(b) As in Example 9(b) (using $\frac{1}{3}$ for $\frac{7}{20}$),

$$\tfrac{1}{3}(1000)\approx 333 \text{ million.}$$

(c) As in Example 9(c),

$$\frac{7}{20}(970)=\frac{7}{20}\cdot\frac{970}{1}=\frac{679}{2}$$
$$=339\frac{1}{2} \text{ million, or } 339{,}500{,}000.$$

1.1 Section Exercises

1. True; the number above the fraction bar is called the numerator and the number below the fraction bar is called the denominator.

2. True; 5 divides the 31 six times with a remainder of one, so $\frac{31}{5}=6\frac{1}{5}$.

3. False; this is an improper fraction. Its value is 1.

4. False; the number 1 is neither prime nor composite.

5. False; the fraction $\frac{13}{39}$ can be written in lowest terms as $\frac{1}{3}$ since $\frac{13}{39}=\frac{13\cdot 1}{13\cdot 3}=\frac{1}{3}$.

6. False; the reciprocal of $\frac{6}{2}=3$ is $\frac{2}{6}=\frac{1}{3}$.

7. False; *product* refers to multiplication, so the product of 10 and 2 is 20. The *sum* of 10 and 2 is 12.

8. False; *difference* refers to subtraction, so the difference between 10 and 2 is 8. The *quotient* of 10 and 2 is 5.

9. Since 19 has only itself and 1 as factors, it is a prime number.

10. Since 31 has only itself and 1 as factors, it is a prime number.

11.
$$\begin{aligned}30&=2\cdot 15\\&=2\cdot 3\cdot 5\end{aligned}$$

Since 30 has factors other than itself and 1, it is a composite number.

12.
$$\begin{aligned}50&=2\cdot 25\\&=2\cdot 5\cdot 5,\end{aligned}$$

so 50 is a composite number.

13.
$$\begin{aligned}64&=2\cdot 32\\&=2\cdot 2\cdot 16\\&=2\cdot 2\cdot 2\cdot 8\\&=2\cdot 2\cdot 2\cdot 2\cdot 4\\&=2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\end{aligned}$$

Since 64 has factors other than itself and 1, it is a composite number.

14.
$$\begin{aligned}81&=3\cdot 27\\&=3\cdot 3\cdot 9\\&=3\cdot 3\cdot 3\cdot 3\end{aligned}$$

Since 81 has factors other than itself and 1, it is a composite number.

15. As stated in the text, the number 1 is neither prime nor composite, by agreement.

16. The number 0 is not a natural number, so it is neither prime nor composite.

17. $57 = 3 \cdot 19$, so 57 is a composite number.

18. $51 = 3 \cdot 17$, so 51 is a composite number.

19. Since 79 has only itself and 1 as factors, it is a prime number.

20. Since 83 has only itself and 1 as factors, it is a prime number.

21. $$\begin{aligned}124 &= 2 \cdot 62\\ &= 2 \cdot 2 \cdot 31,\end{aligned}$$

so 124 is a composite number.

22. $$\begin{aligned}138 &= 2 \cdot 69\\ &= 2 \cdot 3 \cdot 23,\end{aligned}$$

so 138 is a composite number.

23. $$\begin{aligned}500 &= 2 \cdot 250\\ &= 2 \cdot 2 \cdot 125\\ &= 2 \cdot 2 \cdot 5 \cdot 25\\ &= 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5,\end{aligned}$$

so 500 is a composite number.

24. $$\begin{aligned}700 &= 2 \cdot 350\\ &= 2 \cdot 2 \cdot 175\\ &= 2 \cdot 2 \cdot 5 \cdot 35\\ &= 2 \cdot 2 \cdot 5 \cdot 5 \cdot 7,\end{aligned}$$

so 700 is a composite number.

25. $$\begin{aligned}3458 &= 2 \cdot 1729\\ &= 2 \cdot 7 \cdot 247\\ &= 2 \cdot 7 \cdot 13 \cdot 19\end{aligned}$$

Since 3458 has factors other than itself and 1, it is a composite number.

26. $$\begin{aligned}1025 &= 5 \cdot 205\\ &= 5 \cdot 5 \cdot 41\end{aligned}$$

Since 1025 has factors other than itself and 1, it is a composite number.

27. $\frac{8}{16} = \frac{1 \cdot 8}{2 \cdot 8} = \frac{1}{2}$

28. $\frac{4}{12} = \frac{1 \cdot 4}{3 \cdot 4} = \frac{1}{3}$

29. $\frac{15}{18} = \frac{3 \cdot 5}{3 \cdot 6} = \frac{5}{6}$

30. $\frac{16}{20} = \frac{4 \cdot 4}{5 \cdot 4} = \frac{4}{5}$

31. $\frac{64}{100} = \frac{4 \cdot 16}{4 \cdot 25} = \frac{16}{25}$

32. $\frac{55}{200} = \frac{5 \cdot 11}{5 \cdot 40} = \frac{11}{40}$

33. $\frac{18}{90} = \frac{1 \cdot 18}{5 \cdot 18} = \frac{1}{5}$

34. $\frac{16}{64} = \frac{1 \cdot 16}{4 \cdot 16} = \frac{1}{4}$

35. $\frac{144}{120} = \frac{6 \cdot 24}{5 \cdot 24} = \frac{6}{5}$

36. $\frac{132}{77} = \frac{12 \cdot 11}{7 \cdot 11} = \frac{12}{7}$

37. $\frac{16}{24} = \frac{2 \cdot 8}{3 \cdot 8} = \frac{2}{3}$

Therefore, **C** is correct.

38. **A.** $\frac{15}{27} = \frac{3 \cdot 5}{3 \cdot 9} = \frac{5}{9}$

B. $\frac{30}{54} = \frac{6 \cdot 5}{6 \cdot 9} = \frac{5}{9}$

C. $\frac{40}{74} = \frac{2 \cdot 20}{2 \cdot 37} = \frac{20}{37}$

D. $\frac{55}{99} = \frac{11 \cdot 5}{11 \cdot 9} = \frac{5}{9}$

Therefore, **C** is correct.

39. $\frac{4}{5} \cdot \frac{6}{7} = \frac{4 \cdot 6}{5 \cdot 7} = \frac{24}{35}$

40. $\frac{5}{9} \cdot \frac{2}{7} = \frac{5 \cdot 2}{9 \cdot 7} = \frac{10}{63}$

41. $\frac{2}{3} \cdot \frac{15}{16} = \frac{2 \cdot 15}{3 \cdot 16} = \frac{2 \cdot 3 \cdot 5}{3 \cdot 2 \cdot 8} = \frac{5}{8}$

42. $\frac{3}{5} \cdot \frac{20}{21} = \frac{3 \cdot 20}{5 \cdot 21} = \frac{3 \cdot 5 \cdot 4}{5 \cdot 3 \cdot 7} = \frac{4}{7}$

43. $\frac{1}{10} \cdot \frac{12}{5} = \frac{1 \cdot 12}{10 \cdot 5} = \frac{1 \cdot 2 \cdot 6}{2 \cdot 5 \cdot 5} = \frac{6}{25}$

44. $\frac{1}{8} \cdot \frac{10}{7} = \frac{1 \cdot 10}{8 \cdot 7} = \frac{1 \cdot 2 \cdot 5}{2 \cdot 4 \cdot 7} = \frac{5}{28}$

45. $$\begin{aligned}\frac{15}{4} \cdot \frac{8}{25} &= \frac{15 \cdot 8}{4 \cdot 25}\\ &= \frac{3 \cdot 5 \cdot 4 \cdot 2}{4 \cdot 5 \cdot 5}\\ &= \frac{3 \cdot 2}{5}\\ &= \frac{6}{5}, \text{ or } 1\frac{1}{5}\end{aligned}$$

46. $$\begin{aligned}\frac{21}{8} \cdot \frac{4}{7} &= \frac{21 \cdot 4}{8 \cdot 7}\\ &= \frac{3 \cdot 7 \cdot 4}{4 \cdot 2 \cdot 7}\\ &= \frac{3}{2}, \text{ or } 1\frac{1}{2}\end{aligned}$$

47. $21 \cdot \frac{3}{7} = \frac{21 \cdot 3}{1 \cdot 7}$

$= \frac{3 \cdot 7 \cdot 3}{1 \cdot 7}$

$= \frac{3 \cdot 3}{1} = 9$

48. $36 \cdot \frac{4}{9} = \frac{36 \cdot 4}{1 \cdot 9}$

$= \frac{4 \cdot 9 \cdot 4}{1 \cdot 9}$

$= \frac{4 \cdot 4}{1} = 16$

49. $3\frac{1}{4} \cdot 1\frac{2}{3}$

Change both mixed numbers to improper fractions.

$3\frac{1}{4} = 3 + \frac{1}{4} = \frac{12}{4} + \frac{1}{4} = \frac{13}{4}$

$1\frac{2}{3} = 1 + \frac{2}{3} = \frac{3}{3} + \frac{2}{3} = \frac{5}{3}$

$3\frac{1}{4} \cdot 1\frac{2}{3} = \frac{13}{4} \cdot \frac{5}{3}$

$= \frac{13 \cdot 5}{4 \cdot 3}$

$= \frac{65}{12}$, or $5\frac{5}{12}$

50. $2\frac{2}{3} \cdot 1\frac{3}{5}$

Change both mixed numbers to improper fractions.

$2\frac{2}{3} = 2 + \frac{2}{3} = \frac{6}{3} + \frac{2}{3} = \frac{8}{3}$

$1\frac{3}{5} = 1 + \frac{3}{5} = \frac{5}{5} + \frac{3}{5} = \frac{8}{5}$

$2\frac{2}{3} \cdot 1\frac{3}{5} = \frac{8}{3} \cdot \frac{8}{5}$

$= \frac{8 \cdot 8}{3 \cdot 5}$

$= \frac{64}{15}$, or $4\frac{4}{15}$

51. $2\frac{3}{8} \cdot 3\frac{1}{5}$

Change both mixed numbers to improper fractions.

$2\frac{3}{8} = 2 + \frac{3}{8} = \frac{16}{8} + \frac{3}{8} = \frac{19}{8}$

$3\frac{1}{5} = 3 + \frac{1}{5} = \frac{15}{5} + \frac{1}{5} = \frac{16}{5}$

$2\frac{3}{8} \cdot 3\frac{1}{5} = \frac{19}{8} \cdot \frac{16}{5}$

$= \frac{19 \cdot 16}{8 \cdot 5}$

$= \frac{19 \cdot 2 \cdot 8}{8 \cdot 5}$

$= \frac{38}{5}$, or $7\frac{3}{5}$

52. $3\frac{3}{5} \cdot 7\frac{1}{6} = \frac{18}{5} \cdot \frac{43}{6}$

$= \frac{18 \cdot 43}{5 \cdot 6}$

$= \frac{3 \cdot 6 \cdot 43}{5 \cdot 6}$

$= \frac{3 \cdot 43}{5}$

$= \frac{129}{5}$, or $25\frac{4}{5}$

53. $\frac{5}{4} \div \frac{3}{8} = \frac{5}{4} \cdot \frac{8}{3}$ *Multiply by the reciprocal of the second fraction.*

$= \frac{5 \cdot 8}{4 \cdot 3}$

$= \frac{5 \cdot 4 \cdot 2}{4 \cdot 3}$

$= \frac{5 \cdot 2}{3}$

$= \frac{10}{3}$, or $3\frac{1}{3}$

54. $\frac{7}{5} \div \frac{3}{10} = \frac{7}{5} \cdot \frac{10}{3}$ *Multiply by the reciprocal of the second fraction.*

$= \frac{7 \cdot 10}{5 \cdot 3}$

$= \frac{7 \cdot 2 \cdot 5}{5 \cdot 3}$

$= \frac{14}{3}$, or $4\frac{2}{3}$

55. $\frac{32}{5} \div \frac{8}{15} = \frac{32}{5} \cdot \frac{15}{8}$ *Multiply by the reciprocal of the second fraction.*

$= \frac{32 \cdot 15}{5 \cdot 8}$

$= \frac{8 \cdot 4 \cdot 3 \cdot 5}{1 \cdot 5 \cdot 8}$

$= \frac{4 \cdot 3}{1} = 12$

56. $\frac{24}{7} \div \frac{6}{21} = \frac{24}{7} \cdot \frac{21}{6}$ *Multiply by the reciprocal of the second fraction.*

$= \frac{24 \cdot 21}{7 \cdot 6}$

$= \frac{4 \cdot 6 \cdot 3 \cdot 7}{1 \cdot 7 \cdot 6}$

$= \frac{4 \cdot 3}{1} = 12$

57. $\frac{3}{4} \div 12 = \frac{3}{4} \cdot \frac{1}{12}$ *Multiply by the reciprocal of 12.*

$= \frac{3 \cdot 1}{4 \cdot 12}$

$= \frac{3 \cdot 1}{4 \cdot 3 \cdot 4}$

$= \frac{1}{4 \cdot 4} = \frac{1}{16}$

58. $\frac{2}{5} \div 30 = \frac{2}{5} \cdot \frac{1}{30}$ *Multiply by the reciprocal of 30.*

$= \frac{2 \cdot 1}{5 \cdot 30}$

$= \frac{2 \cdot 1}{5 \cdot 2 \cdot 15}$

$= \frac{1}{5 \cdot 15} = \frac{1}{75}$

59. $6 \div \frac{3}{5} = \frac{6}{1} \cdot \frac{5}{3}$ *Multiply by the reciprocal of the second fraction.*

$= \frac{6 \cdot 5}{1 \cdot 3}$

$= \frac{2 \cdot 3 \cdot 5}{1 \cdot 3}$

$= \frac{2 \cdot 5}{1} = 10$

60. $8 \div \frac{4}{9} = \frac{8}{1} \cdot \frac{9}{4}$ *Multiply by the reciprocal of the second fraction.*

$= \frac{8 \cdot 9}{1 \cdot 4}$

$= \frac{2 \cdot 4 \cdot 9}{1 \cdot 4}$

$= \frac{2 \cdot 9}{1} = 18$

61. $6\frac{3}{4} \div \frac{3}{8}$

Change the first number to an improper fraction.

$6\frac{3}{4} = 6 + \frac{3}{4} = \frac{24}{4} + \frac{3}{4} = \frac{27}{4}$

$6\frac{3}{4} \div \frac{3}{8} = \frac{27}{4} \cdot \frac{8}{3}$ *Multiply by the reciprocal of the second fraction.*

$= \frac{27 \cdot 8}{4 \cdot 3}$

$= \frac{3 \cdot 9 \cdot 2 \cdot 4}{4 \cdot 3}$

$= \frac{9 \cdot 2}{1} = 18$

62. $5\frac{3}{5} \div \frac{7}{10} = \frac{28}{5} \cdot \frac{10}{7}$ *Multiply by the reciprocal of the second fraction.*

$= \frac{28 \cdot 10}{5 \cdot 7}$

$= \frac{4 \cdot 7 \cdot 2 \cdot 5}{5 \cdot 7}$

$= \frac{4 \cdot 2}{1} = 8$

63. $2\frac{1}{2} \div 1\frac{5}{7}$

Change both mixed numbers to improper fractions.

$2\frac{1}{2} = 2 + \frac{1}{2} = \frac{4}{2} + \frac{1}{2} = \frac{5}{2}$

$1\frac{5}{7} = 1 + \frac{5}{7} = \frac{7}{7} + \frac{5}{7} = \frac{12}{7}$

$2\frac{1}{2} \div 1\frac{5}{7} = \frac{5}{2} \div \frac{12}{7}$

$= \frac{5}{2} \cdot \frac{7}{12}$ *Multiply by the reciprocal of the second fraction.*

$= \frac{5 \cdot 7}{2 \cdot 12}$

$= \frac{35}{24}$, or $1\frac{11}{24}$

64. $2\frac{2}{9} \div 1\frac{2}{5}$

Change both mixed numbers to improper fractions.

$$2\frac{2}{9} = 2 + \frac{2}{9} = \frac{18}{9} + \frac{2}{9} = \frac{20}{9}$$
$$1\frac{2}{5} = 1 + \frac{2}{5} = \frac{5}{5} + \frac{2}{5} = \frac{7}{5}$$
$$2\frac{2}{9} \div 1\frac{2}{5} = \frac{20}{9} \div \frac{7}{5}$$
$$= \frac{20}{9} \cdot \frac{5}{7}$$ *Multiply by the reciprocal of the second fraction.*
$$= \frac{20 \cdot 5}{9 \cdot 7}$$
$$= \frac{100}{63}, \text{ or } 1\frac{37}{63}$$

65. $2\frac{5}{8} \div 1\frac{15}{32}$

Change both mixed numbers to improper fractions.

$$2\frac{5}{8} = 2 + \frac{5}{8} = \frac{16}{8} + \frac{5}{8} = \frac{21}{8}$$
$$1\frac{15}{32} = 1 + \frac{15}{32} = \frac{32}{32} + \frac{15}{32} = \frac{47}{32}$$
$$2\frac{5}{8} \div 1\frac{15}{32} = \frac{21}{8} \div \frac{47}{32}$$
$$= \frac{21}{8} \cdot \frac{32}{47}$$
$$= \frac{21 \cdot 32}{8 \cdot 47}$$
$$= \frac{21 \cdot 8 \cdot 4}{8 \cdot 47}$$
$$= \frac{21 \cdot 4}{47}$$
$$= \frac{84}{47}, \text{ or } 1\frac{37}{47}$$

66. $2\frac{3}{10} \div 1\frac{4}{5} = \frac{23}{10} \div \frac{9}{5}$
$$= \frac{23}{10} \cdot \frac{5}{9}$$
$$= \frac{23 \cdot 5}{2 \cdot 5 \cdot 9}$$
$$= \frac{23}{18}, \text{ or } 1\frac{5}{8}$$

67. A common denominator for $\frac{p}{q}$ and $\frac{r}{s}$ must be a multiple of both denominators, q and s. Such a number is $q \cdot s$. Therefore, **A** is correct.

68. We need to multiply 8 by 3 to get 24 in the denominator, so we must multiply 5 by 3 as well.

$$\frac{5}{8} = \frac{5 \cdot 3}{8 \cdot 3} = \frac{15}{24}$$

69. $\frac{7}{15} + \frac{4}{15} = \frac{7+4}{15} = \frac{11}{15}$

70. $\frac{2}{9} + \frac{5}{9} = \frac{2+5}{9} = \frac{7}{9}$

71. $\frac{7}{12} + \frac{1}{12} = \frac{7+1}{12}$
$$= \frac{8}{12}$$
$$= \frac{2 \cdot 4}{3 \cdot 4} = \frac{2}{3}$$

72. $\frac{3}{16} + \frac{5}{16} = \frac{3+5}{16} = \frac{8}{16} = \frac{1}{2}$

73. $\frac{5}{9} + \frac{1}{3}$

Since $9 = 3 \cdot 3$, and 3 is prime, the LCD (least common denominator) is $3 \cdot 3 = 9$.

$$\frac{1}{3} = \frac{1}{3} \cdot \frac{3}{3} = \frac{3}{9}$$

Now add the two fractions with the same denominator.

$$\frac{5}{9} + \frac{1}{3} = \frac{5}{9} + \frac{3}{9} = \frac{8}{9}$$

74. $\frac{4}{15} + \frac{1}{5}$

To add $\frac{4}{15}$ and $\frac{1}{5}$, first find the LCD. Since $15 = 3 \cdot 5$ and 5 is prime, the LCD is 15.

$$\frac{4}{15} + \frac{1}{5} = \frac{4}{15} + \frac{1 \cdot 3}{5 \cdot 3}$$
$$= \frac{4}{15} + \frac{3}{15}$$
$$= \frac{4+3}{15} = \frac{7}{15}$$

75. $\frac{3}{8} + \frac{5}{6}$

Since $8 = 2 \cdot 2 \cdot 2$ and $6 = 2 \cdot 3$, the LCD is $2 \cdot 2 \cdot 2 \cdot 3 = 24$.

$$\frac{3}{8} = \frac{3}{8} \cdot \frac{3}{3} = \frac{9}{24} \text{ and } \frac{5}{6} \cdot \frac{4}{4} = \frac{20}{24}$$

Now add fractions with the same denominator.

$$\frac{3}{8} + \frac{5}{6} = \frac{9}{24} + \frac{20}{24} = \frac{29}{24}, \text{ or } 1\frac{5}{24}$$

76. $\frac{5}{6}+\frac{2}{9}$

Since $6=2\cdot 3$ and $9=3\cdot 3$, the LCD is $2\cdot 3\cdot 3=18$.

$$\frac{5}{6}=\frac{5}{6}\cdot\frac{3}{3}=\frac{15}{18} \text{ and } \frac{2}{9}\cdot\frac{2}{2}=\frac{4}{18}$$

Now add fractions with the same denominator.

$$\frac{5}{6}+\frac{2}{9}=\frac{15}{18}+\frac{4}{18}=\frac{19}{18}, \quad \text{or} \quad 1\frac{1}{18}$$

77. $3\frac{1}{8}+2\frac{1}{4}$

$$3\frac{1}{8}=3+\frac{1}{8}=\frac{24}{8}+\frac{1}{8}=\frac{25}{8}$$

$$2\frac{1}{4}=2+\frac{1}{4}=\frac{8}{4}+\frac{1}{4}=\frac{9}{4}$$

$$3\frac{1}{8}+2\frac{1}{4}=\frac{25}{8}+\frac{9}{4}$$

Since $8=2\cdot 2\cdot 2$ and $4=2\cdot 2$, the LCD is $2\cdot 2\cdot 2$ or 8.

$$\begin{aligned}3\frac{1}{8}+2\frac{1}{4}&=\frac{25}{8}+\frac{9\cdot 2}{4\cdot 2}\\&=\frac{25}{8}+\frac{18}{8}\\&=\frac{43}{8}, \quad \text{or} \quad 5\frac{3}{8}\end{aligned}$$

78. $4\frac{2}{3}+2\frac{1}{6}$

$$4\frac{2}{3}=4+\frac{2}{3}=\frac{12}{3}+\frac{2}{3}=\frac{14}{3}$$

$$2\frac{1}{6}=2+\frac{1}{6}=\frac{12}{6}+\frac{1}{6}=\frac{13}{6}$$

Since $6=2\cdot 3$, the LCD is 6.

$$\begin{aligned}4\frac{2}{3}+2\frac{1}{6}&=\frac{14\cdot 2}{3\cdot 2}+\frac{13}{6}\\&=\frac{28}{6}+\frac{13}{6}\\&=\frac{41}{6}, \quad \text{or} \quad 6\frac{5}{6}\end{aligned}$$

79. $3\frac{1}{4}+1\frac{4}{5}$

$$3\frac{1}{4}=3+\frac{1}{4}=\frac{12}{4}+\frac{1}{4}=\frac{13}{4}$$

$$1\frac{4}{5}=1+\frac{4}{5}=\frac{5}{5}+\frac{4}{5}=\frac{9}{5}$$

Since $4=2\cdot 2$, and 5 is prime, the LCD is $2\cdot 2\cdot 5=20$.

$$\begin{aligned}3\frac{1}{4}+1\frac{4}{5}&=\frac{13\cdot 5}{4\cdot 5}+\frac{9\cdot 4}{5\cdot 4}\\&=\frac{65}{20}+\frac{36}{20}\\&=\frac{101}{20}, \quad \text{or} \quad 5\frac{1}{20}\end{aligned}$$

80. $5\frac{3}{4}+1\frac{1}{3}$

To add $5\frac{3}{4}$ and $1\frac{1}{3}$, first change to improper fractions; then find the LCD, which is 12.

$$\begin{aligned}5\frac{3}{4}+1\frac{1}{3}&=\frac{23}{4}+\frac{4}{3}\\&=\frac{23\cdot 3}{4\cdot 3}+\frac{4\cdot 4}{3\cdot 4}\\&=\frac{69}{12}+\frac{16}{12}\\&=\frac{85}{12}, \quad \text{or} \quad 7\frac{1}{12}\end{aligned}$$

81. $\frac{7}{9}-\frac{2}{9}=\frac{7-2}{9}=\frac{5}{9}$

82. $\frac{8}{11}-\frac{3}{11}=\frac{8-3}{11}=\frac{5}{11}$

83. $$\begin{aligned}\frac{13}{15}-\frac{3}{15}&=\frac{13-3}{15}\\&=\frac{10}{15}\\&=\frac{2\cdot 5}{3\cdot 5}=\frac{2}{3}\end{aligned}$$

84. $$\begin{aligned}\frac{11}{12}-\frac{3}{12}&=\frac{11-3}{12}\\&=\frac{8}{12}\\&=\frac{2\cdot 4}{3\cdot 4}=\frac{2}{3}\end{aligned}$$

85. $\frac{7}{12}-\frac{1}{3}$

Since $12=4\cdot 3$ (12 is a multiple of 3), the LCD is 12.

$$\frac{1}{3}\cdot\frac{4}{4}=\frac{4}{12}$$

Now subtract fractions with the same denominator.

$$\frac{7}{12}-\frac{1}{3}=\frac{7}{12}-\frac{4}{12}=\frac{3}{12}=\frac{1\cdot 3}{4\cdot 3}=\frac{1}{4}$$

86. $\frac{5}{6}-\frac{1}{2}$

Since $6=3\cdot 2$ (6 is a multiple of 2), the LCD is 6.

$$\frac{1}{2}\cdot\frac{3}{3}=\frac{3}{6}$$

Now subtract fractions with the same denominator.

$$\frac{5}{6}-\frac{1}{2}=\frac{5}{6}-\frac{3}{6}=\frac{2}{6}=\frac{1\cdot 2}{3\cdot 2}=\frac{1}{3}$$

87. $\frac{7}{12}-\frac{1}{9}$

Since $12 = 2\cdot 2\cdot 3$ and $9 = 3\cdot 3$, the LCD is $2\cdot 2\cdot 3\cdot 3 = 36$.

$$\frac{7}{12}=\frac{7}{12}\cdot\frac{3}{3}=\frac{21}{36} \text{ and } \frac{1}{9}\cdot\frac{4}{4}=\frac{4}{36}$$

Now subtract fractions with the same denominator.

$$\frac{7}{12}-\frac{1}{9}=\frac{21}{36}-\frac{4}{36}=\frac{17}{36}$$

88. $\frac{11}{16}-\frac{1}{12}=\frac{11\cdot 3}{16\cdot 3}-\frac{1\cdot 4}{12\cdot 4}$ *The LCD of 12 and 16 is 48.*

$$=\frac{33}{48}-\frac{4}{48}$$
$$=\frac{29}{48}$$

89. $4\frac{3}{4}-1\frac{2}{5}$

$$4\frac{3}{4}=4+\frac{3}{4}=\frac{16}{4}+\frac{3}{4}=\frac{19}{4}$$
$$1\frac{2}{5}=1+\frac{2}{5}=\frac{5}{5}+\frac{2}{5}=\frac{7}{5}$$

Since $4 = 2\cdot 2$, and 5 is prime, the LCD is $2\cdot 2\cdot 5 = 20$.

$$4\frac{3}{4}-1\frac{2}{5}=\frac{19\cdot 5}{4\cdot 5}-\frac{7\cdot 4}{5\cdot 4}$$
$$=\frac{95}{20}-\frac{28}{20}$$
$$=\frac{67}{20}, \text{ or } 3\frac{7}{20}$$

90. $3\frac{4}{5}-1\frac{4}{9}$

Change both numbers to improper fractions; then add, using 45 as the common denominator.

$$3\frac{4}{5}-1\frac{4}{9}=\frac{19}{5}-\frac{13}{9}$$
$$=\frac{19\cdot 9}{5\cdot 9}-\frac{13\cdot 5}{9\cdot 5}$$
$$=\frac{171}{45}-\frac{65}{45}$$
$$=\frac{106}{45}, \text{ or } 2\frac{16}{45}$$

91. $6\frac{1}{4}-5\frac{1}{3}$

$$6\frac{1}{4}=6+\frac{1}{4}=\frac{24}{4}+\frac{1}{4}=\frac{25}{4}$$
$$5\frac{1}{3}=5+\frac{1}{3}=\frac{15}{3}+\frac{1}{3}=\frac{16}{3}$$

Since $4 = 2\cdot 2$, and 3 is prime, the LCD is $2\cdot 2\cdot 3 = 12$.

$$6\frac{1}{4}-5\frac{1}{3}=\frac{25}{4}-\frac{16}{3}$$
$$=\frac{25\cdot 3}{4\cdot 3}-\frac{16\cdot 4}{3\cdot 4}$$
$$=\frac{75}{12}-\frac{64}{12}$$
$$=\frac{11}{12}$$

92. $5\frac{1}{3}-4\frac{1}{2}$

$$5\frac{1}{3}=5+\frac{1}{3}=\frac{15}{3}+\frac{1}{3}=\frac{16}{3}$$
$$4\frac{1}{2}=4+\frac{1}{2}=\frac{8}{2}+\frac{1}{2}=\frac{9}{2}$$

2 and 3 are prime, so the LCD is $2\cdot 3 = 6$.

$$5\frac{1}{3}-4\frac{1}{2}=\frac{16\cdot 2}{3\cdot 2}-\frac{9\cdot 3}{2\cdot 3}$$
$$=\frac{32}{6}-\frac{27}{6}$$
$$=\frac{5}{6}$$

93. Multiply the number of cups of water per serving by the number of servings.

$$\frac{3}{4}\cdot 8=\frac{3}{4}\cdot\frac{8}{1}$$
$$=\frac{3\cdot 8}{4\cdot 1}$$
$$=\frac{3\cdot 2\cdot 4}{4\cdot 1}$$
$$=\frac{3\cdot 2}{1}=6 \text{ cups}$$

For 8 microwave servings, 6 cups of water will be needed.

94. Four stove top servings require $\frac{1}{4}$ tsp, or $\frac{2}{8}$ tsp, of salt. Six stove top servings require $\frac{1}{2}$ tsp, or $\frac{4}{8}$ tsp, of salt. Five is halfway between 4 and 6, and $\frac{3}{8}$ is halfway between $\frac{2}{8}$ and $\frac{4}{8}$. Therefore, 5 stove top servings would require $\frac{3}{8}$ tsp of salt.

95. The difference in length is found by subtracting.

$$\begin{aligned}3\frac{1}{4}-2\frac{1}{8} &= \frac{13}{4}-\frac{17}{8}\\ &= \frac{13\cdot 2}{4\cdot 2}-\frac{17}{8} \quad \textit{LCD = 8}\\ &= \frac{26}{8}-\frac{17}{8}\\ &= \frac{9}{8}, \text{ or } 1\frac{1}{8}\end{aligned}$$

The difference is $1\frac{1}{8}$ inches.

96. The difference in length is found by subtracting.

$$\begin{aligned}4-2\frac{1}{8} &= \frac{4}{1}-\frac{17}{8}\\ &= \frac{4\cdot 8}{1\cdot 8}-\frac{17}{8} \quad \textit{LCD = 8}\\ &= \frac{32}{8}-\frac{17}{8}\\ &= \frac{15}{8}, \text{ or } 1\frac{7}{8}\end{aligned}$$

The difference is $1\frac{7}{8}$ inches.

97. The difference between the two measures is found by subtracting, using 16 as the LCD.

$$\begin{aligned}\frac{3}{4}-\frac{3}{16} &= \frac{3\cdot 4}{4\cdot 4}-\frac{3}{16}\\ &= \frac{12}{16}-\frac{3}{16}\\ &= \frac{12-3}{16}=\frac{9}{16}\end{aligned}$$

The difference is $\frac{9}{16}$ inch.

98. The difference between the two measures is found by subtracting, using 16 as a common denominator.

$$\begin{aligned}\frac{9}{16}-\frac{3}{8} &= \frac{9}{16}-\frac{3\cdot 2}{8\cdot 2}\\ &= \frac{9}{16}-\frac{6}{16}\\ &= \frac{9-6}{16}=\frac{3}{16}\end{aligned}$$

The difference is $\frac{3}{16}$ inch.

99. The perimeter is the sum of the measures of the 5 sides.

$$\begin{aligned}&196+98\frac{3}{4}+146\frac{1}{2}+100\frac{7}{8}+76\frac{5}{8}\\ &= 196+98\frac{6}{8}+146\frac{4}{8}+100\frac{7}{8}+76\frac{5}{8}\\ &= 196+98+146+100+76+\frac{6+4+7+5}{8}\\ &= 616+\frac{22}{8} \quad \left(\frac{22}{8}=2\frac{6}{8}=2\frac{3}{4}\right)\\ &= 618\frac{3}{4} \text{ feet}\end{aligned}$$

The perimeter is $618\frac{3}{4}$ feet.

100. To find the perimeter of a triangle, add the lengths of the three sides.

$$\begin{aligned}5\frac{1}{4}+7\frac{1}{2}+10\frac{1}{8} &= 5\frac{2}{8}+7\frac{4}{8}+10\frac{1}{8}\\ &= 22\frac{7}{8}\end{aligned}$$

The perimeter of the triangle is $22\frac{7}{8}$ feet.

101. Divide the total board length by 3.

$$\begin{aligned}15\frac{5}{8}\div 3 &= \frac{125}{8}\div\frac{3}{1}\\ &= \frac{125}{8}\cdot\frac{1}{3}\\ &= \frac{125\cdot 1}{8\cdot 3}\\ &= \frac{125}{24}, \text{ or } 5\frac{5}{24}\end{aligned}$$

The length of each of the three pieces must be $5\frac{5}{24}$ inches.

102. Divide the total amount of tomato sauce by the number of servings.

$$2\frac{1}{3}\div 7=\frac{7}{3}\div\frac{7}{1}=\frac{7}{3}\cdot\frac{1}{7}=\frac{7\cdot 1}{3\cdot 7}=\frac{1}{3}$$

For 1 serving of barbecue sauce, $\frac{1}{3}$ cup of tomato sauce is needed.

103. To find the number of cakes the caterer can make, divide $15\frac{1}{2}$ by $1\frac{3}{4}$.

$$\begin{aligned}15\frac{1}{2}\div 1\frac{3}{4} &= \frac{31}{2}\div\frac{7}{4}\\ &= \frac{31}{2}\cdot\frac{4}{7}\\ &= \frac{31\cdot 2\cdot 2}{2\cdot 7}\\ &= \frac{62}{7}, \text{ or } 8\frac{6}{7}\end{aligned}$$

There is not quite enough sugar for 9 cakes. The caterer can make 8 cakes with some sugar left over.

104. Divide the total amount of fabric by the amount of fabric needed to cover one chair.

$$\begin{aligned}23\frac{2}{3}\div 2\frac{1}{4} &= \frac{71}{3}\div\frac{9}{4}\\ &= \frac{71}{3}\cdot\frac{4}{9}\\ &= \frac{71\cdot 4}{3\cdot 9}\\ &= \frac{284}{27}, \text{ or } 10\frac{14}{27}\end{aligned}$$

Kyla can cover 10 chairs (there will be some fabric left over).

105. Multiply the amount of fabric it takes to make one costume by the number of costumes.

$$2\frac{3}{8} \cdot 7 = \frac{19}{8} \cdot \frac{7}{1} = \frac{19 \cdot 7}{8 \cdot 1} = \frac{133}{8}, \text{ or } 16\frac{5}{8} \text{ yd}$$

For 7 costumes, $16\frac{5}{8}$ yards of fabric would be needed.

106. Multiply the amount of sugar for one batch times the number of batches.

$$2\frac{2}{3} \cdot 4 = \frac{8}{3} \cdot \frac{4}{1} = \frac{8 \cdot 4}{3 \cdot 1} = \frac{32}{3}, \text{ or } 10\frac{2}{3}$$

$10\frac{2}{3}$ cups of sugar are required to make four batches of cookies.

107. Subtract the heights to find the difference.

$$10\frac{1}{2} - 7\frac{1}{8} = \frac{21}{2} - \frac{57}{8} = \frac{21 \cdot 4}{2 \cdot 4} - \frac{57}{8} \quad \textit{LCD = 8}$$
$$= \frac{84}{8} - \frac{57}{8} = \frac{27}{8}, \text{ or } 3\frac{3}{8}$$

The difference in heights is $3\frac{3}{8}$ inches.

108. Subtract $\frac{3}{8}$ from $\frac{11}{16}$ using 16 as the LCD.

$$\frac{11}{16} - \frac{3}{8} = \frac{11}{16} - \frac{3 \cdot 2}{8 \cdot 2} = \frac{11}{16} - \frac{6}{16} = \frac{5}{16}$$

Thus, $\frac{3}{8}$ inch is $\frac{5}{16}$ inch smaller than $\frac{11}{16}$ inch.

109. The sum of the fractions representing the U.S. foreign-born population from Latin America, Asia, or Europe is

$$\frac{27}{50} + \frac{27}{100} + \frac{7}{50} = \frac{27 \cdot 2}{50 \cdot 2} + \frac{27}{100} + \frac{7 \cdot 2}{50 \cdot 2} = \frac{54 + 27 + 14}{100} = \frac{95}{100}.$$

So the fraction representing the U.S. foreign-born population from other regions is

$$1 - \frac{95}{100} = \frac{100}{100} - \frac{95}{100} = \frac{5}{100} = \frac{1}{20}.$$

110. The sum of the fractions representing the U.S. foreign-born population from Latin America or Asia is

$$\frac{27}{50} + \frac{27}{100} = \frac{27 \cdot 2}{50 \cdot 2} + \frac{27}{100} = \frac{54 + 27}{100} = \frac{81}{100}.$$

111. Multiply the fraction representing the U.S. foreign-born population from Europe, $\frac{7}{50}$, by the total number of foreign-born people in the U.S., approximately 38 million.

$$\frac{7}{50} \cdot 38 = \frac{7}{50} \cdot \frac{38}{1} = \frac{7 \cdot 2 \cdot 19}{2 \cdot 25} = \frac{133}{25}, \text{ or } 5\frac{8}{25}$$

There were approximately $5\frac{8}{25}$ million (or 5,320,000) foreign-born people in the U.S. in 2006 who were born in Europe.

112. **(a)** 12 is $\frac{1}{3}$ of 36, so Slade got a hit in exactly $\frac{1}{3}$ of her at-bats.

(b) 5 is a little less than $\frac{1}{2}$ of 11, so Goldstein got a hit in just less than $\frac{1}{2}$ of his at-bats.

(c) 1 is a little less than $\frac{1}{10}$ of 11, so Goldstein got a home run in just less than $\frac{1}{10}$ of his at-bats.

(d) 9 is a little less than $\frac{1}{4}$ of 40, so Heen got a hit in just less than $\frac{1}{4}$ of her at-bats.

(e) 8 is $\frac{1}{2}$ of 16, and 10 is $\frac{1}{2}$ of 20, so Koven and Wooding each got hits $\frac{1}{2}$ of the times they were at bat.

113. Observe that there are 24 dots in the entire figure, 6 dots in the triangle, 12 dots in the rectangle, and 2 dots in the overlapping region.

(a) $\frac{12}{24} = \frac{1}{2}$ of all the dots are in the rectangle.

(b) $\frac{6}{24} = \frac{1}{4}$ of all the dots are in the triangle.

(c) $\frac{2}{6} = \frac{1}{3}$ of the dots in the triangle are in the overlapping region.

(d) $\frac{2}{12} = \frac{1}{6}$ of the dots in the rectangle are in the overlapping region.

114. $\frac{14}{26}+\frac{98}{99}+\frac{100}{51}+\frac{90}{31}+\frac{13}{27}$

Estimate each fraction. $\frac{14}{26}$ is about $\frac{1}{2}$, $\frac{98}{99}$ is about 1, $\frac{100}{51}$ is about 2, $\frac{90}{31}$ is about 3, and $\frac{13}{27}$ is about $\frac{1}{2}$.

Therefore, the sum is approximately

$$\tfrac{1}{2}+1+2+3+\tfrac{1}{2}=7.$$

The correct choice is **B**.

1.2 Exponents, Order of Operations, and Inequality

1.2 Classroom Examples, Now Try Exercises

1. **(a)** $9^2=9\cdot 9=81$

(b) $\left(\frac{1}{2}\right)^4=\underbrace{\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}}=\frac{1}{16}$

$\frac{1}{2}$ is used as a factor 4 times.

N1. **(a)** $6^2=6\cdot 6=36$

(b) $\left(\frac{4}{5}\right)^3=\underbrace{\frac{4}{5}\cdot\frac{4}{5}\cdot\frac{4}{5}}=\frac{64}{125}$

$\frac{4}{5}$ is used as a factor 3 times.

2. **(a)** $10-6\div 2=10-3$ *Divide.*

$=7$ *Subtract.*

(b) $7\cdot 6-3(8+1)$

$=7\cdot 6-3(9)$ *Add inside parentheses.*

$=42-27$ *Multiply.*

$=15$ *Subtract.*

(c) $2+3^2-5\cdot 2$

$=2+9-5\cdot 2$ *Use the exponent.*

$=2+9-10$ *Multiply.*

$=11-10$ *Add.*

$=1$ *Subtract.*

N2. **(a)** $15-2\cdot 6=15-12$ *Multiply.*

$=3$ *Subtract.*

(b) $6(2+4)-7\cdot 5$

$=6(6)-7\cdot 5$ *Add inside parentheses.*

$=36-35$ *Multiply.*

$=1$ *Subtract.*

(c) $8\cdot 10\div 4-2^3+3\cdot 4^2$

$=8\cdot 10\div 4-8+3\cdot 16$ *Use exponents.*

$=80\div 4-8+48$ *Multiply.*

$=20-8+48$ *Divide.*

$=12+48$ *Subtract.*

$=60$ *Add.*

3. **(a)** $9[(4+8)-3]$

$=9[12-3]$ *Add inside parentheses.*

$=9(9)$ *Subtract inside parentheses.*

$=81$ *Multiply.*

(b) $\frac{2(7+8)+2}{3\cdot 5+1}=\frac{2(15)+2}{3\cdot 5+1}$ *Add inside parentheses.*

$=\frac{30+2}{15+1}$ *Multiply.*

$=\frac{32}{16}$ *Add.*

$=2$ *Divide.*

N3. **(a)** $7[(3^2-1)+4]$

$=7[(9-1)+4]$ *Use the exponent.*

$=7[8+4]$ *Subtract inside parens.*

$=7(12)$ *Add inside parentheses.*

$=84$ *Multiply.*

(b) $\frac{9(14-4)-2}{4+3\cdot 6}=\frac{9(10)-2}{4+3\cdot 6}$ *Subt. inside parentheses.*

$=\frac{90-2}{4+18}$ *Multiply.*

$=\frac{88}{22}$ *Subtract and add.*

$=4$ *Divide.*

4. **(a)** The statement $12>6$ is *true* since 12 is greater than 6. Note that the inequality symbol points to the lesser number.

(b) The statement $28\neq 4\cdot 7$ is *false* because 28 *is equal to* $4\cdot 7$.

(c) The statement $21\leq 21$ is *true* since $21=21$.

(d) Write the fractions with a common denominator. The statement $\frac{1}{3}<\frac{1}{4}$ is equivalent to the statement $\frac{4}{12}<\frac{3}{12}$. Since 4 is *greater* than 3, the original statement is *false.*

N4. **(a)** The statement $12\neq 10-2$ is *true* because 12 *is not equal to* 8.

(b) The statement $5>4\cdot 2$ is *false* because 5 *is less than* 8.

(c) The statement $7\leq 7$ is *true* since $7=7$.

(d) Write the fractions with a common denominator. The statement $\frac{5}{9}>\frac{7}{11}$ is equivalent to the statement $\frac{55}{99}>\frac{63}{99}$. Since 55 is *less* than 63, the original statement is *false.*

5. **(a)** "Nine is equal to eleven minus two" is written $9=11-2$.

(b) "Fourteen is greater than twelve" is written $14>12$.

(c) "Two is greater than or equal to two" is written $2\geq 2$.

N5. **(a)** "Ten is not equal to eight minus two" is written $10 \neq 8 - 2$.

(b) "Fifty is greater than fifteen" is written $50 > 15$.

(c) "Eleven is less than or equal to twenty" is written $11 \leq 20$.

6. $9 \leq 15$ may be written as $15 \geq 9$.

N6. $8 < 9$ may be written as $9 > 8$.

1.2 Section Exercises

1. False; 6^2 means that 6 is used as a factor 2 times, so $6^2 = 6 \cdot 6 = 36$.

2. False; $3^2 = 3 \cdot 3 = 9$.

3. False; 1 raised to *any* power is 1. Here, $1^3 = 1 \cdot 1 \cdot 1 = 1$.

4. False; a number raised to the first power is that number, so $3^1 = 3$.

5. False; $4 + 3(8 - 2) = 4 + 3 \cdot 6 = 4 + 18 = 22$. The common error leading to 42 is adding 4 to 3 and then multiplying by 6. One must follow the rules for order of operations.

6. False; $12 \div 2 \cdot 3 = 6 \cdot 3 = 18$. Multiplications and divisions are performed *in order from left to right*.

7. $3^2 = 3 \cdot 3 = 9$

8. $8^2 = 8 \cdot 8 = 64$

9. $7^2 = 7 \cdot 7 = 49$

10. $4^2 = 4 \cdot 4 = 16$

11. $12^2 = 12 \cdot 12 = 144$

12. $14^2 = 14 \cdot 14 = 196$

13. $4^3 = 4 \cdot 4 \cdot 4 = 64$

14. $5^3 = 5 \cdot 5 \cdot 5 = 125$

15. $10^3 = 10 \cdot 10 \cdot 10 = 1000$

16. $11^3 = 11 \cdot 11 \cdot 11 = 1331$

17. $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$

18. $6^4 = 6 \cdot 6 \cdot 6 \cdot 6 = 1296$

19. $4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1024$

20. $3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$

21. $\left(\frac{1}{6}\right)^2 = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

22. $\left(\frac{1}{3}\right)^2 = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

23. $\left(\frac{2}{3}\right)^4 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{16}{81}$

24. $\left(\frac{3}{4}\right)^3 = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{27}{64}$

25. $(0.4)^3 = (0.4)(0.4)(0.4) = 0.064$

26. $(0.5)^4 = (0.5)(0.5)(0.5)(0.5) = 0.0625$

27.
$$\begin{aligned} 64 \div 4 \cdot 2 &= (64 \div 4) \cdot 2 \\ &= 16 \cdot 2 \\ &= 32 \end{aligned}$$

28.
$$\begin{aligned} 250 \div 5 \cdot 2 &= (250 \div 5) \cdot 2 \\ &= 50 \cdot 2 \\ &= 100 \end{aligned}$$

29.
$$\begin{aligned} 13 + 9 \cdot 5 &= 13 + 45 && \textit{Multiply.} \\ &= 58 && \textit{Add.} \end{aligned}$$

30.
$$\begin{aligned} 11 + 7 \cdot 6 &= 11 + 42 && \textit{Multiply.} \\ &= 53 && \textit{Add.} \end{aligned}$$

31.
$$\begin{aligned} 25.2 - 12.6 \div 4.2 &= 25.2 - 3 && \textit{Divide.} \\ &= 22.2 && \textit{Subtract.} \end{aligned}$$

32.
$$\begin{aligned} 12.4 - 9.3 \div 3.1 &= 12.4 - 3 && \textit{Divide.} \\ &= 9.4 && \textit{Subtract.} \end{aligned}$$

33.
$$\begin{aligned} \frac{1}{4} \cdot \frac{2}{3} + \frac{2}{5} \cdot \frac{11}{3} &= \frac{1}{6} + \frac{22}{15} && \textit{Multiply.} \\ &= \frac{5}{30} + \frac{44}{30} && \textit{LCD = 30} \\ &= \frac{49}{30}, \text{ or } 1\frac{19}{30} && \textit{Add.} \end{aligned}$$

34.
$$\begin{aligned} \frac{9}{4} \cdot \frac{2}{3} + \frac{4}{5} \cdot \frac{5}{3} &= \frac{3}{2} + \frac{4}{3} && \textit{Multiply.} \\ &= \frac{9}{6} + \frac{8}{6} && \textit{LCD = 6} \\ &= \frac{17}{6}, \text{ or } 2\frac{5}{6} && \textit{Add.} \end{aligned}$$

35.
$$\begin{aligned} 9 \cdot 4 - 8 \cdot 3 &= 36 - 24 && \textit{Multiply.} \\ &= 12 && \textit{Subtract.} \end{aligned}$$

36.
$$\begin{aligned} 11 \cdot 4 + 10 \cdot 3 &= 44 + 30 && \textit{Multiply.} \\ &= 74 && \textit{Add.} \end{aligned}$$

37.
$$\begin{aligned} 20 - 4 \cdot 3 + 5 &= 20 - 12 + 5 && \textit{Multiply.} \\ &= 8 + 5 && \textit{Subtract.} \\ &= 13 && \textit{Add.} \end{aligned}$$

38.
$$\begin{aligned} 18 - 7 \cdot 2 + 6 &= 18 - 14 + 6 && \textit{Multiply.} \\ &= 4 + 6 && \textit{Subtract.} \\ &= 10 && \textit{Add.} \end{aligned}$$

39.
$$\begin{aligned} 10 + 40 \div 5 \cdot 2 &= 10 + 8 \cdot 2 && \textit{Divide.} \\ &= 10 + 16 && \textit{Multiply.} \\ &= 26 && \textit{Add.} \end{aligned}$$

40. $12 + 64 \div 8 - 4$
$= 12 + 8 - 4$ *Divide.*
$= 20 - 4$ *Add.*
$= 16$ *Subtract.*

41. $18 - 2(3 + 4) = 18 - 2(7)$ *Add inside parentheses.*
$= 18 - 14$ *Multiply.*
$= 4$ *Subtract.*

42. $30 - 3(4 + 2) = 30 - 3(6)$ *Add inside parentheses.*
$= 30 - 18$ *Multiply.*
$= 12$ *Subtract.*

43. $3(4 + 2) + 8 \cdot 3 = 3 \cdot 6 + 8 \cdot 3$ *Add.*
$= 18 + 24$ *Multiply.*
$= 42$ *Add.*

44. $9(1 + 7) + 2 \cdot 5 = 9 \cdot 8 + 2 \cdot 5$ *Add.*
$= 72 + 10$ *Multiply.*
$= 82$ *Add.*

45. $18 - 4^2 + 3$
$= 18 - 16 + 3$ *Use the exponent.*
$= 2 + 3$ *Subtract.*
$= 5$ *Add.*

46. $22 - 2^3 + 9$
$= 22 - 8 + 9$ *Use the exponent.*
$= 14 + 9$ *Subtract.*
$= 23$ *Add.*

47. $2 + 3[5 + 4(2)]$
$= 2 + 3[5 + 8]$ *Multiply.*
$= 2 + 3[13]$ *Add.*
$= 2 + 39$ *Multiply.*
$= 41$ *Add.*

48. $5 + 4[1 + 7(3)]$
$= 5 + 4[1 + 21]$ *Multiply.*
$= 5 + 4[22]$ *Add.*
$= 5 + 88$ *Multiply.*
$= 93$ *Add.*

49. $5[3 + 4(2^2)]$
$= 5[3 + 4(4)]$ *Use the exponent.*
$= 5(3 + 16)$ *Multiply.*
$= 5(19)$ *Add.*
$= 95$ *Multiply.*

50. $6[2 + 8(3^3)]$
$= 6[2 + 8 \cdot 27]$ *Use the exponent.*
$= 6(2 + 216)$ *Multiply.*
$= 6 \cdot 218$ *Add.*
$= 1308$ *Multiply.*

51. $3^2[(11 + 3) - 4]$
$= 3^2[14 - 4]$ *Add inside parentheses.*
$= 3^2[10]$ *Subtract.*
$= 9[10]$ *Use the exponent.*
$= 90$ *Multiply.*

52. $4^2[(13 + 4) - 8]$
$= 4^2[17 - 8]$ *Add inside parentheses.*
$= 4^2[9]$ *Subtract inside brackets.*
$= 16[9]$ *Use the exponent.*
$= 144$ *Multiply.*

53. Simplify the numerator and denominator separately; then divide.

$$\frac{6(3^2 - 1) + 8}{8 - 2^2} = \frac{6(9 - 1) + 8}{8 - 4} = \frac{6(8) + 8}{4} = \frac{48 + 8}{4} = \frac{56}{4} = 14$$

54. Simplify the numerator and denominator separately; then divide.

$$\frac{2(8^2 - 4) + 8}{29 - 3^3} = \frac{2(64 - 4) + 8}{29 - 27} = \frac{2(60) + 8}{2} = \frac{120 + 8}{2} = \frac{128}{2} = 64$$

55. $$\frac{4(6 + 2) + 8(8 - 3)}{6(4 - 2) - 2^2} = \frac{4(8) + 8(5)}{6(2) - 2^2} = \frac{4(8) + 8(5)}{6(2) - 4} = \frac{32 + 40}{12 - 4} = \frac{72}{8} = 9$$

56. $$\frac{6(5 + 1) - 9(1 + 1)}{5(8 - 6) - 2^3} = \frac{6(6) - 9(2)}{5(2) - 2^3} = \frac{36 - 18}{10 - 8} = \frac{18}{2} = 9$$

57. $9 \cdot 3 - 11 \leq 16$
$27 - 11 \leq 16$
$16 \leq 16$

The statement is true since $16 = 16$ is true.

58. $6 \cdot 5 - 12 \leq 18$
$30 - 12 \leq 18$
$18 \leq 18$

The statement is true since $18 = 18$ is true.

59. $5 \cdot 11 + 2 \cdot 3 \leq 60$
$55 + 6 \leq 60$
$61 \leq 60$

The statement is false since 61 *is greater than* 60.

60. $9 \cdot 3 + 4 \cdot 5 \geq 48$
$27 + 20 \geq 48$
$47 \geq 48$

The statement is false since 47 *is less than* 48.

61. $0 \geq 12 \cdot 3 - 6 \cdot 6$
$0 \geq 36 - 36$
$0 \geq 0$

The statement is true since $0 = 0$ is true.

62. $10 \leq 13 \cdot 2 - 15 \cdot 1$
$10 \leq 26 - 15$
$10 \leq 11$

The statement is true since $10 < 11$ is true.

63. $45 \geq 2[2 + 3(2 + 5)]$
$45 \geq 2[2 + 3(7)]$
$45 \geq 2[2 + 21]$
$45 \geq 2[23]$
$45 \geq 46$

The statement is false since 45 *is less than* 46.

64. $55 \geq 3[4 + 3(4 + 1)]$
$55 \geq 3[4 + 3(5)]$
$55 \geq 3[4 + 15]$
$55 \geq 3[19]$
$55 \geq 57$

The statement is false since 55 *is less than* 57.

65. $[3 \cdot 4 + 5(2)] \cdot 3 > 72$
$[12 + 10] \cdot 3 > 72$
$[22] \cdot 3 > 72$
$66 > 72$

The statement is false since 66 *is less than* 72.

66. $2 \cdot [7 \cdot 5 - 3(2)] \leq 58$
$2 \cdot [35 - 6] \leq 58$
$2[29] \leq 58$
$58 \leq 58$

The statement is true since $58 = 58$ is true.

67. $\dfrac{3 + 5(4 - 1)}{2 \cdot 4 + 1} \geq 3$
$\dfrac{3 + 5(3)}{8 + 1} \geq 3$
$\dfrac{3 + 15}{9} \geq 3$
$\dfrac{18}{9} \geq 3$
$2 \geq 3$

The statement is false since 2 *is less than* 3.

68. $\dfrac{7(3 + 1) - 2}{3 + 5 \cdot 2} \leq 2$
$\dfrac{7(4) - 2}{3 + 10} \leq 2$
$\dfrac{28 - 2}{13} \leq 2$
$\dfrac{26}{13} \leq 2$
$2 \leq 2$

The statement is true since $2 = 2$ is true.

69. $3 \geq \dfrac{2(5 + 1) - 3(1 + 1)}{5(8 - 6) - 4 \cdot 2}$
$3 \geq \dfrac{2(6) - 3(2)}{5(2) - 8}$
$3 \geq \dfrac{12 - 6}{10 - 8}$
$3 \geq \dfrac{6}{2}$
$3 \geq 3$

The statement is true since $3 = 3$ is true.

70. $7 \leq \dfrac{3(8 - 3) + 2(4 - 1)}{9(6 - 2) - 11(5 - 2)}$
$7 \leq \dfrac{3(5) + 2(3)}{9(4) - 11(3)}$
$7 \leq \dfrac{15 + 6}{36 - 33}$
$7 \leq \dfrac{21}{3}$
$7 \leq 7$

The statement is true since $7 = 7$ is true.

71. $3 \cdot 6 + 4 \cdot 2 = 60$

Listed below are some possibilities. We'll use trial and error until we get the desired result.

$$(3 \cdot 6) + 4 \cdot 2 = 18 + 8 = 26 \neq 60$$
$$(3 \cdot 6 + 4) \cdot 2 = 22 \cdot 2 = 44 \neq 60$$
$$3 \cdot (6 + 4 \cdot 2) = 3 \cdot 14 = 42 \neq 60$$
$$3 \cdot (6 + 4) \cdot 2 = 3 \cdot 10 \cdot 2 = 30 \cdot 2 = 60$$

72. $2 \cdot 8 - 1 \cdot 3 = 42$

$2 \cdot (8 - 1) \cdot 3 = 2 \cdot 7 \cdot 3 = 14 \cdot 3 = 42$

73. $10 - 7 - 3 = 6$

$10 - (7 - 3) = 10 - 4 = 6$

74. $15 - 10 - 2 = 7$

$15 - (10 - 2) = 15 - 8 = 7$

75. $8 + 2^2 = 100$

$(8 + 2)^2 = 10^2 = 10 \cdot 10 = 100$

76. $4 + 2^2 = 36$

$(4 + 2)^2 = 6^2 = 6 \cdot 6 = 36$

77. "$5 < 17$" means "five is less than seventeen." The statement is true.

78. "$8 < 12$" means "eight is less than twelve." The statement is true.

79. "$5 \neq 8$" means "five is not equal to eight." The statement is true.

80. "$6 \neq 9$" means "six is not equal to nine." The statement is true.

81. "$7 \geq 14$" means "seven is greater than or equal to fourteen." The statement is false.

82. "$6 \geq 12$" means "six is greater than or equal to twelve." The statement is false.

83. "$15 \leq 15$" means "fifteen is less than or equal to fifteen." The statement is true.

84. "$21 \leq 21$" means "twenty-one is less than or equal to twenty-one." The statement is true.

85. "Fifteen is equal to five plus ten" is written

$$15 = 5 + 10.$$

86. "Twelve is equal to twenty minus eight" is written

$$12 = 20 - 8.$$

87. "Nine is greater than five minus four" is written

$$9 > 5 - 4.$$

88. "Ten is greater than six plus one" is written

$$10 > 6 + 1.$$

89. "Sixteen is not equal to nineteen" is written

$$16 \neq 19.$$

90. "Three is not equal to four" is written

$$3 \neq 4.$$

91. "One-half is less than or equal to two-fourths" is written

$$\frac{1}{2} \leq \frac{2}{4}.$$

92. "One-third is less than or equal to three-ninths" is written

$$\frac{1}{3} \leq \frac{3}{9}.$$

93. $5 < 20$ becomes $20 > 5$ when the inequality symbol is reversed.

94. $30 > 9$ becomes $9 < 30$ when the inequality symbol is reversed.

95. $2.5 \geq 1.3$ becomes $1.3 \leq 2.5$ when the inequality symbol is reversed.

96. $4.1 \leq 5.3$ becomes $5.3 \geq 4.1$ when the inequality symbol is reversed.

97. **(a)** Substitute "40" for "age" in the expression for women.

$$14.7 - 40 \cdot 0.13$$

(b) $14.7 - 40 \cdot 0.13 = 14.7 - 5.2$ *Multiply.*
$= 9.5$ *Subtract.*

(c) 85% of $9.5 = 0.85(9.5) = 8.075$

Walking at 5 mph is associated with 8.0 METs, which is the table value closest to 8.075.

98. **(a)** $14.7 - 55 \cdot 0.11$ (expression for men)

(b) $14.7 - 55 \cdot 0.11 = 14.7 - 6.05$ *Multiply.*
$= 8.65$ *Subtract.*

(c) 85% of $8.65 = 0.85(8.65) = 7.3525$

Swimming is the activity with the closest MET value.

99. Answers will vary.

100. **(a)** The states that had a figure greater than 13.9 are Alaska (16.7), Texas (14.7), California (20.5), and Idaho (17.8).

(b) The states that had a figure that was at most 14.7 are Texas (14.7), Wyoming (12.5), Maine (12.3), and Missouri (13.9).

(c) The states that had a figure *not* less than 13.9, which is the same as ≥ 13.9, are Alaska (16.7), Texas (14.7), California (20.5), Idaho (17.8), and Missouri (13.9).

1.3 Variables, Expressions, and Equations

1.3 Classroom Examples, Now Try Exercises

1. **(a)** $16p = 16 \cdot p$
$= 16 \cdot 3$ *Let p = 3.*
$= 48$ *Multiply.*

(b) $2p^3 = 2 \cdot p^3$
$= 2 \cdot 3^3$ *Let p = 3.*
$= 2 \cdot 27$ *Cube 3.*
$= 54$ *Multiply.*

N1. **(a)** $9k = 9 \cdot k$
$= 9 \cdot 6$ *Let k = 6.*
$= 54$ *Multiply.*

(b) $4k^2 = 4 \cdot k^2$
$= 4 \cdot 6^2$ *Let k = 6.*
$= 4 \cdot 36$ *Square 6.*
$= 144$ *Multiply.*

2. Replace x with 6 and y with 9 in each expression.

(a) $4x + 5y = 4(6) + 5(9)$
$= 24 + 45$ *Multiply.*
$= 69$ *Add.*

(b) $\dfrac{4x - 2y}{x + 1} = \dfrac{4(6) - 2(9)}{6 + 1}$
$= \dfrac{24 - 18}{6 + 1}$ *Multiply.*
$= \dfrac{6}{7}$ *Subtract and add.*

(c) $2x^2 + y^2 = 2 \cdot 6^2 + 9^2$
$= 2 \cdot 36 + 81$ *Use exponents.*
$= 72 + 81$ *Multiply.*
$= 153$ *Add.*

N2. Replace x with 4 and y with 7 in each expression.

(a) $3x + 4y = 3(4) + 4(7)$
$= 12 + 28$ *Multiply.*
$= 40$ *Add.*

(b) $\dfrac{6x - 2y}{2y - 9} = \dfrac{6(4) - 2(7)}{2(7) - 9}$
$= \dfrac{24 - 14}{14 - 9}$ *Multiply.*
$= \dfrac{10}{5} = 2$ *Subtract; reduce.*

(c) $4x^2 - y^2 = 4 \cdot 4^2 - 7^2$
$= 4 \cdot 16 - 49$ *Use exponents.*
$= 64 - 49$ *Multiply.*
$= 15$ *Subtract.*

3. **(a)** Since a number is subtracted *from* 48, write this as $48 - x$ when using x as the variable to represent the number.

(b) "Product" indicates multiplication. Using x as the variable to represent the number, "The product of 6 and a number" translates as $6 \cdot x$ or $6x$.

(c) "The sum of a number and 5" suggests a number plus 5. Using x as the variable to represent the number, "9 multiplied by the sum of a number and 5" translates as $9(x + 5)$.

N3. **(a)** Using x as the variable to represent the number, "the sum of a number and 10" translates as $x + 10$, or $10 + x$.

(b) "A number divided by 7" translates as $x \div 7$, or $\frac{x}{7}$.

(c) "The difference between 9 and a number" translates as $9 - x$. Thus, "the product of 3 and the difference between 9 and a number" translates as $3(9 - x)$.

4. $8p - 11 = 5$
$8 \cdot 2 - 11 \stackrel{?}{=} 5$ *Replace p with 2.*
$16 - 11 \stackrel{?}{=} 5$ *Multiply.*
$5 = 5$ *True*

The number 2 is a solution of the equation.

N4. $8k + 5 = 61$
$8 \cdot 7 + 5 \stackrel{?}{=} 61$ *Replace k with 7.*
$56 + 5 \stackrel{?}{=} 61$ *Multiply.*
$61 = 61$ *True*

The number 7 is a solution of the equation.

5. Using x as the variable to represent the number, "Three times a number is subtracted from 21, giving 15" translates as

$$21 - 3x = 15.$$

Now try each number from the set $\{0, 2, 4, 6, 8, 10\}$.

$\boldsymbol{x = 0}$: $21 - 3(0) \stackrel{?}{=} 15$
$21 = 15$ *False*

$\boldsymbol{x = 2}$: $21 - 3(2) \stackrel{?}{=} 15$
$15 = 15$ *True*

$\boldsymbol{x = 4}$: $21 - 3(4) \stackrel{?}{=} 15$
$9 = 15$ *False*

Similarly, $x = 6, 8,$ or 10 result in false statements. Thus, 2 is the only solution.

N5. Using x as the variable to represent the number, "The sum of a number and nine is equal to the difference between 25 and the number" translates as

$$x + 9 = 25 - x.$$

Now try each number from the set $\{0, 2, 4, 6, 8, 10\}$.

$\boldsymbol{x = 4}$: $4 + 9 \stackrel{?}{=} 25 - 4$
$13 = 21$ *False*

$\boldsymbol{x = 6}$: $6 + 9 \stackrel{?}{=} 25 - 6$
$15 = 19$ *False*

$\boldsymbol{x = 8}$: $8 + 9 \stackrel{?}{=} 25 - 8$
$17 = 17$ *True*

Similarly, $x = 0, 2,$ or 10 result in false statements. Thus, 8 is the only solution.

6. $\dfrac{3x-1}{5}$ has no equals symbol, so this is an *expression*.

N6. **(a)** $2x + 5 = 6$ has an equals symbol, so this is an *equation*.

(b) $2x + 5 - 6$ has no equals symbol, so this is an *expression*.

1.3 Section Exercises

1. The expression $8x^2$ means $8 \cdot x \cdot x$. The correct choice is **B**.

2. If $x = 2$ and $y = 1$, then the value of xy is $2 \cdot 1 = 2$. The correct choice is **C**.

3. The sum of 15 and a number x is represented by the expression $15 + x$. The correct choice is **A**.

4. There is no equals symbol in $6x + 7$ or $6x - 7$, so those are expressions. The correct choices are **B** and **C**.

5. $2x^3 = 2 \cdot x \cdot x \cdot x$, while $2x \cdot 2x \cdot 2x = (2x)^3$. The last expression is equal to $8x^3$.

6. "7 less than a number" is an expression indicating subtraction, $x - 7$, while "7 is less than a number" is a statement relating 7 and x, $7 < x$.

7. The exponent 2 applies only to its base, which is x. (The expression $(5x)^2$ would require multiplying 5 by $x = 4$ first.)

8. (Answers will vary.) Two such pairs are $x = 0$, $y = 6$ and $x = 1$, $y = 4$. To find a pair, choose one number, substitute it for a variable, and then calculate the value for the other variable.

In part (a) of Exercises 9–22, replace x with 4. In part (b), replace x with 6. Then use the order of operations.

9. **(a)** $x + 7 = 4 + 7$
$= 11$

(b) $x + 7 = 6 + 7$
$= 13$

10. **(a)** $x - 3 = 4 - 3$
$= 1$

(b) $x - 3 = 6 - 3$
$= 3$

11. **(a)** $4x = 4(4) = 16$

(b) $4x = 4(6) = 24$

12. **(a)** $6x = 6(4)$
$= 24$

(b) $6x = 6(6)$
$= 36$

13. **(a)** $4x^2 = 4 \cdot 4^2$
$= 4 \cdot 16$
$= 64$

(b) $4x^2 = 4 \cdot 6^2$
$= 4 \cdot 36$
$= 144$

14. **(a)** $5x^2 = 5 \cdot 4^2$
$= 5 \cdot 16$
$= 80$

(b) $5x^2 = 5 \cdot 6^2$
$= 5 \cdot 36$
$= 180$

15. **(a)** $\dfrac{x+1}{3} = \dfrac{4+1}{3}$
$= \dfrac{5}{3}$

(b) $\dfrac{x+1}{3} = \dfrac{6+1}{3}$
$= \dfrac{7}{3}$

16. **(a)** $\dfrac{x-2}{5} = \dfrac{4-2}{5}$
$= \dfrac{2}{5}$

(b) $\dfrac{x-2}{5} = \dfrac{6-2}{5}$
$= \dfrac{4}{5}$

17. **(a)** $\frac{3x-5}{2x} = \frac{3 \cdot 4 - 5}{2 \cdot 4} = \frac{12-5}{8} = \frac{7}{8}$

(b) $\frac{3x-5}{2x} = \frac{3 \cdot 6 - 5}{2 \cdot 6} = \frac{18-5}{12} = \frac{13}{12}$

18. **(a)** $\frac{4x-1}{3x} = \frac{4(4)-1}{3(4)} = \frac{16-1}{12} = \frac{15}{12} = \frac{5}{4}$

(b) $\frac{4x-1}{3x} = \frac{4(6)-1}{3(6)} = \frac{24-1}{18} = \frac{23}{18}$

19. **(a)** $3x^2 + x = 3 \cdot 4^2 + 4 = 3 \cdot 16 + 4 = 48 + 4 = 52$

(b) $3x^2 + x = 3 \cdot 6^2 + 6 = 3 \cdot 36 + 6 = 108 + 6 = 114$

20. **(a)** $2x + x^2 = 2(4) + 4^2 = 8 + 16 = 24$

(b) $2x + x^2 = 2(6) + 6^2 = 12 + 36 = 48$

21. **(a)** $6.459x = 6.459 \cdot 4 = 25.836$

(b) $6.459x = 6.459 \cdot 6 = 38.754$

22. **(a)** $3.275x = 3.275 \cdot 4 = 13.1$

(b) $3.275x = 3.275 \cdot 6 = 19.65$

In part (a) of Exercises 23–38, replace x with 2 and y with 1. In part (b), replace x with 1 and y with 5.

23. **(a)** $8x + 3y + 5 = 8(2) + 3(1) + 5 = 16 + 3 + 5 = 19 + 5 = 24$

(b) $8x + 3y + 5 = 8(1) + 3(5) + 5 = 8 + 15 + 5 = 23 + 5 = 28$

24. **(a)** $4x + 2y + 7 = 4(2) + 2(1) + 7 = 8 + 2 + 7 = 17$

(b) $4x + 2y + 7 = 4(1) + 2(5) + 7 = 4 + 10 + 7 = 21$

25. **(a)** $3(x + 2y) = 3(2 + 2 \cdot 1) = 3(2 + 2) = 3(4) = 12$

(b) $3(x + 2y) = 3(1 + 2 \cdot 5) = 3(1 + 10) = 3(11) = 33$

26. **(a)** $2(2x + y) = 2[2(2) + 1] = 2(4 + 1) = 2(5) = 10$

(b) $2(2x + y) = 2[2(1) + 5] = 2(2 + 5) = 2(7) = 14$

27. **(a)** $x + \frac{4}{y} = 2 + \frac{4}{1} = 2 + 4 = 6$

(b) $x + \frac{4}{y} = 1 + \frac{4}{5} = \frac{5}{5} + \frac{4}{5} = \frac{9}{5}$

28. **(a)** $y+\frac{8}{x}=1+\frac{8}{2}$
$=1+4$
$=5$

(b) $y+\frac{8}{x}=5+\frac{8}{1}$
$=5+8$
$=13$

29. **(a)** $\frac{x}{2}+\frac{y}{3}=\frac{2}{2}+\frac{1}{3}$
$=\frac{6}{6}+\frac{2}{6}$
$=\frac{8}{6}=\frac{4}{3}$

(b) $\frac{x}{2}+\frac{y}{3}=\frac{1}{2}+\frac{5}{3}$
$=\frac{3}{6}+\frac{10}{6}$
$=\frac{13}{6}$

30. **(a)** $\frac{x}{5}+\frac{y}{4}=\frac{2}{5}+\frac{1}{4}$
$=\frac{8}{20}+\frac{5}{20}$
$=\frac{13}{20}$

(b) $\frac{x}{5}+\frac{y}{4}=\frac{1}{5}+\frac{5}{4}$
$=\frac{4}{20}+\frac{25}{20}$
$=\frac{29}{20}$

31. **(a)** $\frac{2x+4y-6}{5y+2}=\frac{2(2)+4(1)-6}{5(1)+2}$
$=\frac{4+4-6}{5+2}$
$=\frac{8-6}{7}$
$=\frac{2}{7}$

(b) $\frac{2x+4y-6}{5y+2}=\frac{2(1)+4(5)-6}{5(5)+2}$
$=\frac{2+20-6}{25+2}$
$=\frac{22-6}{27}$
$=\frac{16}{27}$

32. **(a)** $\frac{4x+3y-1}{x}=\frac{4(2)+3(1)-1}{2}$
$=\frac{8+3-1}{2}$
$=\frac{10}{2}$
$=5$

(b) $\frac{4x+3y-1}{x}=\frac{4(1)+3(5)-1}{1}$
$=\frac{4+15-1}{1}$
$=\frac{18}{1}$
$=18$

33. **(a)** $2y^2+5x=2\cdot 1^2+5\cdot 2$
$=2\cdot 1+5\cdot 2$
$=2+10$
$=12$

(b) $2y^2+5x=2\cdot 5^2+5\cdot 1$
$=2\cdot 25+5\cdot 1$
$=50+5$
$=55$

34. **(a)** $6x^2+4y=6(2)^2+4(1)$
$=6(4)+4$
$=24+4$
$=28$

(b) $6x^2+4y=6(1)^2+4(5)$
$=6(1)+4(5)$
$=6+20$
$=26$

35. **(a)** $\frac{3x+y^2}{2x+3y} = \frac{3(2)+1^2}{2(2)+3(1)}$
$= \frac{3(2)+1}{4+3}$
$= \frac{6+1}{7}$
$= \frac{7}{7}$
$= 1$

(b) $\frac{3x+y^2}{2x+3y} = \frac{3(1)+5^2}{2(1)+3(5)}$
$= \frac{3(1)+25}{2+15}$
$= \frac{3+25}{17}$
$= \frac{28}{17}$

36. **(a)** $\frac{x^2+1}{4x+5y} = \frac{2^2+1}{4(2)+5(1)}$
$= \frac{4+1}{8+5}$
$= \frac{5}{13}$

(b) $\frac{x^2+1}{4x+5y} = \frac{1^2+1}{4(1)+5(5)}$
$= \frac{1+1}{4+25}$
$= \frac{2}{29}$

37. **(a)** $0.841x^2 + 0.32y^2$
$= 0.841 \cdot 2^2 + 0.32 \cdot 1^2$
$= 0.841 \cdot 4 + 0.32 \cdot 1$
$= 3.364 + 0.32$
$= 3.684$

(b) $0.841x^2 + 0.32y^2$
$= 0.841 \cdot 1^2 + 0.32 \cdot 5^2$
$= 0.841 \cdot 1 + 0.32 \cdot 25$
$= 0.841 + 8$
$= 8.841$

38. **(a)** $0.941x^2 + 0.25y^2$
$= 0.941(2)^2 + 0.25(1)^2$
$= 0.941(4) + 0.25(1)$
$= 3.764 + 0.25$
$= 4.014$

(b) $0.941x^2 + 0.25y^2$
$= 0.941(1)^2 + 0.25(5)^2$
$= 0.941(1) + 0.25(25)$
$= 0.941 + 6.25$
$= 7.191$

39. "Twelve times a number" translates as $12 \cdot x$ or $12x$.

40. "Fifteen times a number" translates as $15 \cdot x$ or $15x$.

41. "Added to" indicates addition. "Nine added to a number" translates as $x + 9$.

42. "Six added to a number" translates as $x + 6$.

43. "Four subtracted from a number" translates as $x - 4$.

44. "Seven subtracted from a number" translates as $x - 7$.

45. "A number subtracted from seven" translates as $7 - x$.

46. "A number subtracted from four" translates as $4 - x$.

47. "The difference between a number and 8" translates as $x - 8$.

48. "The difference between 8 and a number" translates as $8 - x$.

49. "18 divided by a number" translates as $\frac{18}{x}$.

50. "A number divided by 18" translates as $\frac{x}{18}$.

51. "The product of 6 and four less than a number" translates as $6(x - 4)$.

52. "The product of 9 and five more than a number" translates as $9(x + 5)$.

53. An expression cannot by solved—it indicates a series of operations to perform. An expression is simplified. An equation is solved.

54. The value for y is 3. If x is 4, then $3x = 12$, and 3 from 12 equals 9.

55. $4m + 2 = 6$; 1

$4(1) + 2 \stackrel{?}{=} 6$ *Let m = 1.*
$4 + 2 \stackrel{?}{=} 6$
$6 = 6$ *True*

Because substituting 1 for m results in a true statement, 1 is a solution of the equation.

56. $2r + 6 = 8;\ 1$

$$\begin{aligned} 2(1) + 6 &\stackrel{?}{=} 8 \quad \textit{Let } r = 1. \\ 2 + 6 &\stackrel{?}{=} 8 \\ 8 &= 8 \quad \textit{True} \end{aligned}$$

The true result shows that 1 is a solution of the equation.

57. $2y + 3(y - 2) = 14;\ 3$

$$\begin{aligned} 2 \cdot 3 + 3(3 - 2) &\stackrel{?}{=} 14 \quad \textit{Let } y = 3. \\ 2 \cdot 3 + 3 \cdot 1 &\stackrel{?}{=} 14 \\ 6 + 3 &\stackrel{?}{=} 14 \\ 9 &= 14 \quad \textit{False} \end{aligned}$$

Because substituting 3 for y results in a false statement, 3 is not a solution of the equation.

58. $6x + 2(x + 3) = 14;\ 2$

$$\begin{aligned} 6(2) + 2(2 + 3) &\stackrel{?}{=} 14 \quad \textit{Let } x = 2. \\ 6(2) + 2(5) &\stackrel{?}{=} 14 \\ 12 + 10 &\stackrel{?}{=} 14 \\ 22 &= 14 \quad \textit{False} \end{aligned}$$

The false result shows that 2 is not a solution of the equation.

59. $6p + 4p + 9 = 11;\ \frac{1}{5}$

$$\begin{aligned} 6\left(\frac{1}{5}\right) + 4\left(\frac{1}{5}\right) + 9 &\stackrel{?}{=} 11 \quad \textit{Let } p = \frac{1}{5}. \\ \frac{6}{5} + \frac{4}{5} + 9 &\stackrel{?}{=} 11 \\ \frac{10}{5} + 9 &\stackrel{?}{=} 11 \\ 2 + 9 &\stackrel{?}{=} 11 \\ 11 &= 11 \quad \textit{True} \end{aligned}$$

The true result shows that $\frac{1}{5}$ is a solution of the equation.

60. $2x + 3x + 8 = 20;\ \frac{12}{5}$

$$\begin{aligned} 2\left(\frac{12}{5}\right) + 3\left(\frac{12}{5}\right) + 8 &\stackrel{?}{=} 20 \quad \textit{Let } x = \frac{12}{5}. \\ \frac{24}{5} + \frac{36}{5} + \frac{40}{5} &\stackrel{?}{=} 20 \\ \frac{100}{5} &\stackrel{?}{=} 20 \\ 20 &= 20 \quad \textit{True} \end{aligned}$$

The true result shows that $\frac{12}{5}$ is a solution of the equation.

61. $3r^2 - 2 = 46;\ 4$

$$\begin{aligned} 3(4)^2 - 2 &\stackrel{?}{=} 46 \quad \textit{Let } r = 4. \\ 3 \cdot 16 - 2 &\stackrel{?}{=} 46 \\ 48 - 2 &\stackrel{?}{=} 46 \\ 46 &= 46 \quad \textit{True} \end{aligned}$$

The true result shows that 4 is a solution of the equation.

62. $2x^2 + 1 = 19;\ 3$

$$\begin{aligned} 2(3)^2 + 1 &\stackrel{?}{=} 19 \quad \textit{Let } x = 3. \\ 2 \cdot 9 + 1 &\stackrel{?}{=} 19 \\ 18 + 1 &\stackrel{?}{=} 19 \\ 19 &= 19 \quad \textit{True} \end{aligned}$$

The true result shows that 3 is a solution of the equation.

63. $\frac{3}{8}x + \frac{1}{4} = 1;\ 2$

$$\begin{aligned} \frac{3}{8}(2) + \frac{1}{4} &\stackrel{?}{=} 1 \quad \textit{Let } x = 2. \\ \frac{3}{4} + \frac{1}{4} &\stackrel{?}{=} 1 \\ 1 &= 1 \quad \textit{True} \end{aligned}$$

The true result shows that 2 is a solution of the equation.

64. $\frac{7}{10}x + \frac{1}{2} = 4;\ 5$

$$\begin{aligned} \frac{7}{10}(5) + \frac{1}{2} &\stackrel{?}{=} 4 \quad \textit{Let } x = 5. \\ \frac{7}{2} + \frac{1}{2} &\stackrel{?}{=} 4 \\ \frac{8}{2} &= 4 \quad \textit{True} \end{aligned}$$

The true result shows that 5 is a solution of the equation.

65. $0.5(x - 4) = 80;\ 20$

$$\begin{aligned} 0.5(20 - 4) &\stackrel{?}{=} 80 \quad \textit{Let } x = 20. \\ 0.5(16) &\stackrel{?}{=} 80 \\ 8 &= 80 \quad \textit{False} \end{aligned}$$

The false result shows that 20 is not a solution of the equation.

66. $0.2(x - 5) = 70;\ 40$

$$\begin{aligned} 0.2(40 - 5) &\stackrel{?}{=} 70 \quad \textit{Let } x = 40. \\ 0.2(35) &\stackrel{?}{=} 70 \\ 7 &= 70 \quad \textit{False} \end{aligned}$$

The false result shows that 40 is not a solution of the equation.

67. "The sum of a number and 8 is 18" translates as

$$x+8=18.$$

Try each number from the given set, $\{2, 4, 6, 8, 10\}$, in turn.

$$\begin{aligned} x+8&=18 && \textit{Given equation} \\ 2+8&=18 && \textit{False} \\ 4+8&=18 && \textit{False} \\ 6+8&=18 && \textit{False} \\ 8+8&=18 && \textit{False} \\ 10+8&=18 && \textit{True} \end{aligned}$$

The only solution is 10.

68. "A number minus three equals 1" translates as

$$x-3=1.$$

Replace x with each number in the given set. The only true statement results when $x=4$, since

$$4-3=1.$$

Thus, 4 is the only solution.

69. "Sixteen minus three-fourths of a number is 13" translates as

$$16-\tfrac{3}{4}x=13.$$

Try each number from the given set, $\{2, 4, 6, 8, 10\}$, in turn.

$$\begin{aligned} 16-\tfrac{3}{4}x&=13 && \textit{Given equation} \\ 16-\tfrac{3}{4}(2)&=13 && \textit{False} \\ 16-\tfrac{3}{4}(4)&=13 && \textit{True} \\ 16-\tfrac{3}{4}(6)&=13 && \textit{False} \\ 16-\tfrac{3}{4}(8)&=13 && \textit{False} \\ 16-\tfrac{3}{4}(10)&=13 && \textit{False} \end{aligned}$$

The only solution is 4.

70. "The sum of six-fifths of a number and 2 is 14" translates as

$$\tfrac{6}{5}x+2=14.$$

Replace x with each number in the given set. The only true statement results as follows.

$$\begin{aligned} \tfrac{6}{5}(10)+2&\stackrel{?}{=}14 && \textit{Let } x = 10. \\ 12+2&\stackrel{?}{=}14 \\ 14&=14 && \textit{True} \end{aligned}$$

The only solution is 10.

71. "One more than twice a number is 5" translates as

$$2x+1=5.$$

Try each number from the given set. The only resulting true equation is

$$2\cdot 2+1=5,$$

So the only solution is 2.

72. "The product of a number and 3 is 6" translates as

$$3x=6.$$

The only true statement results when $x=2$, since

$$3\cdot 2=6.$$

Thus, 2 is the only solution.

73. "Three times a number is equal to 8 more than twice the number" translates as

$$3x=2x+8.$$

Try each number from the given set.

$$\begin{aligned} 3x&=2x+8 && \textit{Given equation} \\ 3(2)&=2(2)+8 && \textit{False} \\ 3(4)&=2(4)+8 && \textit{False} \\ 3(6)&=2(6)+8 && \textit{False} \\ 3(8)&=2(8)+8 && \textit{True} \\ 3(10)&=2(10)+8 && \textit{False} \end{aligned}$$

The only solution is 8.

74. "Twelve divided by a number equals $\frac{1}{3}$ times that number" translates as

$$\tfrac{12}{x}=\tfrac{1}{3}x.$$

The only true statement results as follows.

$$\begin{aligned} \tfrac{12}{6}&\stackrel{?}{=}\tfrac{1}{3}(6) && \textit{Let } x = 6. \\ 2&=2 && \textit{True} \end{aligned}$$

The only solution is 6.

75. There is no equals symbol, so $3x+2(x-4)$ is an expression.

76. There is no equals symbol, so $8y-(3y+5)$ is an expression.

77. There is an equals symbol, so $7t+2(t+1)=4$ is an equation.

78. There is an equals symbol, so $9r+3(r-4)=2$ is an equation.

79. There is an equals symbol, so $x+y=9$ is an equation.

80. There is no equals symbol, so $x+y-9$ is an expression.

81.
$$\begin{aligned} y&=0.212x-347 \\ &=0.212(1943)-347 \\ &=64.916\approx 64.9 \end{aligned}$$

The life expectancy of an American born in 1943 is about 64.9 years.

82. $y = 0.212x - 347$
$= 0.212(1960) - 347$
$= 68.52 \approx 68.5$

The life expectancy of an American born in 1960 is about 68.5 years.

83. $y = 0.212x - 347$
$= 0.212(1985) - 347$
$= 73.82 \approx 73.8$

The life expectancy of an American born in 1985 is about 73.8 years.

84. $y = 0.212x - 347$
$= 0.212(2005) - 347$
$= 78.06 \approx 78.1$

The life expectancy of an American born in 2005 is about 78.1 years.

85. Life expectancy has increased over 13 years during this time.

1.4 Real Numbers and the Number Line

1.4 Classroom Examples, Now Try Exercises

1. **(a)** Since Erin spends \$53 more than she has in her checking account, her balance is -53.

(b) Since the record high was 134° above zero, this temperature is expressed as 134.

N1. Since the deepest point is *below* the water's surface, the depth is -136.

2. In the set $\left\{\frac{5}{8}, -7, -1\frac{3}{5}, 0, 0.\overline{45}, \sqrt{11}, -\pi\right\}$, $\frac{5}{8}$, -7 $\left(\text{or } \frac{-7}{1}\right)$, $-1\frac{3}{5}$ $\left(\text{or } -\frac{8}{5}\right)$, 0 $\left(\text{or } \frac{0}{1}\right)$, and $0.\overline{45}$ $\left(\text{or } \frac{5}{11}\right)$ are rational (since each of these numbers can be written as the quotient of two integers); $\sqrt{11}$ and $-\pi$ are irrational.

N2. $\left\{-7, -\frac{4}{5}, 0, \sqrt{3}, 2.7, \pi, 13\right\}$

(a) The whole numbers are 0, and 13.

(b) The integers are $-7, 0,$ and 13.

(c) The rational numbers are $-7, -\frac{4}{5}, 0, 2.7,$ and 13.

(d) The irrational numbers are $\sqrt{3}$ and π.

3. Since -4 lies to the left of -1 on the number line, -4 is less than -1. Therefore, the statement $-4 \geq -1$ is *false*.

N3. Since -8 lies to the right of -9 on the number line, -8 is greater than -9. Therefore, the statement $-8 \leq -9$ is *false*.

4. **(a)** $|32| = 32$

(b) $|-32| = -(-32) = 32$

(c) $-|-32| = -(32) = -32$

(d) $-|32 - 2| = -|30| = -30$

N4. **(a)** $|4| = 4$

(b) $|-4| = -(-4) = 4$

(c) $-|-4| = -(4) = -4$

5. The largest positive percent increase is 12.9, so the category is *gasoline* and the year is *2005 to 2006*.

N5. The category *new cars* is negative in both years.

1.4 Section Exercises

1. Use the integer 2,866,000 since "increased by 2,866,000" indicates a positive number.

2. Use the integer 409 since "increased by 409" indicates a positive number.

3. Use the integer $-52{,}000$ since "a decrease of 52,000" indicates a negative number.

4. Use the integer -6854 since "a decrease of 6854" indicates a negative number.

5. Use the rational numbers -11.2 and 8.6 since "declined 11.2%" and "rose 8.6%" indicate a negative number and a positive number, respectively.

6. Use the rational numbers -9.4 and 18.8 since "declined 9.4%" and "rose 18.8%" indicate a negative number and a positive number, respectively.

7. Use the rational number 82.60 since "closed up 82.60" indicates a positive number.

8. Use the rational number -54.68 since "closed down 54.68" indicates a negative number.

9. The only integer between 3.6 and 4.6 is 4.

10. A rational number between 2.8 and 2.9 is 2.85. There are others.

11. There is only one whole number that is not positive and that is less than 1: the number 0.

12. A whole number greater than 3.5 is 4. There are others.

13. An irrational number that is between $\sqrt{12}$ and $\sqrt{14}$ is $\sqrt{13}$. There are others.

14. The only real number that is neither negative nor positive is 0.

15. True; every natural number is positive.

16. False; 0 is a whole number that is not positive. In fact, it is the *only* whole number that is not positive.

17. True; every integer is a rational number. For example, 5 can be written as $\frac{5}{1}$.

18. True; every rational number is a real number.

19. False; if a number is rational, it cannot be irrational, and vice versa.

20. True; every terminating decimal is a rational number.

21. Three examples of positive real numbers that are not integers are $\frac{1}{2}$, $\frac{5}{8}$, and $1\frac{3}{4}$. Other examples are 0.7, $4\frac{2}{3}$, and 5.1.

22. Real numbers that are not positive numbers are 0 and all numbers to the left of 0 on the number line. Three examples are -1, $-\frac{3}{4}$, and -5. Other examples are 0, -5, $-\sqrt{7}$, $-1\frac{1}{2}$, and -0.3.

23. Three examples of real numbers that are not whole numbers are $-3\frac{1}{2}$, $-\frac{2}{3}$, and $\frac{3}{7}$. Other examples are -4.3, $-\sqrt{2}$, and $\sqrt{7}$.

24. Rational numbers that are not integers are all real numbers that can be expressed as a quotient of integers (with non-zero denominators) such that in lowest terms the denominator is not 1. Three examples are $\frac{1}{2}$, $-\frac{2}{3}$, and $\frac{2}{7}$. Other examples are -5.6, $-4\frac{3}{4}$, $-\frac{1}{2}$, $\frac{1}{2}$, and 5.2.

25. Three examples of real numbers that are not rational numbers are $\sqrt{5}$, π, and $-\sqrt{3}$. All irrational numbers are real numbers that are not rational.

26. Rational numbers that are not negative numbers are 0 and all rational numbers to the right of zero on the number line. Three examples are $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{5}{2}$. Other examples are 0, $\frac{1}{2}$, 1, $3\frac{1}{4}$, and 5.

27. $\left\{-9,\ -\sqrt{7},\ -1\frac{1}{4},\ -\frac{3}{5},\ 0,\ 0.\overline{1},\ \sqrt{5},\ 3,\ 5.9,\ 7\right\}$

(a) The natural numbers in the given set are 3 and 7, since they are in the natural number set $\{1, 2, 3, \dots\}$.

(b) The set of whole numbers includes the natural numbers and 0. The whole numbers in the given set are 0, 3, and 7.

(c) The integers are the set of numbers $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$. The integers in the given set are -9, 0, 3, and 7.

(d) Rational numbers are the numbers which can be expressed as the quotient of two integers, with denominators not equal to 0.

We can write numbers from the given set in this form as follows:

$$-9 = \frac{-9}{1}, -1\frac{1}{4} = \frac{-5}{4}, -\frac{3}{5} = \frac{-3}{5}, 0 = \frac{0}{1},$$

$$0.\overline{1} = \frac{1}{9}, 3 = \frac{3}{1}, 5.9 = \frac{59}{10}, \text{ and } 7 = \frac{7}{1}.$$

Thus, the rational numbers in the given set are -9, $-1\frac{1}{4}$, $-\frac{3}{5}$, 0, $0.\overline{1}$, 3, 5.9, and 7.

(e) Irrational numbers are real numbers that are not rational. $-\sqrt{7}$ and $\sqrt{5}$ can be represented by points on the number line but cannot be written as a quotient of integers. Thus, the irrational numbers in the given set are $-\sqrt{7}$ and $\sqrt{5}$.

(f) Real numbers are all numbers that can be represented on the number line. All the numbers in the given set are real.

28. $\{-5.3,\ -5,\ -\sqrt{3},\ -1, -\frac{1}{9}, 0,\ 0.\overline{27},\ 1.2,\ 1.8,\ 3,\ \sqrt{11}\}$

(a) The only natural number in the given set is 3.

(b) The whole numbers in the set are 0 and 3.

(c) The integers in the set are -5, -1, 0, and 3.

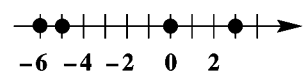
(d) The rational numbers are -5.3, -5, -1, $-\frac{1}{9}$, 0, $0.\overline{27}$, 1.2, 1.8, and 3.

(e) The irrational numbers in the set are $-\sqrt{3}$ and $\sqrt{11}$.

(f) All the numbers in the set are real numbers.

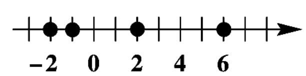
29. Graph 0, 3, -5, and -6.

Place a dot on the number line at the point that corresponds to each number. The order of the numbers from smallest to largest is -6, -5, 0, 3.

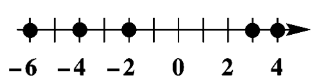


30. Graph 2, 6, -2, and -1.

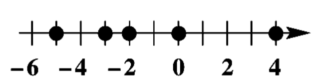
The smallest number, -2, will be the farthest to the left.



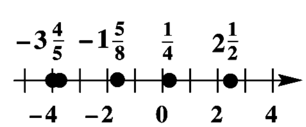
31. Graph -2, -6, -4, 3, and 4.



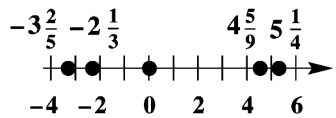
32. Graph -5, -3, -2, 0, and 4.



33. Graph $\frac{1}{4}$, $2\frac{1}{2}$, $-3\frac{4}{5}$, -4, and $-1\frac{5}{8}$.



34. Graph $5\frac{1}{4}$, $4\frac{5}{9}$, $-2\frac{1}{3}$, 0, and $-3\frac{2}{5}$.



35. **(a)** $|-9| = 9$ **(A)**

The distance between -9 and 0 on the number line is 9 units.

(b) $-(-9) = 9$ **(A)**

The opposite of -9 is 9.

(c) $-|-9| = -(9) = -9$ **(B)**

(d) $-|-(-9)| = -|9|$ *Work inside absolute value symbols first*

$= -(9)$

$= -9$ **(B)**

36. The opposite of -5 is <u>5</u>, while the absolute value of -5 is <u>5</u>. The additive inverse of -5 is <u>5</u>, while the additive inverse of the absolute value of -5 is <u>-5</u>.

37. **(a)** The opposite of -7 is found by changing the sign of -7. The opposite of -7 is 7.

(b) The absolute value of -7 is the distance between 0 and -7 on the number line.

$$|-7| = 7$$

The absolute value of -7 is 7.

38. **(a)** The opposite of a number is found by changing the sign of a number, so the opposite of -4 is 4.

(b) The distance between -4 and 0 on the number line is 4 units, so $|-4| = 4$.

39. **(a)** The opposite of 8 is -8.

(b) The distance between 0 and 8 on the number line is 8 units, so the absolute value of 8 is 8.

40. **(a)** The opposite of 10 is -10.

(b) The distance between 10 and 0 on the number line is 10 units, so $|10| = 10$.

41. **(a)** The opposite of a number is found by changing the sign of a number, so the opposite of $-\frac{3}{4}$ is $\frac{3}{4}$.

(b) The distance between $-\frac{3}{4}$ and 0 on the number line is $\frac{3}{4}$ unit, so $\left|-\frac{3}{4}\right| = \frac{3}{4}$.

42. **(a)** The opposite of a number is found by changing the sign of a number, so the opposite of $-\frac{2}{5}$ is $\frac{2}{5}$.

(b) The distance between $-\frac{2}{5}$ and 0 on the number line is $\frac{2}{5}$ unit, so $\left|-\frac{2}{5}\right| = \frac{2}{5}$.

43. Since -6 is a negative number, its absolute value is the additive inverse of -6; that is,

$$|-6| = -(-6) = 6.$$

44. $|-14| = -(-14) = 14$

45. $-|12| = -(12) = -12$

46. $-|19| = -(19) = -19$

47. $-\left|-\frac{2}{3}\right| = -\left[-\left(-\frac{2}{3}\right)\right] = -\left[\frac{2}{3}\right] = -\frac{2}{3}$

48. $-\left|-\frac{4}{5}\right| = -\left[-\left(-\frac{4}{5}\right)\right] = -\left[\frac{4}{5}\right] = -\frac{4}{5}$

49. $|6-3| = |3| = 3$

50. $-|6-3| = -|3| = -3$

51. The statement "Absolute value is always positive." is not true. The absolute value of 0 is 0, and 0 is not positive. We could say that *absolute value is never negative,* or *absolute value is always nonnegative*.

52. If a is negative, $|a| = -a$. This statement is true since both $|a|$ and $-a$ represent the same positive number.

53. $-11, -3$
Since -11 is located to the left of -3 on the number line, -11 is the lesser number.

54. $-8, -13$
Since -13 is located to the left of -8 on the number line, -13 is the lesser number.

55. $-7, -6$
Since -7 is located to the left of -6 on the number line, -7 is the lesser number.

56. $-16, -17$
Since -17 is located to the left of -16 on the number line, -17 is the lesser number.

57. $4, |-5|$
Since $|-5| = 5$, 4 is the lesser of the two numbers.

58. $4, |-3|$
Since $|-3| = 3$, $|-3|$ or 3 is the lesser of the two numbers.

59. $|-3.5|, |-4.5|$
Since $|-3.5| = 3.5$ and $|-4.5| = 4.5$, $|-3.5|$ or 3.5 is the lesser number.

60. $|-8.9|, |-9.8|$
Since $|-8.9| = 8.9$ and $|-9.8| = 9.8$, $|-8.9|$ or 8.9 is the lesser number.

61. $-|-6|, -|-4|$
Since $-|-6| = -6$ and $-|-4| = -4$, $-|-6|$ is to the left of $-|-4|$ on the number line, so $-|-6|$ or -6 is the lesser number.

62. $-|-2|, -|-3|$

$$-|-2| = -2 \quad \text{and} \quad -|-3| = -3$$

Since -3 is to the left of -2 on the number line, $-|-3|$ or -3 is the lesser number.

63. $|5-3|, |6-2|$
Since $|5-3| = |2| = 2$ and $|6-2| = |4| = 4$, $|5-3|$ or 2 is the lesser number.

64. $|7-2|, |8-1|$
Since $|7-2| = |5| = 5$ and $|8-1| = |7| = 7$, $|7-2|$ or 5 is the lesser number.

65. $-5 < -2$
Since -5 is to the *left* of -2 on the number line, -5 is *less than* -2, and the statement $-5 < -2$ is true.

66. $-8 > -2$
Since -8 is to the *left* of -2 on the number line, -8 is *less than* -2, and the statement $-8 > -2$ is false.

67. $-4 \le -(-5)$
Since $-(-5) = 5$ and $-4 < 5$, $-4 \le -(-5)$ is true.

68. $-6 \le -(-3)$
Since $-(-3) = 3$ and $-6 \le 3$, $-6 \le -(-3)$ is true.

69. $|-6| < |-9|$
Since $|-6| = 6$ and $|-9| = 9$, and $6 < 9$, $|-6| < |-9|$ is true.

70. $|-12| < |-20|$
Since $|-12| = 12$ and $|-20| = 20$, and $12 < 20$, $|-12| < |-20|$ is true.

71. $-|8| > |-9|$
Since $-|8| = -8$ and $|-9| = -(-9) = 9$, $-|8| < |-9|$, so $-|8| > |-9|$ is false.

72. $-|12| > |-15|$
Since $-|12| = -12$ and $|-15| = -(-15) = 15$, $-|12| < |-15|$, so $-|12| > |-15|$ is false.

73. $-|-5| \ge -|-9|$
Since $-|-5| = -5$, $-|-9| = -9$, and $-5 > -9$, $-|-5| \ge -|-9|$ is true.

74. $-|-12| \le -|-15|$
Since $-|-12| = -12$, $-|-15| = -15$, and $-12 > -15$, $-|-12| \le -|-15|$ is false.

75. $|6-5| \ge |6-2|$
Since $|6-5| = |1| = 1$ and $|6-2| = |4| = 4$, $|6-5| < |6-2|$, so $|6-5| \ge |6-2|$ is false.

76. $|13-8| \le |7-4|$
Since $|13-8| = |5| = 5$ and $|7-4| = |3| = 3$, $|13-8| > |7-4|$, so $|13-8| \le |7-4|$ is false.

77. The number that represents the greatest percentage increase is 10.6, which corresponds to *fuel and other utilities* from 2004 to 2005.

78. The negative number with the largest absolute value in the table is -0.7, so the greatest percentage decrease is *apparel and upkeep* from 2004 to 2005.

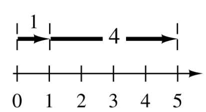
79. The number with the smallest absolute value in the table is -0.4, so the least change corresponds to *apparel and upkeep* from 2006 to 2007.

80. The industry with two negative entries (representing a decrease for both years) is *apparel and upkeep*.

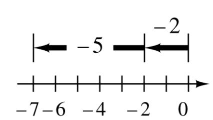
1.5 Adding and Subtracting Real Numbers

1.5 Classroom Examples, Now Try Exercises

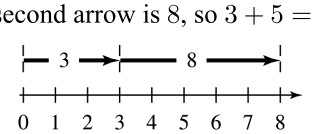
1. **(a)** Start at 0 on a number line. Draw an arrow 1 unit to the right to represent the positive number 1. From the right end of this arrow, draw a second arrow 4 units to the right to represent the addition of a positive number. The number below the end of this second arrow is 5, so $1 + 4 = 5$.



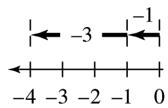
(b) Start at 0 on a number line. Draw an arrow 2 units to the left to represent the negative number -2. From the left end of this arrow, draw a second arrow 5 units to the left to represent the addition of a negative number. The number below the end of this second arrow is -7, so $-2 + (-5) = -7$.



N1. **(a)** Start at 0 on a number line. Draw an arrow 3 units to the right to represent the positive number 3. From the right end of this arrow, draw a second arrow 5 units to the right to represent the addition of a positive number. The number below the end of this second arrow is 8, so $3 + 5 = 8$.



(b) Start at 0 on a number line. Draw an arrow 1 unit to the left to represent the negative number -1. From the left end of this arrow, draw a second arrow 3 units to the left. The number below the end of this second arrow is -4, so $-1 + (-3) = -4$.



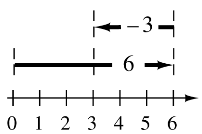
2. $-15 + (-4) = -19$

The sum of two negative numbers is negative.

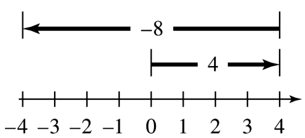
N2. $-6 + (-11) = -17$

The sum of two negative numbers is negative.

3. Start at 0 on a number line. Draw an arrow 6 units to the right. From the right end of this arrow, draw a second arrow 3 units to the left. The number below the end of this second arrow is 3, so $6 + (-3) = 3$.



N3. Start at 0 on a number line. Draw an arrow 4 units to the right. From the right end of this arrow, draw a second arrow 8 units to the left. The number below the end of this second arrow is -4, so $4 + (-8) = -4$.



4. $6 + (-14)$

Since the numbers have different signs, find the difference between their absolute values:

$$14 - 6 = 8.$$

Because -14 has the larger absolute value, the sum is negative:

$$6 + (-14) = -8.$$

N4. $8 + (-17)$

Since the numbers have different signs, find the difference between their absolute values:

$$17 - 8 = 9.$$

Because -17 has the larger absolute value, the sum is negative:

$$8 + (-17) = -9.$$

5. **(a)** $\frac{3}{4} + \left(-\frac{11}{8}\right) = -\frac{5}{8}$

The number with the greater absolute value is $-\frac{11}{8}$, so the sum will be negative. The answer of $-\frac{5}{8}$ is correct.

(b) $-3.8 + 9.5 = 5.7$

The number with the greater absolute value is 9.5, so the sum will be positive. The answer of 5.7 is correct.

N5. **(a)** $\frac{2}{3} + \left(-2\frac{1}{9}\right) = -1\frac{4}{9}$

The number with the greater absolute value is $-2\frac{1}{9}$, so the sum will be negative. The answer of $-1\frac{4}{9}$ is correct.

(b) $3.7 + (-5.7) = -2$

The number with the greater absolute value is -5.7, so the sum will be negative. The answer of -2 is correct.

6. **(a)** $-8 - 5 = -8 + (-5)$ *Add the opposite.*
$= -13$

(b) $-8 - (-12) = -8 + (12)$ *Add the opposite.*
$= 4$

(c) $\frac{5}{4} - (-\frac{3}{7}) = \frac{5}{4} + \frac{3}{7} = \frac{35}{28} + \frac{12}{28} = \frac{47}{28}$, or $1\frac{19}{28}$

N6. **(a)** $-5 - (-11) = -5 + (11)$ *Add the opposite.*
$= 6$

(b) $4 - 15 = 4 + (-15)$ *Add the opposite.*
$= -11$

(c) $-\frac{5}{7} - \frac{1}{3} = -\frac{5}{7} + \left(-\frac{1}{3}\right)$ *Add opposite.*

$$= -\frac{15}{21} + \left(-\frac{7}{21}\right)$$

$$= -\frac{22}{21}, \text{ or } -1\frac{1}{21}$$

7. **(a)** $6 + [(-1 - 4) - 2]$

$$= 6 + \{[-1 + (-4)] - 2\}$$
$$= 6 + (-5 - 2)$$
$$= 6 + [-5 + (-2)]$$
$$= 6 + (-7)$$
$$= -1$$

(b) $\left|-\frac{1}{6}-\left(-\frac{1}{3}\right)\right|-\frac{1}{4}$

$$\begin{aligned}&=\left|-\frac{2}{12}-\left(-\frac{4}{12}\right)\right|-\frac{3}{12}\\&=\left|-\frac{2}{12}+\frac{4}{12}\right|-\frac{3}{12}\\&=\left|\frac{2}{12}\right|-\frac{3}{12}\\&=\frac{2}{12}+\left(-\frac{3}{12}\right)\\&=-\frac{1}{12}\end{aligned}$$

N7. **(a)** $8-[(-3+7)-(3-9)]$

$$\begin{aligned}&=8-[(4)-(3+(-9))]\\&=8-[4-(-6)]\\&=8-[4+6]\\&=8-10\\&=8+(-10)\\&=-2\end{aligned}$$

(b) $3|6-9|-|4-12|$

$$\begin{aligned}&=3|6+(-9)|-|4+(-12)|\\&=3|-3|-|-8|\\&=3(3)-8\\&=9-8\\&=1\end{aligned}$$

8. "7 is increased by the sum of 8 and -3" is written $7+[8+(-3)]$.

$$7+[8+(-3)]=7+5=12$$

N8. "The sum of -3 and 7, increased by 10" is written $(-3+7)+10$.

$$(-3+7)+10=4+10=14$$

9. **(a)** "The difference between -5 and -12" is written $-5-(-12)$.

$$\begin{aligned}-5-(-12)&=-5+12\\&=7\end{aligned}$$

(b) "-2 subtracted from the sum of 4 and -4" is written $[4+(-4)]-(-2)$.

$$\begin{aligned}[4+(-4)]-(-2)&=0-(-2)\\&=0+2\\&=2\end{aligned}$$

N9. **(a)** "The difference between 5 and -8, decreased by 4" is written $[5-(-8)]-4$.

$$\begin{aligned}[5-(-8)]-4&=[5+8]-4\\&=13-4\\&=9\end{aligned}$$

(b) "7 less than -2" is written $-2-7$.

$$\begin{aligned}-2-7&=-2+(-7)\\&=-9\end{aligned}$$

10. The difference between the highest and lowest temperatures is given by

$$\begin{aligned}79-(-56)&=79+56\\&=135.\end{aligned}$$

The difference is 135°F.

N10. The difference between a gain of 226 yards and a loss of 7 yards

$$\begin{aligned}226-(-7)&=226+7\\&=233.\end{aligned}$$

The difference is 233 yards.

11. Subtract the CPI number for 2002 from the CPI number for 2003.

$$119.3-121.4=119.3+(-121.4)=-2.1$$

A negative result indicates a decrease.

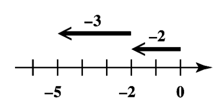
N11. Subtract the CPI number for 2003 from the CPI number for 2004.

$$119.6-119.3=0.3$$

A positive result indicates an increase.

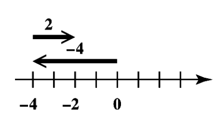
1.5 Section Exercises

1. The sum of two negative numbers will always be a *negative* number. In the illustration, we have $-2+(-3)=-5$.



2. The sum of a number and its opposite will always be *zero* (0).

3. When adding a positive number and a negative number, where the negative number has the greater absolute value, the sum will be a *negative* number. In the illustration, the absolute value of -4 is larger than the absolute value of 2, so the sum is a negative number; that is, $-4+2=-2$.



4. To simplify the expression $8+[-2+(-3+5)]$, one should begin by adding $\underline{-3}$ and $\underline{5}$, according to the rule for order of operations.

5. By the definition of subtraction, in order to perform the subtraction $-6-(-8)$, we must add the opposite of $\underline{-8}$ to $\underline{-6}$ to get $\underline{2}$.

6. "The difference between 7 and 12" translates as $\underline{7-12}$, while "the difference between 12 and 7" translates as $\underline{12-7}$.

7. The expression $x - y$ would have to be *positive* since subtracting a negative number from a positive number is the same as adding a positive number to a positive number, which is a positive number.

8. $y - x = y + (-x)$

If x is a positive number and y is a negative number, $y - x$ will be the sum of two negative numbers, which is a *negative* number.

9. $|x| = x$, since x is a positive number.

$y - |x| = y - x$, which is a *negative* number. (See Exercise 8.)

10. $x + |y|$

Since $|y|$ is positive, $x + |y|$ is the sum of two positive numbers, which is *positive*.

11. $-6 + (-2)$

The sum of two negative numbers is negative.

$$-6 + (-2) = -8$$

12. $-9 + (-2)$

Since the numbers have the same sign, add their absolute values:

$$9 + 2 = 11.$$

Since both numbers are negative, their sum is negative:

$$-9 + (-2) = -11.$$

13. $-5 + (-7)$

Because the numbers have the same sign, add their absolute values:

$$5 + 7 = 12.$$

Because both numbers are negative, their sum is negative:

$$-5 + (-7) = -12.$$

14. $-11 + (-5)$

Because the numbers have the same sign, add their absolute values:

$$11 + 5 = 16.$$

Because both numbers are negative, their sum is negative:

$$-11 + (-5) = -16.$$

15. $6 + (-4)$

To add $6 + (-4)$, find the difference between the absolute values of the numbers.

$$|6| = 6 \text{ and } |-4| = 4$$

$$6 - 4 = 2$$

Since $|6| > |-4|$, the sum will be positive:

$$6 + (-4) = 2.$$

16. $11 + (-8)$

Since the numbers have different signs, find the difference between their absolute values:

$$11 - 8 = 3.$$

Since 11 has the larger absolute value, the answer is positive:

$$11 + (-8) = 3.$$

17. $4 + (-6)$

Since the numbers have different signs, find the difference between their absolute values:

$$6 - 4 = 2.$$

Because -6 has the larger absolute value, the sum is negative:

$$4 + (-6) = -2.$$

18. $3 + (-7)$

Since the numbers have different signs, find the difference between their absolute values:

$$7 - 3 = 4.$$

Since -7 has the larger absolute value, the sum is negative:

$$3 + (-7) = -4.$$

19. $-3.5 + 12.4$

Since the numbers have different signs, find the difference between their absolute values:

$$12.4 - 3.5 = 8.9.$$

Since 12.4 has the larger absolute value, the answer is positive:

$$-3.5 + 12.4 = 8.9.$$

20. $-12.5 + 21.3$

Since the numbers have different signs, find the difference between their absolute values:

$$21.3 - 12.5 = 8.8.$$

Since 21.3 has the larger absolute value, the answer is positive:

$$-12.5 + 21.3 = 8.8.$$

21. $4 + [13 + (-5)]$

Perform the operation inside the brackets first, then add.

$$4 + [13 + (-5)] = 4 + [8] = 12$$

22. $6 + [2 + (-13)] = 6 + [-11] = -5$

23. $8 + [-2 + (-1)] = 8 + [-3] = 5$

24. $12 + [-3 + (-4)] = 12 + [-7] = 5$

25. $-2 + [5 + (-1)] = -2 + [4] = 2$

26. $-8 + [9 + (-2)] = -8 + [7] = -1$

27. $-6 + [6 + (-9)] = -6 + [-3] = -9$

28. $-3 + [3 + (-8)] = -3 + [-5] = -8$

29. $[(-9) + (-3)] + 12 = [-12] + 12 = 0$

30. $[(-8) + (-6)] + 14 = [-14] + 14 = 0$

31. $-\frac{1}{6} + \frac{2}{3} = -\frac{1}{6} + \frac{4}{6} = \frac{3}{6} = \frac{1}{2}$

32. $$\begin{aligned} -\frac{6}{25} + \frac{19}{20} &= -\frac{6 \cdot 4}{25 \cdot 4} + \frac{19 \cdot 5}{20 \cdot 5} \\ &= -\frac{24}{100} + \frac{95}{100} \\ &= \frac{71}{100} \end{aligned}$$

33. Since $8 = 2 \cdot 2 \cdot 2$ and $12 = 2 \cdot 2 \cdot 3$, the LCD is $2 \cdot 2 \cdot 2 \cdot 3 = 24$.

$$\begin{aligned} \frac{5}{8} + \left(-\frac{17}{12}\right) &= \frac{5 \cdot 3}{8 \cdot 3} + \left(-\frac{17 \cdot 2}{12 \cdot 2}\right) \\ &= \frac{15}{24} + \left(-\frac{34}{24}\right) \\ &= -\frac{19}{24} \end{aligned}$$

34. $\frac{9}{10} + \left(-\frac{3}{5}\right) = \frac{9}{10} + \left(-\frac{6}{10}\right) = \frac{3}{10}$

35. $$\begin{aligned} 2\frac{1}{2} + \left(-3\frac{1}{4}\right) &= \frac{5}{2} + \left(-\frac{13}{4}\right) \\ &= \frac{10}{4} + \left(-\frac{13}{4}\right) \\ &= -\frac{3}{4} \end{aligned}$$

36. $$\begin{aligned} -4\frac{3}{8} + 6\frac{1}{2} &= -\frac{35}{8} + \frac{13}{2} \\ &= -\frac{35}{8} + \frac{52}{8} \\ &= \frac{17}{8}, \text{ or } 2\frac{1}{8} \end{aligned}$$

37. $$\begin{aligned} -6.1 + [3.2 + (-4.8)] &= -6.1 + [-1.6] \\ &= -7.7 \end{aligned}$$

38. $$\begin{aligned} -9.4 + [-5.8 + (-1.4)] &= -9.4 + [-7.2] \\ &= -16.6 \end{aligned}$$

39. $$\begin{aligned} [-3 + (-4)] + [5 + (-6)] &= [-7] + [-1] \\ &= -8 \end{aligned}$$

40. $$\begin{aligned} [-8 + (-3)] + [-7 + (-6)] &= [-11] + [-13] \\ &= -24 \end{aligned}$$

41. $$\begin{aligned} [-4 + (-3)] + [8 + (-1)] &= [-7] + [7] \\ &= 0 \end{aligned}$$

42. $$\begin{aligned} [-5 + (-9)] + [16 + (-21)] &= [-14] + [-5] \\ &= -19 \end{aligned}$$

43. $$\begin{aligned} &[-4 + (-6)] + [(-3) + (-8)] + [12 + (-11)] \\ &= ([-10] + [-11]) + [1] \\ &= (-21) + 1 \\ &= -20 \end{aligned}$$

44. $$\begin{aligned} &[-2 + (-11)] + [12 + (-2)] + [18 + (-6)] \\ &= ([-13] + [10]) + 12 \\ &= (-3) + 12 \\ &= 9 \end{aligned}$$

In Exercises 45–68, use the definition of subtraction to find the differences.

45. $4 - 7 = 4 + (-7) = -3$

46. $8 - 13 = 8 + (-13) = -5$

47. $5 - 9 = 5 + (-9) = -4$

48. $6 - 11 = 6 + (-11) = -5$

49. $-7 - 1 = -7 + (-1) = -8$

50. $-9 - 4 = -9 + (-4) = -13$

51. $-8 - 6 = -8 + (-6) = -14$

52. $-9 - 5 = -9 + (-5) = -14$

53. $7 - (-2) = 7 + (2) = 9$

54. $9 - (-2) = 9 + (2) = 11$

55. $-6 - (-2) = -6 + (2) = -4$

56. $-7 - (-5) = -7 + (5) = -2$

57. $$\begin{aligned} 2 - (3 - 5) &= 2 - [3 + (-5)] \\ &= 2 - [-2] \\ &= 2 + (2) \\ &= 4 \end{aligned}$$

58. $$\begin{aligned} -3 - (4 - 11) &= -3 - [4 + (-11)] \\ &= -3 - [-7] \\ &= -3 + (7) \\ &= 4 \end{aligned}$$

59. $$\begin{aligned} \frac{1}{2} - \left(-\frac{1}{4}\right) &= \frac{1}{2} + \frac{1}{4} \\ &= \frac{2}{4} + \frac{1}{4} = \frac{3}{4} \end{aligned}$$

60. $\frac{1}{3} - \left(-\frac{4}{3}\right) = \frac{1}{3} + \frac{4}{3} = \frac{5}{3}$, or $1\frac{2}{3}$

61. $$\begin{aligned}-\frac{3}{4} - \frac{5}{8} &= -\frac{3}{4} + \left(-\frac{5}{8}\right)\\ &= -\frac{6}{8} + \left(-\frac{5}{8}\right)\\ &= -\frac{11}{8}, \text{ or } -1\frac{3}{8}\end{aligned}$$

62. $$\begin{aligned}-\frac{5}{6} - \frac{1}{2} &= -\frac{5}{6} + \left(-\frac{1}{2}\right)\\ &= -\frac{5}{6} + \left(-\frac{3}{6}\right)\\ &= -\frac{8}{6}\\ &= -\frac{4}{3}, \text{ or } -1\frac{1}{3}\end{aligned}$$

63. $$\begin{aligned}&\frac{5}{8} - \left(-\frac{1}{2} - \frac{3}{4}\right)\\ &= \frac{5}{8} - \left[-\frac{1}{2} + \left(-\frac{3}{4}\right)\right]\\ &= \frac{5}{8} - \left[-\frac{2}{4} + \left(-\frac{3}{4}\right)\right]\\ &= \frac{5}{8} - \left(-\frac{5}{4}\right)\\ &= \frac{5}{8} + \frac{5}{4}\\ &= \frac{5}{8} + \frac{10}{8}\\ &= \frac{15}{8}, \text{ or } 1\frac{7}{8}\end{aligned}$$

64. $$\begin{aligned}&\frac{9}{10} - \left(\frac{1}{8} - \frac{3}{10}\right)\\ &= \frac{9}{10} - \left[\frac{1}{8} + \left(-\frac{3}{10}\right)\right]\\ &= \frac{9}{10} - \left[\frac{5}{40} + \left(-\frac{12}{40}\right)\right]\\ &= \frac{9}{10} - \left(-\frac{7}{40}\right)\\ &= \frac{9}{10} + \frac{7}{40}\\ &= \frac{36}{40} + \frac{7}{40}\\ &= \frac{43}{40}, \text{ or } 1\frac{3}{40}\end{aligned}$$

65. $$\begin{aligned}3.4 - (-8.2) &= 3.4 + 8.2\\ &= 11.6\end{aligned}$$

66. $$\begin{aligned}5.7 - (-11.6) &= 5.7 + 11.6\\ &= 17.3\end{aligned}$$

67. $$\begin{aligned}-6.4 - 3.5 &= -6.4 + (-3.5)\\ &= -9.9\end{aligned}$$

68. $$\begin{aligned}-4.4 - 8.6 &= -4.4 + (-8.6)\\ &= -13\end{aligned}$$

69. $$\begin{aligned}(4 - 6) + 12 &= [4 + (-6)] + 12\\ &= [-2] + 12\\ &= 10\end{aligned}$$

70. $$\begin{aligned}(3 - 7) + 4 &= [3 + (-7)] + 4\\ &= [-4] + 4\\ &= 0\end{aligned}$$

71. $$\begin{aligned}(8 - 1) - 12 &= [8 + (-1)] + (-12)\\ &= [7] + (-12)\\ &= -5\end{aligned}$$

72. $$\begin{aligned}(9 - 3) - 15 &= [9 + (-3)] + (-15)\\ &= [6] + (-15)\\ &= -9\end{aligned}$$

73. $$\begin{aligned}6 - (-8 + 3) &= 6 - (-5)\\ &= 6 + 5\\ &= 11\end{aligned}$$

74. $$\begin{aligned}8 - (-9 + 5) &= 8 - (-4)\\ &= 8 + 4\\ &= 12\end{aligned}$$

75. $$\begin{aligned}2 + (-4 - 8) &= 2 + [-4 + (-8)]\\ &= 2 + [-12]\\ &= -10\end{aligned}$$

76. $$\begin{aligned}6 + (-9 - 2) &= 6 + [-9 + (-2)]\\ &= 6 + [-11]\\ &= -5\end{aligned}$$

77. $$\begin{aligned}|-5 - 6| + |9 + 2| &= |-5 + (-6)| + |11|\\ &= |-11| + |11|\\ &= -(-11) + 11\\ &= 11 + 11\\ &= 22\end{aligned}$$

78. $$\begin{aligned}|-4 + 8| + |6 - 1| &= |4| + |5|\\ &= 4 + 5\\ &= 9\end{aligned}$$

79. $$\begin{aligned}&|-8 - 2| - |-9 - 3|\\ &= |-8 + (-2)| - |-9 + (-3)|\\ &= |-10| - |-12|\\ &= -(-10) - [-(-12)]\\ &= 10 - [12]\\ &= -2\end{aligned}$$

80. $|-4-2|-|-8-1|$
$= |-4+(-2)|-|-8+(-1)|$
$= |-6|-|-9|$
$= -(-6)-[-(-9)]$
$= 6-[9]$
$= -3$

81. $\left(-\frac{3}{4}-\frac{5}{2}\right)-\left(-\frac{1}{8}-1\right)$
$= \left(-\frac{3}{4}-\frac{10}{4}\right)-\left(-\frac{1}{8}-\frac{8}{8}\right)$
$= -\frac{13}{4}-\left(-\frac{9}{8}\right)$
$= -\frac{26}{8}+\frac{9}{8}$
$= -\frac{17}{8}$, or $-2\frac{1}{8}$

82. $\left(-\frac{3}{8}-\frac{2}{3}\right)-\left(-\frac{9}{8}-3\right)$
$= \left(-\frac{9}{24}-\frac{16}{24}\right)-\left(-\frac{9}{8}-\frac{24}{8}\right)$
$= -\frac{25}{24}-\left(-\frac{33}{8}\right)$
$= -\frac{25}{24}+\frac{99}{24}$
$= \frac{74}{24}=\frac{37}{12}$, or $3\frac{1}{12}$

83. $\left(-\frac{1}{2}+0.25\right)-\left(-\frac{3}{4}+0.75\right)$
$= \left(-\frac{1}{2}+\frac{1}{4}\right)-\left(-\frac{3}{4}+\frac{3}{4}\right)$
$= \left(-\frac{2}{4}+\frac{1}{4}\right)-0$
$= -\frac{1}{4}$, or -0.25

84. $\left(-\frac{3}{2}-0.75\right)-\left(0.5-\frac{1}{2}\right)$
$= \left(-\frac{3}{2}-\frac{3}{4}\right)-\left(\frac{1}{2}-\frac{1}{2}\right)$
$= \left(-\frac{6}{4}-\frac{3}{4}\right)-0$
$= -\frac{9}{4}$, or -2.25

85. $-9+[(3-2)-(-4+2)]$
$= -9+[1-(-2)]$
$= -9+[1+2]$
$= -9+3$
$= -6$

86. $-8-[(-4-1)+(9-2)]$
$= -8-[(-4+(-1))]+7$
$= -8-[-5+7]$
$= -8-[2]$
$= -8+(-2)$
$= -10$

87. $-3+[(-5-8)-(-6+2)]$
$= -3+[(-5+(-8))-(-4)]$
$= -3+[-13+4]$
$= -3+[-9]$
$= -12$

88. $-4+[(-12+1)-(-1-9)]$
$= -4+[(-11)-(-1+(-9))]$
$= -4+[-11-(-10)]$
$= -4+[-11+10]$
$= -4+[-1]$
$= -5$

89. $-9.1237+[(-4.8099-3.2516)+11.27903]$
$= -9.1237+[(-4.8099+(-3.2516))+11.27903]$
$= -9.1237+[-8.0615+11.27903]$
$= -9.1237+3.21753$
$= -5.90617$

90. $-7.6247-[(-3.9928+1.42773)-(-2.80981)]$
$= -7.6247-[-2.56507+2.80981]$
$= -7.6247-[0.24474]$
$= -7.6247+(-0.24474)$
$= -7.86944$

91. "The sum of -5 and 12 and 6" is written $-5+12+6$.

$$\begin{aligned}-5+12+6 &= [-5+12]+6\\ &= 7+6=13\end{aligned}$$

92. "The sum of -3 and 5 and -12" is written $-3+5+(-12)$.

$$\begin{aligned}-3+5+(-12) &= 2+(-12)\\ &= -10\end{aligned}$$

93. "14 added to the sum of -19 and -4" is written $[-19+(-4)]+14$.

$$\begin{aligned}[-19+(-4)]+14 &= (-23)+14\\ &= -9\end{aligned}$$

94. "-2 added to the sum of -18 and 11" is written $(-18+11)+(-2)$.

$$\begin{aligned}(-18+11)+(-2) &= -7+(-2)\\ &= -9\end{aligned}$$

95. "The sum of -4 and -10, increased by 12" is written$[-4+(-10)]+12$.

$$\begin{aligned}[-4+(-10)]+12&=-14+12\\&=-2\end{aligned}$$

96. "The sum of -7 and -13, increased by 14" is written $[-7+(-13)]+14$.

$$\begin{aligned}[-7+(-13)]+14&=-20+14\\&=-6\end{aligned}$$

97. "$\frac{2}{7}$ more than the sum of $\frac{5}{7}$ and $-\frac{9}{7}$" is written $\left[\frac{5}{7}+\left(-\frac{9}{7}\right)\right]+\frac{2}{7}$.

$$\begin{aligned}\left[\frac{5}{7}+\left(-\frac{9}{7}\right)\right]+\frac{2}{7}&=-\frac{4}{7}+\frac{2}{7}\\&=-\frac{2}{7}\end{aligned}$$

98. "1.85 more than the sum of -1.25 and -4.75" is written $[-1.25+(-4.75)]+1.85$.

$$\begin{aligned}[-1.25+(-4.75)]+1.85&=-6+1.85\\&=-4.15\end{aligned}$$

99. "The difference between 4 and -8" is written $4-(-8)$.

$$4-(-8)=4+8=12$$

100. "The difference between 7 and -14" is written $7-(-14)$. This expression can be simplified as follows.

$$7-(-14)=7+14=21$$

101. "8 less than -2" is written $-2-8$.

$$-2-8=-2+(-8)=-10$$

102. "9 less than -13" is written $-13-9$.

$$-13-9=-13+(-9)=-22$$

103. "The sum of 9 and -4, decreased by 7" is written $[9+(-4)]-7$.

$$[9+(-4)]-7=5+(-7)=-2$$

104. "The sum of 12 and -7, decreased by 14" is written $[12+(-7)]-14$.

$$\begin{aligned}[12+(-7)]-14&=5-14\\&=5+(-14)\\&=-9\end{aligned}$$

105. "12 less than the difference between 8 and -5" is written $[8-(-5)]-12$.

$$\begin{aligned}[8-(-5)]-12&=[8+(5)]-12\\&=13-12\\&=13+(-12)\\&=1\end{aligned}$$

106. "19 less than the difference between 9 and -2" is written $[9-(-2)]-19$.

$$\begin{aligned}[9-(-2)]-19&=(9+2)-19\\&=11-19\\&=11+(-19)\\&=-8\end{aligned}$$

107. $$\begin{aligned}[-5+(-4)]+(-3)&=-9+(-3)\\&=-12\end{aligned}$$

The total number of seats that New York, Pennsylvania, and Ohio are projected to lose is twelve, which can be represented by the signed number -12.

108. $$\begin{aligned}&[-3+(-2)]+[9+5+3+2+2]\\&=[-5]+[14+3+2+2]\\&=-5+[17+2+2]\\&=-5+[19+2]\\&=-5+21\\&=16\end{aligned}$$

The algebraic sum of these changes can be represented by the signed number $+16$.

109. To find the low temperature, start with 44 and subtract 100.

$$\begin{aligned}44-100&=44+(-100)\\&=-56\end{aligned}$$

The low temperature was -56°F.

110. The lowest temperature is represented by -32. The highest temperature is represented by $-32+145$, or 113°F.

111. 33°F lower than -36°F can be represented as

$$\begin{aligned}-36-33&=-36+(-33)\\&=-69.\end{aligned}$$

The record low in Utah is -69°F.

112. $$\begin{aligned}14{,}494-(-282)&=14{,}494+282\\&=14{,}776\end{aligned}$$

The difference between these two elevations is 14,776 feet.

113. $$\begin{aligned}0+(-130)+(-54)&=-130+(-54)\\&=-184\end{aligned}$$

Their new altitude is 184 meters below the surface, which can be represented by the signed number -184.

114. $$\begin{aligned}34{,}000-2100&=34{,}000+(-2100)\\&=31{,}900\end{aligned}$$

The new altitude of the plane is 31,900 feet, which can be represented by the signed number $+31{,}900$.

115. **(a)** $6.9 - (-0.5) = 6.9 + 0.5$
$= 7.4$

The difference is 7.4%.

(b) Americans spent more money than they earned, which means they had to dip into savings or increase borrowing.

116. $236 - (-455) = 236 + 455$
$= 691$

The difference is \$691 billion.

117. $1879 + 869 - 579 + 1004$
$= 2748 - 579 + 1004$ *Add.*
$= 2169 + 1004$ *Subtract.*
$= 3173$ *Add.*

The average was \$3173.

118. $526 - 32 + 112 = 494 + 112$ *Subtract.*
$= 606$ *Add.*

The average was \$606.

119. Add the scores of the four turns to get the final score.

$$-19 + 28 + (-5) + 13 = 9 + (-5) + 13 = 4 + 13 = 17$$

Her final score for the four turns was 17.

120. Add the scores for the five turns to get the final score.

$$-13 + 15 + (-12) + 24 + 14 = 2 + (-12) + 24 + 14 = -10 + 24 + 14 = 14 + 14 = 28$$

His final score for the five turns was 28.

121. Sum of checks:
$\$35.84 + \$26.14 + \$3.12 = \$61.98 + \$3.12$
Sum of deposits: $= \$65.10$
$\$85.00 + \$120.76 = \$205.76$

Final balance = Beginning balance − checks + deposits
$= \$904.89 - \$65.10 + \$205.76$
$= \$839.79 + \$205.76$
$= \$1045.55$

Her account balance at the end of August was \$1045.55.

122. Sum of checks:

$$\$41.29 + \$13.66 + \$84.40 = \$54.95 + \$84.40 = \$139.35$$

Sum of deposits:

$$\$80.59 + \$276.13 = \$356.72$$

Final balance = Beginning balance − checks + deposits
$= \$904.89 - \$139.35 + \$356.72$
$= \$765.54 + \$356.72$
$= \$1122.26$

His account balance at the end of September was \$1122.26.

123.

−870.00	*amount owed*
+ 185.90	*2 return credits* (\$35.90 + \$150.00)
−684.10	
−102.50	*3 purchases* (\$82.50 + \$10.00 + \$10.00)
−786.60	
+ 500.00	*payment*
−286.60	
−37.23	*finance charge*
−323.83	

She still owes \$323.83.

124.

−679.00	*amount owed*
+ 179.84	*3 return credits* (\$36.89 + \$29.40 + \$113.55)
−499.16	
−588.66	*4 purchases* (\$135.78 + \$412.88 + \$20.00 + \$20.00)
−1087.82	
+ 400.00	*payment*
−687.82	
−24.57	*finance charge*
−712.39	

He still owes \$712.39.

125. The outlay for 2005 is \$38.7 billion and the outlay for 2006 is \$69.1 billion. Thus, the *change in outlay* is

$$69.1 - 38.7 = 69.1 + (-38.7) = 30.4$$

billion dollars (an increase).

126. The outlay for 2006 is \$69.1 billion and the outlay for 2007 is \$39.2 billion. Thus, the *change in outlay* is

$$39.2 - 69.1 = 39.2 + (-69.1) = -29.9$$

billion dollars (a decrease since it is negative).

127. The outlay for 2007 is \$39.2 billion and the outlay for 2008 is \$42.3 billion. Thus, the *change in outlay* is

$$42.3 - 39.2 = 42.3 + (-39.2) = 3.1$$

billion dollars (an increase).

128. The outlay for 2005 is \$38.7 billion and the outlay for 2008 is \$42.3 billion. Thus, the *change in outlay* is

$$42.3 - 38.7 = 42.3 + (-38.7) = 3.6$$

billion dollars (an increase).

129. $17{,}400 - (-32{,}995) = 17{,}400 + 32{,}995 = 50{,}395$

The difference between the height of Mt. Foraker and the depth of the Philippine Trench is 50,395 feet.

130. $14{,}110 - (-23{,}376) = 14{,}110 + 23{,}376 = 37{,}486$

The difference between the height of Pikes Peak and the depth of the Java Trench is 37,486 feet.

131. $-23{,}376 - (-24{,}721) = -23{,}376 + 24{,}721 = 1345$

The Cayman Trench is 1345 feet deeper than the Java Trench.

132. $-24{,}721 - (-32{,}995) = -24{,}721 + 32{,}995 = 8274$

The Philippine Trench is 8274 feet deeper than the Cayman Trench.

133. $14{,}246 - 14{,}110 = 14{,}246 + (-14{,}110) = 136$

Mt. Wilson is 136 feet higher than Pikes Peak.

134. $(14{,}246 + 14{,}110) - 17{,}400 = 28{,}356 + (-17{,}400) = 10{,}956$

If Mt. Wilson and Pikes Peak were stacked one on top of the other, they would be 10,956 feet higher than Mt. Foraker.

1.6 Multiplying and Dividing Real Numbers

1.6 Classroom Examples, Now Try Exercises

1. **(a)** $-16\left(\frac{5}{32}\right) = -\left(\frac{\overset{1}{\cancel{16}}}{1} \cdot \frac{5}{\underset{2}{\cancel{32}}}\right) = -\frac{5}{2}$

(b) $4.56(-2) = -(4.56 \cdot 2) = -9.12$

N1. **(a)** $-11(9) = -(11 \cdot 9) = -99$

(b) $3.1(-2.5) = -(3.1 \cdot 2.5) = -7.75$

2. $-\frac{3}{4}\left(-\frac{2}{5}\right) = \frac{3}{4} \cdot \frac{2}{5} = \frac{3 \cdot 2}{4 \cdot 5} = \frac{3 \cdot 2}{2 \cdot 2 \cdot 5} = \frac{3}{10}$

N2. $-\frac{1}{7}\left(-\frac{5}{2}\right) = \frac{1}{7} \cdot \frac{5}{2} = \frac{1 \cdot 5}{7 \cdot 2} = \frac{5}{14}$

3. **(a)** $\frac{-36}{6} = -36 \cdot \frac{1}{6} = -6$

(b) $\frac{-12.56}{-0.4} = -12.56\left(-\frac{1}{0.4}\right) = 31.4$

(c) $\frac{10}{7} \div \left(-\frac{24}{5}\right) = \frac{\overset{5}{\cancel{10}}}{7}\left(-\frac{5}{\underset{12}{\cancel{24}}}\right) = -\frac{25}{84}$

N3. **(a)** $\frac{15}{-3} = 15 \cdot \left(-\frac{1}{3}\right) = -5$

(b) $\frac{9.81}{-0.9} = 9.81\left(-\frac{1}{0.9}\right) = -10.9$

(c) $-\frac{5}{6} \div \frac{17}{9} = -\frac{5}{\underset{2}{\cancel{6}}} \cdot \frac{\overset{3}{\cancel{9}}}{17} = -\frac{15}{34}$

4. **(a)** $\frac{-16}{-2} = 8$

(b) $\frac{-16.4}{2.05} = -8$

(c) $\frac{1}{4} \div \left(-\frac{2}{3}\right) = \frac{1}{4} \cdot \left(-\frac{3}{2}\right) = -\frac{3}{8}$

N4. **(a)** $\frac{-10}{5} = -2$

(b) $\frac{-1.44}{-0.12} = 12$

(c) $-\frac{3}{8} \div \frac{7}{10} = -\frac{3}{\underset{4}{\cancel{8}}} \cdot \frac{\overset{5}{\cancel{10}}}{7} = -\frac{15}{28}$

5. **(a)** $-3(4) - 2(-6) = -12 - (-12) = -12 + 12 = 0$

(b) $\frac{-6(-8) + -3(9)}{-2[4 - (-3)]} = \frac{48 - 27}{-2(4 + 3)} = \frac{21}{-2(7)} = \frac{21}{-14} = -\frac{3}{2}$

N5. **(a)** $-4(6) - (-5)5 = -24 - (-25) = -24 + 25 = 1$

(b) $$\frac{12(-4)-6(-3)}{-4(7-16)} = \frac{-48-(-18)}{-4(-9)} = \frac{-48+18}{36} = \frac{-30}{36} = -\frac{5}{6}$$

6. Replace x with -2 and y with -3.

$$\begin{aligned}2x^2-4y^2 &= 2(-2)^2-4(-3)^2\\ &= 2(4)-4(9)\\ &= 8-36\\ &= -28\end{aligned}$$

N6. Replace x with -4 and y with -3.

$$\frac{3x^2-12}{y} = \frac{3(-4)^2-12}{-3} = \frac{3(16)-12}{-3} = \frac{48-12}{-3} = \frac{36}{-3} = -12$$

7. **(a)** "Three times the difference between 4 and -11" is written $3[4-(-11)]$.

$$3[4-(-11)] = 3(15) = 45$$

(b) "Three-fifths of the sum of 2 and -7" is written $\frac{3}{5}[2+(-7)]$.

$$\tfrac{3}{5}[2+(-7)] = \tfrac{3}{5}(-5) = -3$$

N7. **(a)** "Twice the sum of -10 and 7" is written $2(-10+7)$.

$$2(-10+7) = 2(-3) = -6$$

(b) "40% of the difference between 45 and 15" is written $0.40(45-15)$.

$$0.40(45-15) = 0.40(30) = 12$$

8. "The product of -9 and 2, divided by the difference between 5 and -1" is written $\frac{-9(2)}{5-(-1)}$.

$$\frac{-9(2)}{5-(-1)} = \frac{-18}{6} = -3$$

N8. "The quotient of 21 and the sum of 10 and -7" is written $\frac{21}{10+(-7)}$.

$$\frac{21}{10+(-7)} = \frac{21}{3} = 7$$

9. **(a)** "The quotient of a number and -2 is 6" is written

$$\frac{x}{-2} = 6.$$

Here, x must be a negative number, since the denominator is negative and the quotient is positive. Since $\frac{-12}{-2} = 6$, the solution is -12.

(b) "Twice a number is -6" is written

$$2x = -6.$$

Since $2(-3) = -6$, the solution is -3.

N9. **(a)** "The sum of a number and -4 is 7" is written

$$x + (-4) = 7.$$

Here, x must be 4 more than 7, so the solution is 11.

$$11 + (-4) = 7, \ldots$$

(b) "The difference between -8 and a number is -11" is written

$$-8 - x = -11.$$

If we start at -8 on a number line, we must move 3 units to the left to get to -11, so the solution is 3.

1.6 Section Exercises

1. The product or the quotient of two numbers with the same sign is <u>greater than 0</u>, since the product or quotient of two positive numbers is positive and the product or quotient of two negative numbers is positive.

2. The product or quotient of two numbers with different signs is <u>less than 0</u>, since the product or quotient is negative.

3. If three negative numbers are multiplied, the product is <u>less than 0</u>, since a negative number times a negative number is a positive number, and that positive number times a negative number is a negative number.

4. If two negative numbers are multiplied and then their product is divided by a negative number, the result is <u>less than 0</u>, since the product is a positive number, and a positive number divided by a negative number is a negative number.

5. If a negative number is squared and the result is added to a positive number, the result is <u>greater than 0</u>, since a negative number squared is a positive number, and a positive number added to another positive number is a positive number.

6. The reciprocal of a negative number is <u>less than 0</u>, since it is just the number one divided by a negative number, which is negative.

7. If three positive numbers, five negative numbers, and zero are multiplied, the product is <u>equal to 0</u>. Since one of the numbers is zero, the product is zero (regardless of what the other numbers are).

8. The cube power of a negative number is <u>less than 0</u>. Remember, a negative number raised to an odd power (like 3) is negative.

9. The quotient formed by any nonzero number divided by 0 is <u>undefined</u>, and the quotient formed by 0 divided by any nonzero number is <u>0</u>. Examples include $\frac{1}{0}$, which is undefined, and $\frac{0}{1}$, which equals 0.

10. Look for the expression that has 0 in the denominator. The expression $\frac{4-4}{4-4}$, or $\frac{0}{0}$, is undefined. The correct response is **C**.

11. $5(-6) = -(5 \cdot 6) = -30$

Note that the product of a positive number and a negative number is negative.

12. $-3(4) = -(3 \cdot 4) = -12$

Note that the product of a negative number and a positive number is negative.

13. $-5(-6) = 5 \cdot 6 = 30$

Note that the product of two negative numbers is positive.

14. $-3(-4) = 3 \cdot 4 = 12$

Note that the product of two negative numbers is positive.

15. $-10(-12) = 10 \cdot 12 = 120$

16. $-9(-5) = 9 \cdot 5 = 45$

17. $3(-11) = -(3 \cdot 11) = -33$

18. $3(-15) = -(3 \cdot 15) = -45$

19. $-0.5(0) = 0$

20. $-0.3(0) = 0$

21. $-6.8(0.35) = -(6.8 \cdot 0.35) = -2.38$

22. $-4.6(0.24) = -(4.6 \cdot 0.24) = -1.104$

23.
$$\begin{aligned} -\frac{3}{8} \cdot \left(-\frac{10}{9}\right) &= \frac{3}{8}\left(\frac{10}{9}\right) \\ &= \frac{3 \cdot 10}{8 \cdot 9} \\ &= \frac{3 \cdot (2 \cdot 5)}{(4 \cdot 2) \cdot (3 \cdot 3)} \\ &= \frac{3 \cdot 2 \cdot 5}{4 \cdot 2 \cdot 3 \cdot 3} \\ &= \frac{5}{4 \cdot 3} = \frac{5}{12} \end{aligned}$$

24.
$$\begin{aligned} -\frac{5}{4} \cdot \left(-\frac{5}{8}\right) &= \frac{5}{4} \cdot \frac{5}{8} \\ &= \frac{5 \cdot 5}{4 \cdot 8} \\ &= \frac{25}{32} \end{aligned}$$

25.
$$\begin{aligned} \frac{2}{15}\left(-1\frac{1}{4}\right) &= \frac{2}{15}\left(-\frac{5}{4}\right) \\ &= -\frac{2 \cdot 5}{15 \cdot 4} \\ &= -\frac{2 \cdot 5}{3 \cdot 5 \cdot 2 \cdot 2} \\ &= -\frac{1}{3 \cdot 2} = -\frac{1}{6} \end{aligned}$$

26.
$$\begin{aligned} \frac{3}{7}\left(-1\frac{5}{9}\right) &= \frac{3}{7}\left(-\frac{14}{9}\right) \\ &= -\frac{3 \cdot (2 \cdot 7)}{7 \cdot (3 \cdot 3)} \\ &= -\frac{2}{3} \end{aligned}$$

27. $-8\left(-\frac{3}{4}\right) = 8\left(\frac{3}{4}\right) = \frac{24}{4} = 6$

28. $-6\left(-\frac{5}{3}\right) = 6\left(\frac{5}{3}\right) = \frac{30}{3} = 10$

29. Using only positive integer factors, 32 can be written as $1 \cdot 32$, $2 \cdot 16$, or $4 \cdot 8$. Including the negative integer factors, we see that the integer factors of 32 are -32, -16, -8, -4, -2, -1, 1, 2, 4, 8, 16, and 32.

30. The integer factors of 36 are -36, -18, -12, -9, -6, -4, -3, -2, -1, 1, 2, 3, 4, 6, 9, 12, 18, and 36.

31. The integer factors of 40 are -40, -20, -10, -8, -5, -4, -2, -1, 1, 2, 4, 5, 8, 10, 20, and 40.

32. The integer factors of 50 are -50, -25, -10, -5, -2, -1, 1, 2, 5, 10, 25, and 50.

33. The integer factors of 31 are -31, -1, 1, and 31.

34. The integer factors of 17 are -17, -1, 1, and 17.

35. $\frac{15}{5} = \frac{5 \cdot 3}{5} = \frac{3}{1} = 3$

36. $\frac{35}{5} = \frac{7 \cdot 5}{5} = \frac{7}{1} = 7$

37. $\frac{-42}{6} = -\frac{2 \cdot 3 \cdot 7}{2 \cdot 3} = -7$

Note that the quotient of two numbers having different signs is negative.

38. $\dfrac{-28}{7} = -\dfrac{4 \cdot 7}{7} = -4$

Note that the quotient of two numbers having different signs is negative.

39. $\dfrac{-32}{-4} = \dfrac{4 \cdot 8}{4} = 8$

Note that the quotient of two numbers having the same sign is positive.

40. $\dfrac{-35}{-5} = \dfrac{5 \cdot 7}{5} = 7$

Note that the quotient of two numbers having the same sign is positive.

41. $\dfrac{96}{-16} = -\dfrac{6 \cdot 16}{16} = -6$

42. $\dfrac{38}{-19} = -\dfrac{2 \cdot 19}{19} = -2$

43. Dividing by a fraction (in this case, $-\frac{1}{8}$) is the same as multiplying by the reciprocal of the fraction (in this case, $-\frac{8}{1}$).

$$\begin{aligned}\left(-\frac{4}{3}\right) \div \left(-\frac{1}{8}\right) &= \left(-\frac{4}{3}\right) \cdot \left(-\frac{8}{1}\right) \\ &= \frac{4 \cdot 8}{3 \cdot 1} \\ &= \frac{32}{3}, \text{ or } 10\frac{2}{3}\end{aligned}$$

44. Dividing by a fraction (in this case, $-\frac{1}{3}$) is the same as multiplying by the reciprocal of the fraction (in this case, $-\frac{3}{1}$).

$$\begin{aligned}\left(-\frac{6}{5}\right) \div \left(-\frac{1}{3}\right) &= \left(-\frac{6}{5}\right) \cdot \left(-\frac{3}{1}\right) \\ &= \frac{6 \cdot 3}{5 \cdot 1} \\ &= \frac{18}{5}, \text{ or } 3\frac{3}{5}\end{aligned}$$

45. $\dfrac{-8.8}{2.2} = -\dfrac{4(2.2)}{2.2} = -4$

46. $\dfrac{-4.6}{0.23} = -20$

Note that dividing by a number with absolute value between 0 and 1 gives us a number *larger* than the original numerator.

47. $\dfrac{0}{-5} = 0,$ because 0 divided by any nonzero number is 0.

48. $\dfrac{0}{-9} = 0,$ because 0 divided by any nonzero number is 0.

49. $\dfrac{11.5}{0}$ is *undefined* because we cannot divide by 0.

50. $\dfrac{15.2}{0}$ is *undefined* because we cannot divide by 0.

In Exercises 51–68, use the order of operations.

51. $\begin{aligned}7 - 3 \cdot 6 &= 7 - 18 \\ &= -11\end{aligned}$

52. $\begin{aligned}8 - 2 \cdot 5 &= 8 - 10 \\ &= -2\end{aligned}$

53. $\begin{aligned}-10 - (-4)(2) &= -10 - (-8) \\ &= -10 + 8 \\ &= -2\end{aligned}$

54. $\begin{aligned}-11 - (-3)(6) &= -11 - (-18) \\ &= -11 + 18 \\ &= 7\end{aligned}$

55. $\begin{aligned}-7(3 - 8) &= -7[3 + (-8)] \\ &= -7(-5) = 35\end{aligned}$

56. $\begin{aligned}-5(4 - 7) &= -5[4 + (-7)] \\ &= -5(-3) = 15\end{aligned}$

57. $\begin{aligned}7 + 2(4 - 1) &= 7 + 2(3) \\ &= 7 + 6 \\ &= 13\end{aligned}$

58. $\begin{aligned}5 + 3(6 - 4) &= 5 + 3(2) \\ &= 5 + 6 \\ &= 11\end{aligned}$

59. $\begin{aligned}-4 + 3(2 - 8) &= -4 + 3(-6) \\ &= -4 - 18 \\ &= -22\end{aligned}$

60. $\begin{aligned}-8 + 4(5 - 7) &= -8 + 4(-2) \\ &= -8 - 8 \\ &= -16\end{aligned}$

61. $\begin{aligned}(12 - 14)(1 - 4) &= (-2)(-3) \\ &= 6\end{aligned}$

62. $\begin{aligned}(8 - 9)(4 - 12) &= (-1)(-8) \\ &= 8\end{aligned}$

63. $\begin{aligned}(7 - 10)(10 - 4) &= (-3)(6) \\ &= -18\end{aligned}$

64. $\begin{aligned}(5 - 12)(19 - 4) &= -7(15) \\ &= -105\end{aligned}$

65. $\begin{aligned}(-2 - 8)(-6) + 7 &= (-10)(-6) + 7 \\ &= 60 + 7 \\ &= 67\end{aligned}$

66. $$\begin{aligned}(-9-4)(-2)+10 &= (-13)(-2)+10\\ &= 26+10\\ &= 36\end{aligned}$$

67. $$\begin{aligned}3(-5)+|3-10| &= -15+|-7|\\ &= -15+7\\ &= -8\end{aligned}$$

68. $$\begin{aligned}4(-8)+|4-15| &= -32+|-11|\\ &= -32+11\\ &= -21\end{aligned}$$

69. $$\begin{aligned}\frac{-5(-6)}{9-(-1)} &= \frac{30}{10}\\ &= \frac{3\cdot 10}{10} = 3\end{aligned}$$

70. $$\begin{aligned}\frac{-12(-5)}{7-(-5)} &= \frac{60}{12}\\ &= \frac{5\cdot 12}{12} = 5\end{aligned}$$

71. $$\begin{aligned}\frac{-21(3)}{-3-6} &= \frac{-63}{-3+(-6)}\\ &= \frac{-63}{-9} = 7\end{aligned}$$

72. $$\begin{aligned}\frac{-40(3)}{-2-3} &= \frac{-120}{-5}\\ &= \frac{5\cdot 24}{5} = 24\end{aligned}$$

73. $$\begin{aligned}\frac{-10(2)+6(2)}{-3-(-1)} &= \frac{-20+12}{-3+1}\\ &= \frac{-8}{-2} = 4\end{aligned}$$

74. $$\begin{aligned}\frac{-12(4)+5(3)}{-14-(-3)} &= \frac{-48+15}{-14+3}\\ &= \frac{-33}{-11} = 3\end{aligned}$$

75. $$\frac{3^2-4^2}{7(-8+9)} = \frac{9-16}{7(1)} = \frac{-7}{7} = -1$$

76. $$\begin{aligned}\frac{5^2-7^2}{2(3+3)} &= \frac{25-49}{2(6)}\\ &= \frac{-24}{2(6)}\\ &= -\frac{2\cdot 2\cdot 6}{2\cdot 6}\\ &= -2\end{aligned}$$

77. $$\begin{aligned}\frac{8(-1)-|(-4)(-3)|}{-6-(-1)} &= \frac{-8-|12|}{-6+1}\\ &= \frac{-8-12}{-5}\\ &= \frac{-20}{-5} = 4\end{aligned}$$

78. $$\begin{aligned}\frac{-27(-2)-|6\cdot 4|}{-2(3)-2(2)} &= \frac{54-|24|}{-6-4}\\ &= \frac{54-24}{-10}\\ &= \frac{30}{-10}\\ &= -3\end{aligned}$$

79. $$\begin{aligned}&\frac{-13(-4)-(-8)(-2)}{(-10)(2)-4(-2)}\\ &= \frac{52-16}{-20-(-8)}\\ &= \frac{36}{-20+8}\\ &= \frac{36}{-12} = -3\end{aligned}$$

80. $$\begin{aligned}\frac{-5(2)+[3(-2)-4]}{-3-(-1)} &= \frac{-10+[-6+(-4)]}{-3+(1)}\\ &= \frac{-10+[-10]}{-2}\\ &= \frac{-20}{-2} = 10\end{aligned}$$

In Exercises 81–92, replace x with 6, y with -4, and a with 3. Then use the order of operations to evaluate the expression.

81. $$\begin{aligned}5x-2y+3a &= 5(6)-2(-4)+3(3)\\ &= 30-(-8)+9\\ &= 30+8+9\\ &= 38+9\\ &= 47\end{aligned}$$

82. $$\begin{aligned}6x-5y+4a &= 6(6)-5(-4)+4(3)\\ &= 36+20+12\\ &= 56+12\\ &= 68\end{aligned}$$

83. $$\begin{aligned}(2x+y)(3a) &= [2(6)+(-4)][3(3)]\\ &= [12+(-4)](9)\\ &= (8)(9)\\ &= 72\end{aligned}$$

84. $$\begin{aligned}(5x-2y)(-2a) &= [5(6)-2(-4)][-2(3)]\\ &= (30+8)(-6)\\ &= (38)(-6)\\ &= -228\end{aligned}$$

85. $$\begin{aligned}&\left(\frac{1}{3}x-\frac{4}{5}y\right)\left(-\frac{1}{5}a\right)\\ &= \left[\frac{1}{3}(6)-\frac{4}{5}(-4)\right]\left[-\frac{1}{5}(3)\right]\\ &= \left[2-\left(-\frac{16}{5}\right)\right]\left(-\frac{3}{5}\right)\\ &= \left(2+\frac{16}{5}\right)\left(-\frac{3}{5}\right)\\ &= \left(\frac{10}{5}+\frac{16}{5}\right)\left(-\frac{3}{5}\right)\\ &= \left(\frac{26}{5}\right)\left(-\frac{3}{5}\right)\\ &= -\frac{78}{25}\end{aligned}$$

86. $$\begin{aligned}&\left(\frac{5}{6}x+\frac{3}{2}y\right)\left(-\frac{1}{3}a\right)\\ &= \left[\frac{5}{6}(6)+\frac{3}{2}(-4)\right]\left[-\frac{1}{3}(3)\right]\\ &= [5+(-6)](-1)\\ &= (-1)(-1)\\ &= 1\end{aligned}$$

87. $$\begin{aligned}&(-5+x)(-3+y)(3-a)\\ &= (-5+6)[-3+(-4)][3-3]\\ &= (1)(-7)(0)\\ &= 0\end{aligned}$$

88. $$\begin{aligned}&(6-x)(5+y)(3+a)\\ &= (6-6)[5+(-4)](3+3)\\ &= 0(1)(6)\\ &= 0\end{aligned}$$

89. $$\begin{aligned}-2y^2+3a &= -2(-4)^2+3(3)\\ &= -2(16)+9\\ &= -32+9\\ &= -23\end{aligned}$$

90. $$\begin{aligned}5x-4a^2 &= 5(6)-4(3)^2\\ &= 30-4(9)\\ &= 30-36\\ &= -6\end{aligned}$$

91. $$\begin{aligned}\frac{2y^2-x}{a+10} &= \frac{2(-4)^2-(6)}{3+10}\\ &= \frac{2(16)-6}{13}\\ &= \frac{32-6}{13}\\ &= \frac{26}{13}\\ &= 2\end{aligned}$$

92. $$\begin{aligned}\frac{xy+8a}{x-y} &= \frac{6(-4)+8(3)}{6-(-4)}\\ &= \frac{-24+24}{6+4}\\ &= \frac{0}{10}\\ &= 0\end{aligned}$$

93. "The product of -9 and 2, added to 9" is written $9+(-9)(2)$.

$$\begin{aligned}9+(-9)(2) &= 9+(-18)\\ &= -9\end{aligned}$$

94. "The product of 4 and -7, added to -12" is written $-12+4(-7)$.

$$\begin{aligned}-12+4(-7) &= -12+(-28)\\ &= -40\end{aligned}$$

95. "Twice the product of -1 and 6, subtracted from -4" is written $-4-2[(-1)(6)]$.

$$\begin{aligned}-4-2[(-1)(6)] &= -4-2(-6)\\ &= -4-(-12)\\ &= -4+12=8\end{aligned}$$

96. "Twice the product of -8 and 2, subtracted from -1" is written $-1-2(-8)(2)$.

$$\begin{aligned}-1-2[(-8)(2)] &= -1-2(-16)\\ &= -1-(-32)\\ &= -1+32\\ &= 31\end{aligned}$$

97. "Nine subtracted from the product of 1.5 and -3.2 is written $(1.5)(-3.2)-9$.

$$\begin{aligned}(1.5)(-3.2)-9 &= -4.8-9\\ &= -4.8+(-9)\\ &= -13.8\end{aligned}$$

98. "Three subtracted from the product of 4.2 and -8.5 is written $(4.2)(-8.5) - 3$.

$$\begin{aligned}(4.2)(-8.5) - 3 &= -35.7 - 3\\ &= -35.7 + (-3)\\ &= -38.7\end{aligned}$$

99. "The product of 12 and the difference between 9 and -8" is written $12[9 - (-8)]$.

$$\begin{aligned}12[9 - (-8)] &= 12[9 + 8]\\ &= 12(17) = 204\end{aligned}$$

100. "The product of -3 and the difference between 3 and -7" is written $-3[3 - (-7)]$.

$$\begin{aligned}-3[3 - (-7)] &= -3[3 + 7]\\ &= -3(10) = -30\end{aligned}$$

101. "The quotient of -12 and the sum of -5 and -1" is written

$$\frac{-12}{-5 + (-1)},$$

and

$$\frac{-12}{-5 + (-1)} = \frac{-12}{-6} = 2.$$

102. "The quotient of -20 and the sum of -8 and -2" is written

$$\frac{-20}{-8 + (-2)},$$

and

$$\frac{-20}{-8 + (-2)} = \frac{-20}{-10} = 2.$$

103. "The sum of 15 and -3, divided by the product of 4 and -3" is written

$$\frac{15 + (-3)}{4(-3)},$$

and

$$\frac{15 + (-3)}{4(-3)} = \frac{12}{-12} = -1.$$

104. "The sum of -18 and -6, divided by the product of 2 and -4" is written

$$\frac{-18 + (-6)}{2(-4)},$$

and

$$\frac{-18 + (-6)}{2(-4)} = \frac{-24}{-8} = 3.$$

105. "Two-thirds of the difference between 8 and -1" is written

$$\tfrac{2}{3}[8 - (-1)],$$

and

$$\tfrac{2}{3}[8 - (-1)] = \tfrac{2}{3}[8 + (1)] = \tfrac{2}{3}[9] = 6.$$

106. "Three-fourths of the sum of -8 and 12" is written

$$\tfrac{3}{4}(-8 + 12),$$

and

$$\tfrac{3}{4}(-8 + 12) = \tfrac{3}{4}(4) = 3.$$

107. "20% of the product of -5 and 6" is written

$$0.20(-5 \cdot 6),$$

and

$$0.20(-5 \cdot 6) = 0.20(-30) = -6.$$

108. "30% of the product of -8 and 5" is written

$$0.30(-8 \cdot 5),$$

and

$$0.30(-8 \cdot 5) = 0.30(-40) = -12.$$

109. "The sum of $\frac{1}{2}$ and $\frac{5}{8}$, times the difference between $\frac{3}{5}$ and $\frac{1}{3}$" is written

$$\left(\tfrac{1}{2} + \tfrac{5}{8}\right)\left(\tfrac{3}{5} - \tfrac{1}{3}\right),$$

and

$$\begin{aligned}\left(\tfrac{1}{2} + \tfrac{5}{8}\right)\left(\tfrac{3}{5} - \tfrac{1}{3}\right) &= \left(\tfrac{4}{8} + \tfrac{5}{8}\right)\left(\tfrac{9}{15} - \tfrac{5}{15}\right)\\ &= \tfrac{9}{8}\left(\tfrac{4}{15}\right)\\ &= \frac{3 \cdot 3 \cdot 4}{2 \cdot 4 \cdot 3 \cdot 5} = \frac{3}{10}.\end{aligned}$$

110. "The sum of $\frac{3}{4}$ and $\frac{1}{2}$, times the difference between $\frac{2}{3}$ and $\frac{1}{6}$" is written

$$\left(\tfrac{3}{4} + \tfrac{1}{2}\right)\left(\tfrac{2}{3} - \tfrac{1}{6}\right),$$

and

$$\begin{aligned}\left(\tfrac{3}{4} + \tfrac{1}{2}\right)\left(\tfrac{2}{3} - \tfrac{1}{6}\right) &= \left(\tfrac{3}{4} + \tfrac{2}{4}\right)\left(\tfrac{4}{6} - \tfrac{1}{6}\right)\\ &= \tfrac{5}{4}\left(\tfrac{3}{6}\right)\\ &= \frac{5 \cdot 3}{4 \cdot 2 \cdot 3} = \frac{5}{8}.\end{aligned}$$

111. "The product of $-\frac{1}{2}$ and $\frac{3}{4}$, divided by $-\frac{2}{3}$" is written $\dfrac{-\frac{1}{2}\left(\frac{3}{4}\right)}{-\frac{2}{3}}$. Simplifying gives us:

$$\begin{aligned}\frac{-\frac{1}{2}\left(\frac{3}{4}\right)}{-\frac{2}{3}} &= \frac{-\frac{3}{8}}{-\frac{2}{3}}\\ &= -\frac{3}{8} \cdot \left(-\frac{3}{2}\right)\\ &= \frac{9}{16}\end{aligned}$$

112. "The product of $-\frac{2}{3}$ and $-\frac{1}{5}$, divided by $\frac{1}{7}$" is written $\dfrac{-\frac{2}{3}\left(-\frac{1}{5}\right)}{\frac{1}{7}}$. Simplifying gives us:

$$\begin{aligned}\frac{-\frac{2}{3}\left(-\frac{1}{5}\right)}{\frac{1}{7}} &= \frac{\frac{2}{15}}{\frac{1}{7}}\\ &= \frac{2}{15} \cdot \frac{7}{1}\\ &= \frac{14}{15}\end{aligned}$$

113. "The quotient of a number and 3 is -3" is written

$$\frac{x}{3} = -3.$$

The solution is -9, since

$$\frac{-9}{3} = -3.$$

114. "The quotient of a number and 4 is -1" is written

$$\frac{x}{4} = -1.$$

The solution is -4, since

$$\frac{-4}{4} = -1.$$

115. "6 less than a number is 4" is written

$$x - 6 = 4.$$

The solution is 10, since

$$10 - 6 = 4.$$

116. "7 less than a number is 2" is written

$$x - 7 = 2.$$

The solution is 9, since

$$9 - 7 = 2.$$

117. "When 5 is added to a number, the result is -5" is written

$$x + 5 = -5.$$

The solution is -10, since

$$-10 + 5 = -5.$$

118. "When 6 is added to a number, the result is -3" is written

$$x + 6 = -3.$$

The solution is -9, since

$$-9 + 6 = -3.$$

119. Add the numbers and divide by 5.

$$\frac{(23 + 18 + 13) + [(-4) + (-8)]}{5}$$
$$= \frac{54 - 12}{5}$$
$$= \frac{42}{5}, \text{ or } 8\frac{2}{5}$$

120. Add the numbers and divide by 5.

$$\frac{(18 + 12) + 0 + [(-4) + (-10)]}{5}$$
$$= \frac{30 - 14}{5}$$
$$= \frac{16}{5}, \text{ or } 3\frac{1}{5}$$

121. Add the numbers and divide by 4.

$$\frac{(29 + 8) + [(-15) + (-6)]}{4}$$
$$= \frac{37 - 21}{4}$$
$$= \frac{16}{4} = 4$$

122. Add the numbers and divide by 4.

$$\frac{(34 + 9) + [(-17) + (-2)]}{4}$$
$$= \frac{43 - 19}{4}$$
$$= \frac{24}{4} = 6$$

123. Add the integers from -10 to 14.

$$(-10) + (-9) + \cdots + 14 = 50$$

[the 3 dots indicate that the pattern continues]

There are 25 integers from -10 to 14 (10 negative, zero, and 14 positive). Thus, the average is

$$\frac{50}{25} = 2.$$

124. Add the even integers from -18 to 4.

$$(-18) + (-16) + \cdots + 4 = -84$$

There are 12 integers from -18 to 4 (9 negative, zero, and 2 positive). Thus, the average is

$$\frac{-84}{12} = -7.$$

125. **(a)** 3,473,986 is divisible by 2 because its last digit, 6, is divisible by 2.

(b) 4,336,879 is not divisible by 2 because its last digit, 9, is not divisible by 2.

126. **(a)** 4,799,232 is divisible by 3 because the sum of its digits,

$$4 + 7 + 9 + 9 + 2 + 3 + 2 = 36,$$

is divisible by 3.

(b) 2,443,871 is not divisible by 3 because the sum of its digits,

$$2 + 4 + 4 + 3 + 8 + 7 + 1 = 29,$$

is not divisible by 3.

127. **(a)** 6,221,464 is divisible by 4 because the number formed by its last two digits, 64, is divisible by 4.

(b) 2,876,335 is not divisible by 4 because the number formed by its last two digits, 35, is not divisible by 4.

128. **(a)** 3,774,595 is divisible by 5 because its last digit, 5, is divisible by 5.

(b) 9,332,123 is not divisible by 5 because its last digit, 3, is not divisible by 5.

129. **(a)** 1,524,822 is divisible by 2 because its last digit, 2, is divisible by 2. It is also divisible by 3 because the sum of its digits,

$$1+5+2+4+8+2+2=24,$$

is divisible by 3.

Because 1,524,822 is divisible by *both* 2 and 3, it is divisible by 6.

(b) 2,873,590 is divisible by 2 because its last digit, 0, is divisible by 2. However, it is not divisible by 3 because the sum of its digits,

$$2+8+7+3+5+9+0=34,$$

is not divisible by 3.

Because 2,873,590 is not divisible by *both* 2 and 3, it is not divisible by 6.

130. **(a)** 2,923,296 is divisible by 8 because the number formed by its last three digits, 296, is divisible by 8.

(b) 7,291,623 is not divisible by 8 because the number formed by its last three digits, 623, is not divisible by 8.

131. **(a)** 4,114,107 is divisible by 9 because the sum of its digits,

$$4+1+1+4+1+0+7=18,$$

is divisible by 9.

(b) 2,287,321 is not divisible by 9 because the sum of its digits,

$$2+2+8+7+3+2+1=25,$$

is not divisible by 9.

132. **(a)** 4,253,520 is divisible by 3 because the sum of its digits,

$$4+2+5+3+5+2+0=21,$$

is divisible by 3. It is also divisible by 4 because the number formed by its last two digits, 20, is divisible by 4.

Because 4,253,520 is divisible by *both* 3 and 4, it is divisible by 12.

(b) 4,249,474 is not divisible by 3 because the sum of its digits,

$$4+2+4+9+4+7+4=34,$$

is not divisible by 3. Because a number is not divisible by 12 unless it is divisible by *both* 3 and 4, this is sufficient to show that the number is not divisible by 12.

Summary Exercises on Operations with Real Numbers

1. $$\begin{aligned}14-3\cdot 10 &= 14-30\\ &= 14+(-30)\\ &= -16\end{aligned}$$

2. $$\begin{aligned}-3(8)-4(-7) &= -24-(-28)\\ &= -24+28\\ &= 4\end{aligned}$$

3. $$\begin{aligned}(3-8)(-2)-10 &= (-5)(-2)-10\\ &= 10-10\\ &= 0\end{aligned}$$

4. $$\begin{aligned}-6(7-3) &= -6(4)\\ &= -24\end{aligned}$$

5. $$\begin{aligned}7+3(2-10) &= 7+3(-8)\\ &= 7-24\\ &= -17\end{aligned}$$

6. $$\begin{aligned}-4[(-2)(6)-7] &= -4[-12-7]\\ &= -4[-19]\\ &= 76\end{aligned}$$

7. $$\begin{aligned}(-4)(7)-(-5)(2) &= (-28)-(-10)\\ &= -28+(10)\\ &= -18\end{aligned}$$

8. $$\begin{aligned}-5[-4-(-2)(-7)] &= -5[-4-(14)]\\ &= -5[-18]\\ &= 90\end{aligned}$$

9. $$\begin{aligned}40-(-2)[8-9] &= 40-(-2)[-1]\\ &= 40-(2)\\ &= 38\end{aligned}$$

10. $$\begin{aligned}\frac{5(-4)}{-7-(-2)} &= \frac{-20}{-7+2}\\ &= \frac{-20}{-5} = 4\end{aligned}$$

11. $$\begin{aligned}\frac{-3-(-9+1)}{-7-(-6)} &= \frac{-3-(-8)}{-7+6}\\ &= \frac{-3+8}{-1}\\ &= \frac{5}{-1} = -5\end{aligned}$$

12. $$\begin{aligned}\frac{5(-8+3)}{13(-2)+(-7)(-3)} &= \frac{5(-5)}{-26+21}\\ &= \frac{-25}{-5} = 5\end{aligned}$$

13. $\dfrac{6^2-8}{-2(2)+4(-1)}=\dfrac{36-8}{-4+(-4)}$
$=\dfrac{28}{-8}$
$=-\dfrac{4\cdot 7}{2\cdot 4}=-\dfrac{7}{2}$, or $-3\dfrac{1}{2}$

14. $\dfrac{16(-8+5)}{15(-3)+(-7-4)(-3)}=\dfrac{16(-3)}{-45+(-11)(-3)}$
$=\dfrac{-48}{-45+33}$
$=\dfrac{-48}{-12}=4$

15. $\dfrac{9(-6)-3(8)}{4(-7)+(-2)(-11)}=\dfrac{-54-24}{-28+22}$
$=\dfrac{-78}{-6}=13$

16. $\dfrac{2^2+4^2}{5^2-3^2}=\dfrac{4+16}{25-9}$
$=\dfrac{20}{16}=\dfrac{5}{4}$, or $1\dfrac{1}{4}$

17. $\dfrac{(2+4)^2}{(5-3)^2}=\dfrac{(6)^2}{(2)^2}$
$=\dfrac{36}{4}=9$

18. $\dfrac{4^3-3^3}{-5(-4+2)}=\dfrac{64-27}{-5(-2)}$
$=\dfrac{37}{10}$, or $3\dfrac{7}{10}$

19. $\dfrac{-9(-6)+(-2)(27)}{3(8-9)}=\dfrac{(54)+(-54)}{3(-1)}$
$=\dfrac{0}{-3}=0$

20. $|-4(9)|-|-11|=|-36|-11$
$=36-11$
$=25$

21. $\dfrac{6(-10+3)}{15(-2)-3(-9)}=\dfrac{6(-7)}{(-30)-(-27)}$
$=\dfrac{-42}{-30+27}$
$=\dfrac{-42}{-3}=14$

22. $\dfrac{3^2-5^2}{(-9)^2-9^2}=\dfrac{9-25}{81-81}$
$=\dfrac{-16}{0}$, which is *undefined*.

23. $\dfrac{(-10)^2+10^2}{-10(5)}=\dfrac{100+100}{-50}$
$=\dfrac{200}{-50}=-4$

24. $-\dfrac{3}{4}\div\left(-\dfrac{5}{8}\right)=-\dfrac{3}{4}\cdot\left(-\dfrac{8}{5}\right)$
$=\dfrac{3\cdot 2\cdot 4}{4\cdot 5}$
$=\dfrac{3\cdot 2}{5}=\dfrac{6}{5}$, or $1\dfrac{1}{5}$

25. $\dfrac{1}{2}\div\left(-\dfrac{1}{2}\right)=\dfrac{1}{2}\cdot\left(-\dfrac{2}{1}\right)$
$=-\dfrac{2}{2}=-1$

26. $\dfrac{8^2-12}{(-5)^2+2(6)}=\dfrac{64-12}{25+12}$
$=\dfrac{52}{37}$, or $1\dfrac{15}{37}$

27. $\left[\dfrac{5}{8}-\left(-\dfrac{1}{16}\right)\right]+\dfrac{3}{8}=\left[\dfrac{10}{16}+\dfrac{1}{16}\right]+\dfrac{6}{16}$
$=\left[\dfrac{11}{16}\right]+\dfrac{6}{16}$
$=\dfrac{17}{16}$, or $1\dfrac{1}{16}$

28. $\left(\dfrac{1}{2}-\dfrac{1}{3}\right)-\dfrac{5}{6}=\left(\dfrac{3}{6}-\dfrac{2}{6}\right)-\dfrac{5}{6}$
$=\left(\dfrac{1}{6}\right)-\dfrac{5}{6}$
$=-\dfrac{4}{6}=-\dfrac{2}{3}$

29. $-0.9(-3.7)=0.9(3.7)$
$=3.33$

30. $-5.1(-0.2)=5.1(0.2)$
$=1.02$

31. $-3^2-2^2=-(3^2)-(2^2)$
$=-9-4$
$=-13$

32. $|-2(3)+4|-|-2|=|-6+4|-2$
$=|-2|-2$
$=2-2=0$

33. $40+2[-5-3]=40+2[-8]$
$=40-16$
$=24$

In Exercises 34–42, replace x with -2, y with 3, and a with 4. Then use the order of operations to evaluate the expression.

34. $$\begin{aligned} -x + y - 3a &= -(-2) + 3 - 3(4) \\ &= 2 + 3 - 12 \\ &= 5 - 12 \\ &= -7 \end{aligned}$$

35. $$\begin{aligned} (x+6)^3 - y^3 &= (-2+6)^3 - 3^3 \\ &= (4)^3 - 27 \\ &= 64 - 27 \\ &= 37 \end{aligned}$$

36. $$\begin{aligned} (x-y) - (a-2y) &= (-2-3) - (4 - 2 \cdot 3) \\ &= (-5) - (4-6) \\ &= -5 - (-2) \\ &= -5 + 2 \\ &= -3 \end{aligned}$$

37. $$\begin{aligned} \left(\frac{1}{2}x + \frac{2}{3}y\right)\left(-\frac{1}{4}a\right) &= \left(\frac{1}{2}(-2) + \frac{2}{3}(3)\right)\left(-\frac{1}{4}(4)\right) \\ &= (-1+2)(-1) \\ &= (1)(-1) \\ &= -1 \end{aligned}$$

38. $$\begin{aligned} \frac{2x+3y}{a-xy} &= \frac{2(-2)+3(3)}{4-(-2)(3)} \\ &= \frac{-4+9}{4-(-6)} \\ &= \frac{5}{4+6} \\ &= \frac{5}{10} = \frac{1}{2} \end{aligned}$$

39. $$\begin{aligned} \frac{x^2 - y^2}{x^2 + y^2} &= \frac{(-2)^2 - 3^2}{(-2)^2 + 3^2} \\ &= \frac{4-9}{4+9} \\ &= \frac{-5}{13} = -\frac{5}{13} \end{aligned}$$

40. $$\begin{aligned} -x^2 + 3y &= -(-2)^2 + 3(3) \\ &= -(4) + 9 \\ &= 5 \end{aligned}$$

41. $$\begin{aligned} \left(\frac{x}{y}\right)^3 = \left(\frac{-2}{3}\right)^3 &= \left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right) \\ &= -\frac{8}{27} \end{aligned}$$

42. $$\left(\frac{a}{x}\right)^2 = \left(\frac{4}{-2}\right)^2 = (-2)^2 = (-2)(-2) = 4$$

1.7 Properties of Real Numbers

1.7 Classroom Examples, Now Try Exercises

1. **(a)** $x + 2 = 2 + \underline{x}$

(b) $5x = x \cdot \underline{5}$

N1. **(a)** $7 + (-3) = -3 + \underline{7}$

(b) $(-5)4 = 4 \cdot \underline{(-5)}$

2. **(a)** $-5 + (2 + 8) = \underline{(-5+2)} + 8$

(b) $10[(-8) \cdot (-3)] = \underline{[10 \cdot (-8)] \cdot (-3)}$

N2. **(a)** $-9 + (3 + 7) = \underline{(-9+3)} + 7$

(b) $5[(-4) \cdot 9] = \underline{[5 \cdot (-4)] \cdot 9}$

3. $(2 \cdot 4)6 = (4 \cdot 2)6$

While the same numbers are grouped inside the two pairs of parentheses, the order of the numbers has been changed. This illustrates a *commutative* property.

N3. $5 + (7 + 6) = 5 + (6 + 7)$

While the same numbers are grouped inside the two pairs of parentheses, the order of the numbers has been changed. This illustrates a *commutative* property.

4. **(a)** $$\begin{aligned} &43 + 26 + 17 + 24 + 6 \\ &= (43 + 17) + (26 + 24) + 6 \\ &= 60 + 50 + 6 \\ &= 110 + 6 \\ &= 116 \end{aligned}$$

(b) $\frac{1}{2}(67)(2) = \frac{1}{2}(2)(67) = 1(67) = 67$

N4. **(a)** $$\begin{aligned} &8 + 54 + 7 + 6 + 32 \\ &= (8 + 32) + (54 + 6) + 7 \\ &= 40 + 60 + 7 \\ &= 100 + 7 \\ &= 107 \end{aligned}$$

(b) $5(37)(20) = 5(20)(37) = 100(37) = 3700$

5. **(a)** $5 + \underline{0} = 5$ *Additive identity*

(b) $\underline{1} \cdot \frac{1}{3} = \frac{1}{3}$ *Multiplicative identity*

N5. **(a)** $\frac{2}{5} \cdot \underline{1} = \frac{2}{5}$ *Multiplicative identity*

(b) $8 + \underline{0} = 8$ *Additive identity*

6. **(a)** $\frac{36}{48} = \frac{3 \cdot 12}{4 \cdot 12}$ *Factor.*
$= \frac{3}{4} \cdot \frac{12}{12}$ *Write as a product.*
$= \frac{3}{4} \cdot 1$ *Divide.*
$= \frac{3}{4}$ *Identity property*

(b) $\frac{3}{8} - \frac{5}{24} = \frac{3}{8} \cdot 1 - \frac{5}{24}$ *Identity property*
$= \frac{3}{8} \cdot \frac{3}{3} - \frac{5}{24}$ *Use 1 = $\frac{3}{3}$ to get a common denominator.*
$= \frac{9}{24} - \frac{5}{24}$ *Multiply.*
$= \frac{4}{24}$ *Subtract.*
$= \frac{1}{6}$ *Reduce.*

N6. **(a)** $\frac{16}{20} = \frac{4 \cdot 4}{5 \cdot 4}$ *Factor.*
$= \frac{4}{5} \cdot \frac{4}{4}$ *Write as a product.*
$= \frac{4}{5} \cdot 1$ *Divide.*
$= \frac{4}{5}$ *Identity property*

(b) $\frac{2}{5} + \frac{3}{20} = \frac{2}{5} \cdot 1 + \frac{3}{20}$ *Identity property*
$= \frac{2}{5} \cdot \frac{4}{4} + \frac{3}{20}$ *Use 1 = $\frac{4}{4}$ to get a common denominator.*
$= \frac{8}{20} + \frac{3}{20}$ *Multiply.*
$= \frac{11}{20}$ *Add.*

7. **(a)** $\underline{-6} + 6 = 0$ *Inverse property*

(b) $-\frac{1}{9} \cdot \underline{(-9)} = 1$ *Inverse property*

N7. **(a)** $10 + \underline{-10} = 0$ *Inverse property*

(b) $-9 \cdot \underline{\left(-\frac{1}{9}\right)} = 1$ *Inverse property*

8. $\frac{1}{2} + 3y + \left(-\frac{1}{2}\right)$
$= \left(\frac{1}{2} + 3y\right) + \left(-\frac{1}{2}\right)$ *Order of operations*
$= \left(3y + \frac{1}{2}\right) + \left(-\frac{1}{2}\right)$ *Commutative property*
$= 3y + \left[\frac{1}{2} + \left(-\frac{1}{2}\right)\right]$ *Associative property*
$= 3y + 0$ *Inverse property*
$= 3y$ *Identity property*

N8. $-\frac{1}{3}x + 7 + \frac{1}{3}x$
$= \left(-\frac{1}{3}x + 7\right) + \frac{1}{3}x$ *Order of operations*
$= \left[7 + \left(-\frac{1}{3}x\right)\right] + \frac{1}{3}x$ *Commutative property*
$= 7 + \left[\left(-\frac{1}{3}x\right) + \frac{1}{3}x\right]$ *Associative property*
$= 7 + 0$ *Inverse property*
$= 7$ *Identity property*

9. **(a)** $4(3+7) = 4 \cdot 3 + 4 \cdot 7$ *Distributive property*
$= 12 + 28$ *Multiply.*
$= 40$ *Add.*

(b) $-6(x + y - z) = -6x + (-6)y + (-6)(-z)$
$= -6x - 6y + 6z$

(c) $3a - 3b = 3(a - b)$

N9. **(a)** $-5(4x + 1) = -5 \cdot 4x + (-5 \cdot 1)$ *Dist. prop.*
$= -20x - 5$ *Multiply.*

(b) $6(2r + t - 5z) = 6(2r) + 6t + 6(-5z)$
$= 12r + 6t - 30z$

(c) $5x - 5y = 5(x - y)$

10. **(a)** $-(-5y + 8) = -1(-5y + 8)$
$= 5y - 8$

(b) $-(x - y - z) = -1(x - y - z)$
$= -x + y + z$

N10. **(a)** $-(2 - r) = -1(2 - r)$
$= -2 + r$

(b) $-(2x - 5y - 7) = -1(2x - 5y - 7)$
$= -2x + 5y + 7$

1.7 Section Exercises

1. **(a)** **B**, since 0 is the identity element for addition.

(b) **F**, since 1 is the identity element for multiplication.

(c) **C**, since $-a$ is the additive inverse of a.

(d) **I**, since $\frac{1}{a}$ is the multiplicative inverse, or reciprocal, of any nonzero number a.

(e) **B**, since 0 is the only number that is equal to its negative; that is, $0 = -0$.

(f) **D** and **F**, since -1 has reciprocal $\frac{1}{(-1)} = -1$ and 1 has a reciprocal $\frac{1}{(1)} = 1$; that is, -1 and 1 are their own multiplicative inverses.

(g) **B**, since the multiplicative inverse of a number a is $\frac{1}{a}$ and the only number that we *cannot* divide by is 0.

(h) **A**

(i) **G**, since we can consider $(5 \cdot 4)$ to be one number, $(5 \cdot 4) \cdot 3$ is the same as $3 \cdot (5 \cdot 4)$ by the commutative property.

(j) **H**

2. The commutative property allows us to change the <u>order</u> of the terms in a sum or the factors in a product. The associative property allows us to change the <u>grouping</u> of the terms in a sum or the factors in a product.

3. "Washing your face" and "brushing your teeth" *are* commutative.

4. "Putting on your left sock" and "putting on your right sock" *are* commutative.

5. "Preparing a meal" and "eating a meal" *are not* commutative.

6. "Starting a car" and "driving away in a car" *are not* commutative.

7. "Putting on your socks" and "putting on your shoes" *are not* commutative.

8. "Getting undressed" and "taking a shower" *are not* commutative.

9. "(Foreign sales) clerk" is a clerk dealing with foreign sales, whereas "foreign (sales clerk)" is a sales clerk who is foreign.

10. "(Defective merchandise) counter" is a location at which we would return an item that does not work, whereas "defective (merchandise counter)" is a broken place where items are bought and sold.

11. $-15 + 9 = 9 + \underline{(-15)}$
by the *commutative property of addition*.

12. $6 + (-2) = -2 + \underline{6}$
by the *commutative property of addition*.

13. $-8 \cdot 3 = \underline{3} \cdot (-8)$
by the *commutative property of multiplication*.

14. $-12 \cdot 4 = 4 \cdot \underline{(-12)}$
by the *commutative property of multiplication*.

15. $(3 + 6) + 7 = 3 + (\underline{6} + 7)$
by the *associative property of addition*.

16. $(-2 + 3) + 6 = -2 + (\underline{3} + 6)$
by the *associative property of addition*.

17. $7 \cdot (2 \cdot 5) = (\underline{7} \cdot 2) \cdot 5$
by the *associative property of multiplication*.

18. $8 \cdot (6 \cdot 4) = (8 \cdot \underline{6}) \cdot 4$
by the *associative property of multiplication*.

19.
$$\begin{aligned} 25 - (6 - 2) &= 25 - (4) \\ &= 21 \\ (25 - 6) - 2 &= 19 - 2 \\ &= 17 \end{aligned}$$

Since $21 \neq 17$, this example shows that subtraction is not associative.

20.
$$\begin{aligned} 180 \div (15 \div 3) &= 180 \div 5 \\ &= 36 \\ (180 \div 15) \div 3 &= 12 \div 3 \\ &= 4 \end{aligned}$$

Since $36 \neq 4$, this example shows that division is not associative.

21.

Number	Additive inverse	Multiplicative inverse
5	-5	$\frac{1}{5}$
-10	10	$-\frac{1}{10}$
$-\frac{1}{2}$	$\frac{1}{2}$	-2
$\frac{3}{8}$	$-\frac{3}{8}$	$\frac{8}{3}$
$x\,(x \neq 0)$	$-x$	$\frac{1}{x}$
$-y\,(y \neq 0)$	y	$-\frac{1}{y}$

In general, a number and its additive inverse have <u>opposite</u> signs. A number and its multiplicative inverse have <u>the same</u> signs.

22. Jack recognized the identity property of addition.

23. $4 + 15 = 15 + 4$

The order of the two numbers has been changed, so this is an example of the commutative property of addition: $a + b = b + a$.

24. $3 + 12 = 12 + 3$

The order of the two numbers has been changed, so this is an example of the commutative property of addition: $a + b = b + a$.

25. $5 \cdot (13 \cdot 7) = (5 \cdot 13) \cdot 7$

The numbers are in the same order but grouped differently, so this is an example of the associative property of multiplication: $(ab)c = a(bc)$.

26. $-4 \cdot (2 \cdot 6) = (-4 \cdot 2) \cdot 6$

The numbers are in the same order but grouped differently, so this is an example of the associative property of multiplication: $(ab)c = a(bc)$.

27. $-6 + (12 + 7) = (-6 + 12) + 7$

The numbers are in the same order but grouped differently, so this is an example of the associative property of addition: $(a + b) + c = a + (b + c)$.

28. $(-8 + 13) + 2 = -8 + (13 + 2)$

The numbers are in the same order but grouped differently, so this is an example of the associative property of addition: $(a + b) + c = a + (b + c)$.

29. $-9 + 9 = 0$

The sum of the two numbers is 0, so they are additive inverses (or opposites) of each other. This is an example of the additive inverse property: $a + (-a) = 0$.

30. $1 + (-1) = 0$

The sum of the two numbers is 0, so they are additive inverses (or opposites) of each other. This is an example of the additive inverse property: $a + (-a) = 0$.

31. $\frac{2}{3}\left(\frac{3}{2}\right) = 1$

The product of the two numbers is 1, so they are multiplicative inverses (or reciprocals) of each other. This is an example of the multiplicative inverse property: $a \cdot \frac{1}{a} = 1\ (a \neq 0)$.

32. $\frac{5}{8}\left(\frac{8}{5}\right) = 1$

The product of the two numbers is 1, so they are multiplicative inverses (or reciprocals) of each other. This is an example of the multiplicative inverse property: $a \cdot \frac{1}{a} = 1\ (a \neq 0)$.

33. $1.75 + 0 = 1.75$

The sum of a number and 0 is the original number. This is an example of the identity property of addition: $a + 0 = a$.

34. $-8.45 + 0 = -8.45$

The sum of a number and 0 is the original number. This is an example of the identity property of addition: $a + 0 = a$.

35. $(4 + 17) + 3 = 3 + (4 + 17)$

The order of the numbers has been changed, but not the grouping, so this is an example of the commutative property of addition: $a + b = b + a$.

36. $(-8 + 4) + 12 = 12 + (-8 + 4)$

The order of the numbers has been changed, but not the grouping, so this is an example of the commutative property of addition: $a + b = b + a$.

37. $2(x + y) = 2x + 2y$

The number 2 outside the parentheses is "distributed" over the x and y. This is an example of the distributive property.

38. $9(t + s) = 9t + 9s$

The number 9 outside the parentheses is "distributed" over the t and s. This is an example of the distributive property.

39. $-\frac{5}{9} = -\frac{5}{9} \cdot \frac{3}{3} = -\frac{15}{27}$

$\frac{3}{3}$ is a form of the number 1. We use it to rewrite $-\frac{5}{9}$ as $-\frac{15}{27}$. This is an example of the identity property of multiplication.

40. $-\frac{7}{12} = -\frac{7}{12} \cdot \frac{7}{7} = -\frac{49}{84}$

$\frac{7}{7}$ is a form of the number 1. We use it to rewrite $-\frac{7}{12}$ as $-\frac{49}{84}$. This is an example of the identity property of multiplication.

41. $4(2x) + 4(3y) = 4(2x + 3y)$

This is an example of the distributive property. The number 4 is "distributed " over $2x$ and $3y$.

42. $6(5t) - 6(7r) = 6(5t - 7r)$

This is an example of the distributive property. The number 6 is "distributed " over $5t$ and $7r$.

43.
$$\begin{aligned} 97 + 13 + 3 + 37 &= (97 + 3) + (13 + 37) \\ &= 100 + 50 \\ &= 150 \end{aligned}$$

44.
$$\begin{aligned} 49 + 199 + 1 + 1 &= (49 + 1) + (199 + 1) \\ &= 50 + 200 \\ &= 250 \end{aligned}$$

45.
$$\begin{aligned} 1999 + 2 + 1 + 8 &= (1999 + 1) + (2 + 8) \\ &= 2000 + 10 \\ &= 2010 \end{aligned}$$

46.
$$\begin{aligned} 2998 + 3 + 2 + 17 &= (2998 + 2) + (3 + 17) \\ &= 3000 + 20 \\ &= 3020 \end{aligned}$$

47.
$$\begin{aligned} 159 + 12 + 141 + 88 &= (159 + 141) + (12 + 88) \\ &= 300 + 100 \\ &= 400 \end{aligned}$$

48.
$$\begin{aligned} &106 + 8 + (-6) + (-8) \\ &= [106 + (-6)] + [8 + (-8)] \\ &= 100 + 0 \\ &= 100 \end{aligned}$$

49. $843 + 627 + (-43) + (-27)$
$= [843 + (-43)] + [627 + (-27)]$
$= 800 + 600$
$= 1400$

50. $1846 + 1293 + (-46) + (-93)$
$= [1846 + (-46)] + [1293 + (-93)]$
$= 1800 + 1200$
$= 3000$

51. $5(47)(2) = 5(2)(47) = 10(47) = 470$

52. $2(79)(5) = 2(5)(79) = 10(79) = 790$

53. $-4 \cdot 5 \cdot 93 \cdot 5 = -4 \cdot 5 \cdot 5 \cdot 93$
$= -20 \cdot 5 \cdot 93$
$= -100 \cdot 93$
$= -9300$

54. $2 \cdot 25 \cdot 67 \cdot (-2) = -2 \cdot 2 \cdot 25 \cdot 67$
$= -4 \cdot 25 \cdot 67$
$= -100 \cdot 67$
$= -6700$

55. $6t + 8 - 6t + 3$
$= 6t + 8 + (-6t) + 3$ *Definition of subtraction*
$= (6t + 8) + (-6t) + 3$ *Order of operations*
$= (8 + 6t) + (-6t) + 3$ *Commutative property*
$= 8 + [6t + (-6t)] + 3$ *Associative property*
$= 8 + 0 + 3$ *Inverse property*
$= (8 + 0) + 3$ *Order of operations*
$= 8 + 3$ *Identity property*
$= 11$ *Add.*

56. $9r + 12 - 9r + 1$
$= 9r + 12 + (-9r) + 1$ *Definition of subtraction*
$= (9r + 12) + (-9r) + 1$ *Order of operations*
$= (12 + 9r) + (-9r) + 1$ *Commutative property*
$= 12 + [9r + (-9r)] + 1$ *Associative property*
$= 12 + 0 + 1$ *Inverse property*
$= (12 + 0) + 1$ *Order of operations*
$= 12 + 1$ *Identity property*
$= 13$ *Add.*

57. $\frac{2}{3}x - 11 + 11 - \frac{2}{3}x$
$= \frac{2}{3}x + (-11) + 11 + \left(-\frac{2}{3}x\right)$
Definition of subtraction
$= \left[\frac{2}{3}x + (-11)\right] + 11 + \left(-\frac{2}{3}x\right)$
Order of operations
$= \frac{2}{3}x + (-11 + 11) + \left(-\frac{2}{3}x\right)$
Associative property
$= \frac{2}{3}x + 0 + \left(-\frac{2}{3}x\right)$ *Inverse property*
$= \left(\frac{2}{3}x + 0\right) + \left(-\frac{2}{3}x\right)$
Order of operations
$= \frac{2}{3}x + \left(-\frac{2}{3}x\right)$ *Identity property*
$= 0$ *Inverse property*

58. $\frac{1}{5}y + 4 - 4 - \frac{1}{5}y$
$= \frac{1}{5}y + 4 + (-4) + \left(-\frac{1}{5}y\right)$
Definition of subtraction
$= \left(\frac{1}{5}y + 4\right) + (-4) + \left(-\frac{1}{5}y\right)$
Order of operations
$= \frac{1}{5}y + [4 + (-4)] + \left(-\frac{1}{5}y\right)$
Associative property
$= \frac{1}{5}y + 0 + \left(-\frac{1}{5}y\right)$ *Inverse property*
$= \left(\frac{1}{5}y + 0\right) + \left(-\frac{1}{5}y\right)$ *Order of operations*
$= \frac{1}{5}y + \left(-\frac{1}{5}y\right)$ *Identity property*
$= 0$ *Inverse property*

59. $\left(\frac{9}{7}\right)(-0.38)\left(\frac{7}{9}\right)$
$= \left[\left(\frac{9}{7}\right)(-0.38)\right]\left(\frac{7}{9}\right)$ *Order of operations*
$= \left[(-0.38)\left(\frac{9}{7}\right)\right]\left(\frac{7}{9}\right)$ *Commutative property*

$= (-0.38)\left[\left(\frac{9}{7}\right)\left(\frac{7}{9}\right)\right]$ *Associative property*

$= (-0.38)(1)$ *Inverse property*

$= -0.38$ *Identity property*

60. $\left(\frac{4}{5}\right)(-0.73)\left(\frac{5}{4}\right)$

$= \left[\left(\frac{4}{5}\right)(-0.73)\right]\left(\frac{5}{4}\right)$ *Order of operations*

$= \left[(-0.73)\left(\frac{4}{5}\right)\right]\left(\frac{5}{4}\right)$ *Commutative property*

$= (-0.73)\left[\left(\frac{4}{5}\right)\left(\frac{5}{4}\right)\right]$ *Associative property*

$= (-0.73)(1)$ *Inverse property*

$= -0.73$ *Identity property*

61. $t + (-t) + \frac{1}{2}(2)$

$= t + (-t) + 1$ *Inverse property*
$= [t + (-t)] + 1$ *Order of operations*
$= 0 + 1$ *Inverse property*
$= 1$ *Identity property*

62. $w + (-w) + \frac{1}{4}(4)$

$= w + (-w) + 1$ *Inverse property*
$= [w + (-w)] + 1$ *Order of operations*
$= 0 + 1$ *Inverse property*
$= 1$ *Identity property*

63. $-3(4 - 6)$

When distributing a negative number over a quantity, be careful not to "lose" a negative sign. The problem should be worked in the following way.

$$\begin{aligned} -3(4-6) &= -3(4) - 3(-6) \\ &= -12 + 18 \\ &= 6 \end{aligned}$$

64. $\frac{3}{4} = \frac{3}{4} \cdot 1 = \frac{3}{4} \cdot \frac{3}{3} = \frac{9}{12}$

To rewrite $\frac{3}{4}$ as $\frac{9}{12}$, use the fact that $\frac{3}{3}$ is another name for the multiplicative identity element, 1.

65. $$\begin{aligned} 5(9+8) &= 5 \cdot 9 + 5 \cdot 8 \\ &= 45 + 40 \\ &= 85 \end{aligned}$$

66. $$\begin{aligned} 6(11+8) &= 6 \cdot 11 + 6 \cdot 8 \\ &= 66 + 48 \\ &= 114 \end{aligned}$$

67. $$\begin{aligned} 4(t+3) &= 4 \cdot t + 4 \cdot 3 \\ &= 4t + 12 \end{aligned}$$

68. $$\begin{aligned} 5(w+4) &= 5 \cdot w + 5 \cdot 4 \\ &= 5w + 20 \end{aligned}$$

69. $$\begin{aligned} 7(z-8) &= 7[z + (-8)] \\ &= 7z + 7(-8) \\ &= 7z - 56 \end{aligned}$$

70. $$\begin{aligned} 8(x-6) &= 8[x + (-6)] \\ &= 8x + 8(-6) \\ &= 8x - 48 \end{aligned}$$

71. $$\begin{aligned} -8(r+3) &= -8(r) + (-8)(3) \\ &= -8r + (-24) \\ &= -8r - 24 \end{aligned}$$

72. $$\begin{aligned} -11(x+4) &= -11(x) + (-11)(4) \\ &= -11x - 44 \end{aligned}$$

73. $-\frac{1}{4}(8x + 3)$

$= -\frac{1}{4}(8x) + (-\frac{1}{4})(3)$

$= \left[(-\frac{1}{4}) \cdot 8\right]x - \frac{3}{4}$

$= -2x - \frac{3}{4}$

74. $-\frac{1}{3}(9x + 5)$

$= -\frac{1}{3}(9x) + (-\frac{1}{3})(5)$

$= \left[(-\frac{1}{3}) \cdot 9\right]x - \frac{5}{3}$

$= -3x - \frac{5}{3}$

75. $$\begin{aligned} -5(y-4) &= -5(y) + (-5)(-4) \\ &= -5y + 20 \end{aligned}$$

76. $$\begin{aligned} -9(g-4) &= -9(g) + (-9)(-4) \\ &= -9g + 36 \end{aligned}$$

77. $-\frac{4}{3}(12y + 15z)$

$= -\frac{4}{3}(12y) + (-\frac{4}{3})(15z)$

$= \left[(-\frac{4}{3}) \cdot 12\right]y + \left[(-\frac{4}{3}) \cdot 15\right]z$

$= -16y + (-20)z$

$= -16y - 20z$

78. $-\frac{2}{5}(10b+20a)$
$= -\frac{2}{5}(10b) + (-\frac{2}{5})(20a)$
$= \left[(-\frac{2}{5})\cdot 10\right]b + \left[(-\frac{2}{5})\cdot 20\right]a$
$= -4b + (-8a)$
$= -4b - 8a$

79. $8z + 8w = 8(z+w)$

80. $4s + 4r = 4(s+r)$

81. $7(2v) + 7(5r) = 7(2v+5r)$

82. $13(5w) + 13(4p) = 13(5w+4p)$

83. $8(3r+4s-5y)$
$= 8(3r) + 8(4s) + 8(-5y)$ *Distributive property*
$= (8\cdot 3)r + (8\cdot 4)s + [8(-5)]y$ *Associative property*
$= 24r + 32s - 40y$ *Multiply.*

84. $2(5u-3v+7w)$
$= 2(5u) + 2(-3v) + 2(7w)$ *Distributive property*
$= (2\cdot 5)u + [2(-3)]v + (2\cdot 7)w$ *Associative property*
$= 10u - 6v + 14w$ *Multiply.*

85. $-3(8x+3y+4z)$
$= -3(8x) + (-3)(3y) + (-3)(4z)$ *Distributive property*
$= (-3\cdot 8)x + (-3\cdot 3)y + (-3\cdot 4)z$ *Associative property*
$= -24x - 9y - 12z$ *Multiply.*

86. $-5(2x-5y+6z)$
$= -5(2x) + (-5)(-5y) + (-5)(6z)$ *Distributive property*
$= (-5\cdot 2)x + [-5(-5)]y + (-5\cdot 6)z$ *Associative property*
$= -10x + 25y - 30z$ *Multiply.*

87. $5x + 15 = 5x + 5\cdot 3$
$= 5(x+3)$

88. $9p + 18 = 9p + 9\cdot 2$
$= 9(p+2)$

89. $-(4t+3m)$
$= -1(4t+3m)$ *Identity property*
$= -1(4t) + (-1)(3m)$ *Distributive property*
$= (-1\cdot 4)t + (-1\cdot 3)m$ *Associative property*
$= -4t - 3m$ *Multiply.*

90. $-(9x+12y)$
$= -1(9x+12y)$ *Identity property*
$= -1(9x) + (-1)(12y)$ *Distributive property*
$= (-1\cdot 9)x + (-1\cdot 12)y$ *Associative property*
$= -9x - 12y$ *Multiply.*

91. $-(-5c-4d)$
$= -1(-5c-4d)$ *Identity property*
$= -1(-5c) + (-1)(-4d)$ *Distributive property*
$= (-1\cdot -5)c + (-1\cdot -4)d$ *Associative property*
$= 5c + 4d$ *Multiply.*

92. $-(-13x-15y)$
$= -1(-13x-15y)$
$= -1(-13x) + (-1)(-15y)$
$= (-1\cdot -13)x + (-1\cdot -15)y$
$= 13x + 15y$

93. $-(-q+5r-8s)$
$= -1(-q+5r-8s)$
$= -1(-q) + (-1)(5r) + (-1)(-8s)$
$= (-1\cdot -1)q + (-1\cdot 5)r + (-1\cdot -8)s$
$= q - 5r + 8s$

94. $-(-z+5w-9y)$
$= -1(-z+5w-9y)$
$= -1(-z) + (-1)(5w) + (-1)(-9y)$
$= (-1\cdot -1)z + (-1\cdot 5)w + (-1\cdot -9)y$
$= z - 5w + 9y$

1.8 Simplifying Expressions

1.8 Classroom Examples, Now Try Exercises

1. **(a)** $5(4x-3y) = 5(4x) - 5(3y)$
$= (5\cdot 4)x - (5\cdot 3)y$
$= 20x - 15y$

(b) $-(7-6k)+9 = -1(7-6k)+9$
$= -1(7) - 1(-6k) + 9$
$= -7 + 6k + 9$
$= -7 + 9 + 6k$
$= 2 + 6k$

N1. **(a)** $3(2x-4y) = 3(2x) - 3(4y)$
$= (3\cdot 2)x - (3\cdot 4)y$
$= 6x - 12y$

(b) $-4-(-3y+5) = -4-1(-3y+5)$
$= -4-1(-3y)-1(5)$
$= -4+3y+(-5)$
$= -4+(-5)+3y$
$= -9+3y, \text{ or } 3y-9$

2. (a) $5z+9z-4z = (5+9-4)z = 10z$

(b) $4r-r = 4r-1r = (4-1)r = 3r$

(c) $8p+8p^2$ cannot be simplified. $8p$ and $8p^2$ are unlike terms and cannot be combined.

N2. (a) $4x+6x-7x = (4+6-7)x = 3x$

(b) $z+z = 1z+1z = (1+1)z = 2z$

(c) $4p^2-3p^2 = (4-3)p^2 = 1p^2, \text{ or } p^2$

3. (a) $-(3+5k)+7k = -1(3+5k)+7k$
$= -1(3)-1(5k)+7k$
$= -3-5k+7k$
$= -3+2k$

(b) $7z-2-(1+z) = 7z-2-1(1+z)$
$= 7z-2-1-z$
$= 6z-3$

N3. (a) $5k-6-(3-4k)$
$= 5k-6-1(3-4k)$
$= 5k-6-1(3)-1(-4k)$
$= 5k-6-3+4k$
$= 9k-9$

(b) $\frac{1}{4}x-\frac{2}{3}(x-9) = \frac{1}{4}x-\frac{2}{3}(x)-\frac{2}{3}(-9)$
$= \frac{3}{12}x-\frac{8}{12}x+6$
$= -\frac{5}{12}x+6$

4. "Three times a number, subtracted from the sum of the number and 8" is written $(x+8)-3x$.

$$(x+8)-3x = x+8-3x = -2x+8$$

N4. "Twice a number, subtracted from the sum of the number and 5" is written $(x+5)-2x$.

$$(x+5)-2x = x+5-2x = -x+5, \text{ or } 5-x$$

1.8 Section Exercises

1. $-(6x-3) = -1(6x-3)$
$= -1(6x)-1(-3)$
$= -6x+3$
The correct response is **B**.

2. The numerical coefficient of $5x^3y^7$ is 5.
The correct response is **A**.

3. Examples **A**, **B**, and **D** are pairs of *unlike* terms since either the variables or their powers are different. Example **C** is a pair of *like* terms, since both terms have the same variables (r and y) and the same exponents (both variables are to the first power). Note that we can use the commutative property to rewrite $6yr$ as $6ry$.

4. "Six times a number" translates as $6x$, and "the product of eleven and the number" translates as $11x$. Thus, the correct translation of "six times a number, subtracted from the product of eleven and the number" is **B**, $11x-6x$.

5. $4r+19-8 = 4r+11$

6. $7t+18-4 = 7t+14$

7. $5+2(x-3y) = 5+2(x)+2(-3y)$
$= 5+2x-6y$

8. $8+3(s-6t) = 8+3s+3(-6t)$
$= 8+3s-18t$

9. $-2-(5-3p) = -2-1(5-3p)$
$= -2-1(5)-1(-3p)$
$= -2-5+3p$
$= -7+3p$

10. $-10-(7-14r)$
$= -10-1(7-14r)$
$= -10-1(7)-1(-14r)$
$= -10-7+14r$
$= -17+14r$

11. $6+(4-3x)-8 = 6+4-3x-8$
$= 10-3x-8$
$= 10-8-3x$
$= 2-3x$

12. $-12+(7-8x)+6 = -12+7-8x+6$
$= -5-8x+6$
$= -5+6-8x$
$= 1-8x$

13. The numerical coefficient of the term $-12k$ is -12.

14. The numerical coefficient of the term $-11y$ is -11.

15. The numerical coefficient of the term $3m^2$ is 3.

16. The numerical coefficient of the term $9n^6$ is 9.

17. Because xw can be written as $1 \cdot xw$, the numerical coefficient of the term xw is 1.

18. Because pq can be written as $1 \cdot pq$, the numerical coefficient of the term pq is 1.

19. Since $-x = -1x$, the numerical coefficient of the term $-x$ is -1.

20. Since $-t = -1t$, the numerical coefficient of the term $-t$ is -1.

21. Since $\frac{x}{2} = \frac{1}{2}x$, the numerical coefficient of the term $\frac{x}{2}$ is $\frac{1}{2}$.

22. Since $\frac{x}{6} = \frac{1}{6}x$, the numerical coefficient of the term $\frac{x}{6}$ is $\frac{1}{6}$.

23. Since $\frac{2x}{5} = \frac{2}{5}x$, the numerical coefficient of the term $\frac{2x}{5}$ is $\frac{2}{5}$.

24. Since $\frac{8x}{9} = \frac{8}{9}x$, the numerical coefficient of the term $\frac{8x}{9}$ is $\frac{8}{9}$.

25. The numerical coefficient of the term 10 is 10.

26. The numerical coefficient of the term 15 is 15.

27. $8r$ and $-13r$ are *like* terms since they have the same variable with the same exponent (which is understood to be 1).

28. $-7x$ and $12x$ are *like* terms since they have the same variable with the same exponent (which is understood to be 1).

29. $5z^4$ and $9z^3$ are *unlike* terms. Although both have the variable z, the exponents are not the same.

30. $8x^5$ and $-10x^3$ are *unlike* terms. Although both have the variable x, the exponents are not the same.

31. All numerical terms (constants) are considered like terms, so 4, 9, and -24 are *like* terms.

32. All numerical terms (constants) are considered like terms, so 7, 17, and -83 are *like* terms.

33. x and y are *unlike* terms because they do not have the same variable.

34. t and s are *unlike* terms because they do not have the same variable.

35. The student made a sign error when applying the distributive property.

$$\begin{aligned} 7x - 2(3 - 2x) &= 7x - 2(3) - 2(-2x) \\ &= 7x - 6 + 4x \\ &= 11x - 6 \end{aligned}$$

The correct answer is $11x - 6$.

36. The student incorrectly started by adding $3 + 2$. First, 2 must be multiplied by $4x - 5$.

$$\begin{aligned} 3 + 2(4x - 5) &= 3 + 2(4x) + 2(-5) \\ &= 3 + 8x - 10 \\ &= 8x - 7 \end{aligned}$$

37. $$\begin{aligned} 7y + 6y &= (7 + 6)y \\ &= 13y \end{aligned}$$

38. $$\begin{aligned} 5m + 2m &= (5 + 2)m \\ &= 7m \end{aligned}$$

39. $$\begin{aligned} -6x - 3x &= (-6 - 3)x \\ &= -9x \end{aligned}$$

40. $$\begin{aligned} -4z - 8z &= (-4 - 8)z \\ &= -12z \end{aligned}$$

41. $$\begin{aligned} 12b + b &= 12b + 1b \\ &= (12 + 1)b \\ &= 13b \end{aligned}$$

42. $$\begin{aligned} 19x + x &= 19x + 1x \\ &= (19 + 1)x \\ &= 20x \end{aligned}$$

43. $$\begin{aligned} 3k + 8 + 4k + 7 &= 3k + 4k + 8 + 7 \\ &= (3 + 4)k + 15 \\ &= 7k + 15 \end{aligned}$$

44. $$\begin{aligned} 1 + 15z + 2 + 4z &= 1 + 2 + 15z + 4z \\ &= 3 + (15 + 4)z \\ &= 3 + 19z \end{aligned}$$

45. $$\begin{aligned} &-5y + 3 - 1 + 5 + y - 7 \\ &= (-5y + 1y) + (3 + 5) + (-1 - 7) \\ &= (-5 + 1)y + (8) + (-8) \\ &= -4y + 8 - 8 \\ &= -4y \end{aligned}$$

46. $$\begin{aligned} &2k - 7 - 5k + 7k - 3 - k \\ &= (2k - 5k + 7k - 1k) + (-7 - 3) \\ &= (2 - 5 + 7 - 1)k + (\,-10) \\ &= 3k - 10 \end{aligned}$$

47. $$\begin{aligned} &-2x + 3 + 4x - 17 + 20 \\ &= (-2x + 4x) + (3 - 17 + 20) \\ &= (-2 + 4)x + 6 \\ &= 2x + 6 \end{aligned}$$

48. $$\begin{aligned} &r - 6 - 12r - 4 + 6r \\ &= (1r - 12r + 6r) + (-6 - 4) \\ &= (1 - 12 + 6)r + (-10) \\ &= -5r - 10 \end{aligned}$$

49. $$\begin{aligned} &16 - 5m - 4m - 2 + 2m \\ &= (16 - 2) + (-5m - 4m + 2m) \\ &= 14 + (-5 - 4 + 2)m \\ &= 14 - 7m \end{aligned}$$

50. $$\begin{aligned} &6 - 3z - 2z - 5 + z - 3z \\ &= (6 - 5) + (-3z - 2z + 1z - 3z) \\ &= 1 + (-3 - 2 + 1 - 3)z \\ &= 1 - 7z \end{aligned}$$

51. $$\begin{aligned} &-10 + x + 4x - 7 - 4x \\ &= (-10 - 7) + (1x + 4x - 4x) \\ &= -17 + (1 + 4 - 4)x \\ &= -17 + 1x \\ &= -17 + x \end{aligned}$$

52. $-p+10p-3p-4-5p$
$=(-1p+10p-3p-5p)+(-4)$
$=(-1+10-3-5)p-4$
$=1p-4$
$=p-4$

53. $1+7x+11x-1+5x$
$=(1-1)+(7x+11x+5x)$
$=0+(7+11+5)x$
$=23x$

54. $-r+2-5r+3+4r$
$=(-1r-5r+4r)+(2+3)$
$=(-1-5+4)r+5$
$=-2r+5$

55. $-\frac{4}{3}+2t+\frac{1}{3}t-8-\frac{8}{3}t$
$=\left(2t+\frac{1}{3}t-\frac{8}{3}t\right)+\left(-\frac{4}{3}-8\right)$
$=\left(2+\frac{1}{3}-\frac{8}{3}\right)t+\left(-\frac{4}{3}-8\right)$
$=\left(\frac{6}{3}+\frac{1}{3}-\frac{8}{3}\right)t+\left(-\frac{4}{3}-\frac{24}{3}\right)$
$=-\frac{1}{3}t-\frac{28}{3}$

56. $-\frac{5}{6}+8x+\frac{1}{6}x-7-\frac{7}{6}$
$=\left(8x+\frac{1}{6}x\right)+\left(-\frac{5}{6}-7-\frac{7}{6}\right)$
$=\left(\frac{48}{6}x+\frac{1}{6}x\right)+\left(-\frac{5}{6}-\frac{42}{6}-\frac{7}{6}\right)$
$=\frac{49}{6}x-\frac{54}{6}$
$=\frac{49}{6}x-9$

57. $6y^2+11y^2-8y^2=(6+11-8)y^2$
$=9y^2$

58. $-9m^3+3m^3-7m^3=(-9+3-7)m^3$
$=-13m^3$

59. $2p^2+3p^2-8p^3-6p^3$
$=(2p^2+3p^2)+(-8p^3-6p^3)$
$=(2+3)p^2+(-8-6)p^3$
$=5p^2-14p^3$ or $-14p^3+5p^2$

60. $5y^3+6y^3-3y^2-4y^2$
$=(5y^3+6y^3)+(-3y^2-4y^2)$
$=(5+6)y^3+(-3-4)y^2$
$=11y^3-7y^2$

61. $2(4x+6)+3=2(4x)+2(6)+3$
$=8x+12+3$
$=8x+15$

62. $4(6y-9)+7=4(6y)+4(-9)+7$
$=24y-36+7$
$=24y-29$

63. $100[0.05(x+3)]$
$=[100(0.05)](x+3)$ *Associative property*
$=5(x+3)$
$=5(x)+5(3)$ *Distributive property*
$=5x+15$

64. $100[0.06(x+5)]$
$=[100(0.06)](x+5)$ *Associative property*
$=6(x+5)$
$=6(x)+6(5)$ *Distributive property*
$=6x+30$

65. $-6-4(y-7)$
$=-6-4(y)+(-4)(-7)$ *Distributive property*
$=-6-4y+28$
$=-4y+22$

66. $-4-5(t-13)$
$=-4-5(t)+(-5)(-13)$ *Distributive property*
$=-4-5t+65$
$=-5t+61$

67. $-\frac{4}{3}(y-12)-\frac{1}{6}y$
$=-\frac{4}{3}y-\frac{4}{3}(-12)-\frac{1}{6}y$
$=-\frac{4}{3}y+16-\frac{1}{6}y$
$=-\frac{4}{3}y-\frac{1}{6}y+16$
$=\left(-\frac{8}{6}-\frac{1}{6}\right)y+16$
$=-\frac{3}{2}y+16$ $\left[-\frac{9}{6}=-\frac{3}{2}\right]$

68. $-\frac{7}{5}(t-15)-\frac{1}{2}t=-\frac{7}{5}t-\frac{7}{5}(-15)-\frac{1}{2}t$
$=-\frac{14}{10}t+21-\frac{5}{10}t$
$=-\frac{19}{10}t+21$

69. $-5(5y-9)+3(3y+6)$
$= -5(5y)+(-5)(-9)+3(3y)+3(6)$
Distributive property
$= -25y+45+9y+18$
$= (-25y+9y)+(45+18)$
$= (-25+9)y+63$
$= -16y+63$

70. $-3(2t+4)+8(2t-4)$
$= -3(2t)+(-3)(4)+8(2t)+8(-4)$
Distributive property
$= -6t-12+16t-32$
$= (-6t+16t)+(-12-32)$
$= (-6+16)t+(-44)$
$= 10t-44$

71. $-3(2r-3)+2(5r+3)$
$= -3(2r)+(-3)(-3)+2(5r)+2(3)$
Distributive property
$= -6r+9+10r+6$
$= (-6r+10r)+(9+6)$
$= (-6+10)r+15$
$= 4r+15$

72. $-4(5y-7)+3(2y-5)$
$= -4(5y)+(-4)(-7)+3(2y)+3(-5)$
Distributive property
$= -20y+28+6y-15$
$= (-20y+6y)+(28-15)$
$= (-20+6)y+13$
$= -14y+13$

73. $8(2k-1)-(4k-3)$
$= 8(2k-1)-1(4k-3)$
Replace $-$ *with* -1.
$= 8(2k)+8(-1)+(-1)(4k)+(-1)(-3)$
$= 16k-8-4k+3$
$= 12k-5$

74. $6(3p-2)-(5p+1)$
$= 6(3p-2)-1(5p+1)$
Replace $-$ *with* -1.
$= 6(3p)+6(-2)+(-1)(5p)+(-1)(1)$
$= 18p-12-5p-1$
$= 13p-13$

75. $-2(-3k+2)-(5k-6)-3k-5$
$= -2(-3k)+(-2)(2)-1(5k-6)-3k-5$
$= 6k-4+(-1)(5k)+(-1)(-6)-3k-5$
$= 6k-4-5k+6-3k-5$
$= -2k-3$

76. $-2(3r-4)-(6-r)+2r-5$
$= -2(3r)+(-2)(-4)-1(6-r)+2r-5$
$= -6r+8+(-1)(6)+(-1)(-r)+2r-5$
$= -6r+8-6+r+2r-5$
$= -3r-3$

77. $-4(-3x+3)-(6x-4)-2x+1$
$= -4(-3x+3)-1(6x-4)-2x+1$
$= 12x-12-6x+4-2x+1$
Distributive property
$= (12x-6x-2x)+(-12+4+1)$
Group like terms.
$= 4x-7$ *Combine like terms.*

78. $-5(8x+2)-(5x-3)-3x+17$
$= -5(8x+2)-1(5x-3)-3x+17$
$= -40x-10-5x+3-3x+17$
$= -48x+10$

79. $-7.5(2y+4)-2.9(3y-6)$
$= -7.5(2y)-7.5(4)-2.9(3y)-2.9(-6)$
Distributive property
$= -15y-30-8.7y+17.4$ *Multiply.*
$= -23.7y-12.6$ *Combine like terms.*

80. $8.4(6t-6)+2.4(9-3t)$
$= 8.4(6t)+8.4(-6)+2.4(9)+2.4(-3t)$
$= 50.4t-50.4+21.6-7.2t$
$= 43.2t-28.$

81. "Five times a number, added to the sum of the number and three" is written $(x+3)+5x$.

$$\begin{aligned}(x+3)+5x &= x+3+5x\\ &= (x+5x)+3\\ &= 6x+3\end{aligned}$$

82. "Six times a number, added to the sum of the number and six" is written $(x+6)+6x$.

$$\begin{aligned}(x+6)+6x &= x+6+6x\\ &= (x+6x)+6\\ &= 7x+6\end{aligned}$$

83. "A number multiplied by -7, subtracted from the sum of 13 and six times the number" is written $(13+6x)-(-7x)$.

$$\begin{aligned}(13+6x)-(-7x) &= 13+6x+7x\\ &= 13+13x\end{aligned}$$

84. "A number multiplied by 5, subtracted from the sum of 14 and eight times the number" is written $(14+8x)-5x$.

$$\begin{aligned}(14+8x)-5x &= 14+8x-5x\\ &= 14+3x\end{aligned}$$

85. "Six times a number added to -4, subtracted from twice the sum of three times the number and 4" is written $2(3x+4)-(-4+6x)$.

$$\begin{aligned}&2(3x+4)-(-4+6x)\\&=2(3x+4)-1(-4+6x)\\&=6x+8+4-6x\\&=6x+(-6x)+8+4\\&=0+12=12\end{aligned}$$

86. "Nine times a number added to 6, subtracted from triple the sum of 12 and 8 times the number" is written $3(12+8x)-(6+9x)$.

$$\begin{aligned}&3(12+8x)-(6+9x)\\&=3(12+8x)-1(6+9x)\\&=36+24x-6-9x\\&=30+15x\end{aligned}$$

87. For widgets, the fixed cost is \$1000 and the variable cost is \$5 per widget, so the cost to produce x widgets is

$$1000+5x \text{ (dollars).}$$

88. For gadgets, the fixed cost is \$750 and the variable cost is \$3 per gadget, so the cost to produce y gadgets is

$$750+3y \text{ (dollars).}$$

89. The total cost to make x widgets and y gadgets is

$$1000+5x+750+3y \text{ (dollars).}$$

90.
$$\begin{aligned}&1000+5x+750+3y\\&=(1000+750)+5x+3y\\&=1750+5x+3y,\end{aligned}$$

so the total cost to make x widgets and y gadgets is

$$1750+5x+3y \text{ (dollars).}$$

Chapter 1 Review Exercises

1.
$$\begin{aligned}\frac{8}{5}\div\frac{32}{15}&=\frac{8}{5}\cdot\frac{15}{32}\\&=\frac{8\cdot(3\cdot5)}{5\cdot(8\cdot4)}\\&=\frac{8\cdot3\cdot5}{5\cdot8\cdot4}\\&=\frac{3}{4}\end{aligned}$$

2.
$$\begin{aligned}2\frac{4}{5}\cdot1\frac{1}{4}&=\frac{14}{5}\cdot\frac{5}{4}\\&=\frac{2\cdot7\cdot5}{5\cdot2\cdot2}\\&=\frac{7}{2},\text{ or }3\frac{1}{2}\end{aligned}$$

3.
$$\begin{aligned}\frac{5}{8}-\frac{1}{6}&=\frac{5\cdot3}{8\cdot3}-\frac{1\cdot4}{6\cdot4}\quad \textit{LCD = 24}\\&=\frac{15}{24}-\frac{4}{24}\\&=\frac{11}{24}\end{aligned}$$

4.
$$\begin{aligned}\frac{3}{8}+3\frac{1}{2}-\frac{3}{16}&=\frac{3}{8}+\frac{7}{2}-\frac{3}{16}\\&=\frac{3\cdot2}{8\cdot2}+\frac{7\cdot8}{2\cdot8}-\frac{3}{16}\quad \textit{LCD = 16}\\&=\frac{6}{16}+\frac{56}{16}-\frac{3}{16}\\&=\frac{62}{16}-\frac{3}{16}\\&=\frac{59}{16},\text{ or }3\frac{11}{16}\end{aligned}$$

5.
$$\begin{aligned}\frac{1}{6}\cdot7618&=\frac{1}{6}\cdot\frac{7618}{1}\\&=\frac{2\cdot3809}{2\cdot3}\\&=\frac{3809}{3}\approx1269.7\end{aligned}$$

About 1270 thousand luxury cars were sold in the United States in 2007.

6. Since the entire pie chart represents $\frac{3}{3}$, this leaves $\frac{3}{3}-\frac{1}{3}=\frac{2}{3}$ of the cars that are *not* small.

$$\begin{aligned}\frac{2}{3}\cdot7618&=\frac{2}{3}\cdot\frac{7618}{1}\\&=\frac{2\cdot7618}{3\cdot1}\\&=\frac{15{,}236}{3}\approx5078.7\end{aligned}$$

About 5079 thousand cars sold in the United States in 2007 were *not* small.

7. $5^4=5\cdot5\cdot5\cdot5=625$

8. $\left(\frac{3}{5}\right)^3=\frac{3}{5}\cdot\frac{3}{5}\cdot\frac{3}{5}=\frac{27}{125}$

9.
$$\begin{aligned}(0.02)^2&=(0.02)(0.02)\\&=0.0004\end{aligned}$$

10.
$$\begin{aligned}(0.1)^3&=(0.1)(0.1)(0.1)\\&=0.001\end{aligned}$$

11. $8\cdot5-13=40-13=27$

12.
$$\begin{aligned}16+12\div4-2&=16+(12\div4)-2\\&=16+3-2\\&=19-2\\&=17\end{aligned}$$

13. $20 - 2(5+3) = 20 - 2(8)$
$= 20 - 16$
$= 4$

14. $7\left[3 + 6(3^2)\right] = 7[3 + 6(9)]$
$= 7(3 + 54)$
$= 7(57)$
$= 399$

15. $\dfrac{9(4^2 - 3)}{4 \cdot 5 - 17} = \dfrac{9(16-3)}{20-17}$
$= \dfrac{9(13)}{3}$
$= \dfrac{3 \cdot 3 \cdot 13}{3} = 39$

16. $\dfrac{6(5-4) + 2(4-2)}{3^2 - (4+3)} = \dfrac{6(1) + 2(2)}{9 - (4+3)}$
$= \dfrac{6+4}{9-7}$
$= \dfrac{10}{2} = 5$

17. $12 \cdot 3 - 6 \cdot 6 = 36 - 36 = 0$

Since $0 = 0$ is true, so is $0 \leq 0$, and therefore, the statement "$12 \cdot 3 - 6 \cdot 6 \leq 0$" is true.

18. $3[5(2) - 3] = 3(10 - 3) = 3(7) = 21$

Therefore, the statement "$3[5(2) - 3] > 20$" is true.

19. $4^2 - 8 = 16 - 8 = 8$

Since $9 \leq 8$ is false, the statement "$9 \leq 4^2 - 8$" is false.

20. "Thirteen is less than seventeen" is written $13 < 17$.

21. "Five plus two is not equal to ten" is written $5 + 2 \neq 10$.

22. "Two-thirds is greater than or equal to four-sixths" is written $\frac{2}{3} \geq \frac{4}{6}$.

In Exercises 23–26, replace x with 6 and y with 3.

23. $2x + 6y = 2(6) + 6(3)$
$= 12 + 18 = 30$

24. $4(3x - y) = 4[3(6) - 3]$
$= 4(18 - 3)$
$= 4(15) = 60$

25. $\dfrac{x}{3} + 4y = \dfrac{6}{3} + 4(3)$
$= 2 + 12 = 14$

26. $\dfrac{x^2 + 3}{3y - x} = \dfrac{6^2 + 3}{3(3) - 6}$
$= \dfrac{36 + 3}{9 - 6}$
$= \dfrac{39}{3} = 13$

27. "Six added to a number" translates as $x + 6$.

28. "A number subtracted from eight" translates as $8 - x$.

29. "Nine subtracted from six times a number" translates as $6x - 9$.

30. "Three-fifths of a number added to 12" translates as $12 + \frac{3}{5}x$.

31. $5x + 3(x+2) = 22;\ 2$
$5x + 3(x+2) = 5(2) + 3(2+2)$ *Let x = 2.*
$= 5(2) + 3(4)$
$= 10 + 12 = 22$

Since the left side and the right side are equal, 2 is a solution of the given equation.

32. $\dfrac{t+5}{3t} = 1;\ 6$

$\dfrac{t+5}{3t} = \dfrac{6+5}{3(6)}$ *Let t = 6.*
$= \dfrac{11}{18}$

Since the left side, $\frac{11}{18}$, is not equal to the right side, 1, 6 is not a solution of the equation.

33. "Six less than twice a number is 10" is written

$$2x - 6 = 10.$$

Letting x equal $0,\ 2,\ 4,\ 6,$ and 10 results in a false statement, so those values are not solutions.

Since $2(8) - 6 = 16 - 6 = 10$, the solution is 8.

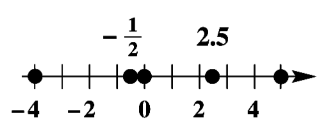
34. "The product of a number and 4 is 8" is written

$$4x = 8.$$

Since $4(2) = 8$, the solution is 2.

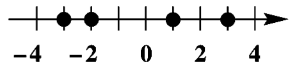
35. $-4, -\frac{1}{2}, 0, 2.5, 5$

Graph these numbers on a number line. They are already arranged in order from smallest to largest.



36. $-2, |-3|, -3, |-1|$

Recall that $|-3| = 3$ and $|-1| = 1$. From smallest to largest, the numbers are $-3, -2, |-1|, |-3|$.



37. Since $\frac{4}{3}$ is the quotient of two integers, it is a *rational number*. Since all rational numbers are also real numbers, $\frac{4}{3}$ is a *real number*.

38. Since the decimal representation of $0.\overline{63}$ repeats, it is a *rational number*. Since all rational numbers are also real numbers, $0.\overline{63}$ is a *real number*.

39. Since 19 is a *natural number*, it is also a *whole number* and an *integer*. We can write it as $\frac{19}{1}$, so it is a *rational number* and hence, a *real number*.

40. Since the decimal representation of $\sqrt{6}$ does not terminate or repeat, it is an *irrational number*. Since all irrational numbers are also real numbers, $\sqrt{6}$ is a *real number*.

41. $-10, 5$
Since any negative number is less than any positive number, -10 is the lesser number.

42. $-8, -9$
Since -9 is to the left of -8 on the number line, -9 is the lesser number.

43. $-\dfrac{2}{3}, -\dfrac{3}{4}$

To compare these fractions, use a common denominator.

$$-\frac{2}{3} = -\frac{8}{12}, \quad -\frac{3}{4} = -\frac{9}{12}$$

Since $-\frac{9}{12}$ is to the left of $-\frac{8}{12}$ on the number line, $-\frac{3}{4}$ is the lesser number.

44. $0, -|23|$
Since $-|23| = -23$ and $-23 < 0$, $-|23|$ is the lesser number.

45. $12 > -13$
This statement is true since 12 is to the right of -13 on the number line.

46. $0 > -5$
This statement is true since 0 is to the right of -5 on the number line.

47. $-9 < -7$
This statement is true since -9 is to the left of -7 on the number line.

48. $-13 \geq -13$
This is a true statement since $-13 = -13$.

49. **(a)** The opposite of the number -9 is its negative; that is, $-(-9) = 9$.

(b) Since $-9 < 0$, the absolute value of the number -9 is $|-9| = -(-9) = 9$.

50. 0 **(a)** $-0 = 0$ **(b)** $|0| = 0$

51. 6 **(a)** $-(6) = -6$ **(b)** $|6| = 6$

52. $-\frac{5}{7}$ **(a)** $-(-\frac{5}{7}) = \frac{5}{7}$

(b) $\left|-\frac{5}{7}\right| = -(-\frac{5}{7}) = \frac{5}{7}$

53. $|-12| = -(-12) = 12$

54. $-|3| = -3$

55. $-|-19| = -[-(-19)] = -19$

56. $-|9-2| = -|7| = -7$

57. $-10 + 4 = -6$

58. $14 + (-18) = -4$

59. $-8 + (-9) = -17$

60. $$\begin{aligned}\frac{4}{9} + \left(-\frac{5}{4}\right) &= \frac{4 \cdot 4}{9 \cdot 4} + \left(-\frac{5 \cdot 9}{4 \cdot 9}\right) \quad \textit{LCD = 36} \\ &= \frac{16}{36} + \left(-\frac{45}{36}\right) \\ &= -\frac{29}{36}\end{aligned}$$

61. $-13.5 + (-8.3) = -21.8$

62. $$\begin{aligned}(-10 + 7) + (-11) &= (-3) + (-11) \\ &= -14\end{aligned}$$

63. $$\begin{aligned}&[-6 + (-8) + 8] + [9 + (-13)] \\ &= \{[-6 + (-8)] + 8\} + (-4) \\ &= [(-14) + 8] + (-4) \\ &= (-6) + (-4) = -10\end{aligned}$$

64. $$\begin{aligned}&(-4 + 7) + (-11 + 3) + (-15 + 1) \\ &= (3) + (-8) + (-14) \\ &= [3 + (-8)] + (-14) \\ &= (-5) + (-14) = -19\end{aligned}$$

65. $-7 - 4 = -7 + (-4) = -11$

66. $-12 - (-11) = -12 + (11) = -1$

67. $5 - (-2) = 5 + (2) = 7$

68. $$\begin{aligned}-\frac{3}{7} - \frac{4}{5} &= -\frac{3 \cdot 5}{7 \cdot 5} - \frac{4 \cdot 7}{5 \cdot 7} \\ &= -\frac{15}{35} - \frac{28}{35} \quad \textit{LCD = 35} \\ &= -\frac{15}{35} + \left(-\frac{28}{35}\right) \\ &= -\frac{43}{35}, \text{ or } -1\frac{8}{35}\end{aligned}$$

69. $2.56-(-7.75)=2.56+(7.75)$
$=10.31$

70. $(-10-4)-(-2)=[-10+(-4)]+2$
$=(-14)+(2)$
$=-12$

71. $(-3+4)-(-1)=(-3+4)+1$
$=1+1$
$=2$

72. $-(-5+6)-2=-(1)+(-2)$
$=-1+(-2)$
$=-3$

73. "19 added to the sum of -31 and 12" is written

$$(-31+12)+19=(-19)+19$$
$$=0.$$

74. "13 more than the sum of -4 and -8" is written

$$[-4+(-8)]+13=-12+13$$
$$=1.$$

75. "The difference between -4 and -6" is written

$$-4-(-6)=-4+6$$
$$=2.$$

76. "Five less than the sum of 4 and -8" is written

$$[4+(-8)]-5=(-4)+(-5)$$
$$=-9.$$

77. $x+(-2)=-4$

Because $(-2)+(-2)=-4$,
the solution is -2.

78. $12+x=11$

Because $12+(-1)=1$,
the solution is -1.

79. $-23.75+50.00=26.25$

He now has a positive balance of \$26.25.

80. $-26+16=-10$

The high temperature was -10°F.

81. $-28+13-14=(-28+13)-14$
$=(-28+13)+(-14)$
$=-15+(-14)$
$=-29$

His present financial status is $-$\$29.

82. $-3-7=-3+(-7)$
$=-10$

The new temperature is -10°.

83. $8-12+42=[8+(-12)]+42$
$=-4+42$
$=38$

The total net yardage is 38.

84. To get the closing value for the previous day, we can add the amount it was down to the amount at which it closed.

$$47.92+9496.28=9544.20$$

85. $(-12)(-3)=36$

86. $15(-7)=-(15\cdot 7)$
$=-105$

87. $-\frac{4}{3}\left(-\frac{3}{8}\right)=\frac{4}{3}\cdot\frac{3}{8}$
$=\frac{4\cdot 3}{3\cdot 4\cdot 2}$
$=\frac{1}{2}$

88. $(-4.8)(-2.1)=10.08$

89. $5(8-12)=5[8+(-12)]$
$=5(-4)=-20$

90. $(5-7)(8-3)=[5+(-7)][8+(-3)]$
$=(-2)(5)=-10$

91. $2(-6)-(-4)(-3)=-12-(12)$
$=-12+(-12)$
$=-24$

92. $3(-10)-5=-30+(-5)=-35$

93. $\frac{-36}{-9}=\frac{4\cdot 9}{9}=4$

94. $\frac{220}{-11}=-\frac{20\cdot 11}{11}=-20$

95. $-\frac{1}{2}\div\frac{2}{3}=-\frac{1}{2}\cdot\frac{3}{2}=-\frac{3}{4}$

96. $-33.9\div(-3)=\frac{-33.9}{-3}=11.3$

97. $\frac{-5(3)-1}{8-4(-2)}=\frac{-15+(-1)}{8-(-8)}$
$=\frac{-16}{8+8}$
$=\frac{-16}{16}=-1$

98. $$\frac{5(-2)-3(4)}{-2[3-(-2)]-1}=\frac{-10-12}{-2(3+2)-1}=\frac{-10+(-12)}{-2(5)-1}=\frac{-22}{-10+(-1)}=\frac{-22}{-11}=2$$

99. $$\frac{10^2-5^2}{8^2+3^2-(-2)}=\frac{100-25}{64+9+2}=\frac{75}{75}=1$$

100. $$\frac{(0.6)^2+(0.8)^2}{(-1.2)^2-(-0.56)}=\frac{0.36+0.64}{1.44+0.56}=\frac{1.00}{2.00}=0.5$$

In Exercises 101–104, replace x with -5, y with 4, and z with -3.

101. $$6x-4z=6(-5)-4(-3)=-30-(-12)=-30+12=-18$$

102. $$5x+y-z=5(-5)+(4)-(-3)=(-25+4)+3=-21+3=-18$$

103. $$5x^2=5(-5)^2=5(25)=125$$

104. $$z^2(3x-8y)=(-3)^2[3(-5)-8(4)]=9(-15-32)=9[-15+(-32)]=9(-47)=-423$$

105. "Nine less than the product of -4 and 5" is written

$$-4(5)-9=-20+(-9)=-29.$$

106. "Five-sixths of the sum of 12 and -6" is written

$$\tfrac{5}{6}[12+(-6)]=\tfrac{5}{6}(6)=5.$$

107. "The quotient of 12 and the sum of 8 and -4" is written

$$\frac{12}{8+(-4)}=\frac{12}{4}=3.$$

108. "The product of -20 and 12, divided by the difference between 15 and -15" is written

$$\frac{-20(12)}{15-(-15)}=\frac{-240}{15+15}=\frac{-240}{30}=-8.$$

109. "8 times a number is -24" is written

$$8x=-24.$$

If $x=-3$,

$$8x=8(-3)=-24.$$

The solution is -3.

110. "The quotient of a number and 3 is -2" is written

$$\frac{x}{3}=-2.$$

If $x=-6$,

$$\frac{x}{3}=\frac{-6}{3}=-2.$$

The solution is -6.

111. Find the average of the eight numbers.

$$\frac{26+38+40+20+4+14+96+18}{8}=\frac{256}{8}=\frac{8\cdot 32}{8}=32$$

112. Find the average of the six numbers.

$$\frac{-12+28+(-36)+0+12+(-10)}{6}=\frac{-18}{6}=-3$$

113. $6+0=6$

This is an example of an identity property.

114. $5\cdot 1=5$

This is an example of an identity property.

115. $-\frac{2}{3}\left(-\frac{3}{2}\right)=1$

This is an example of an inverse property.

116. $17+(-17)=0$

This is an example of an inverse property.

117. $5+(-9+2)=[5+(-9)]+2$

This is an example of an associative property.

118. $w(xy)=(wx)y$

This is an example of an associative property.

119. $3x+3y=3(x+y)$

This is an example of the distributive property.

120. $(1+2)+3=3+(1+2)$

This is an example of a commutative property.

121. $7y+14=7y+7\cdot 2$
$=7(y+2)$

122. $-12(4-t)=-12(4)-(-12)(t)$
$=-48+12t$

123. $3(2s)+3(5y)=3(2s+5y)$

124. $-(-4r+5s)=-1(-4r+5s)$
$=(-1)(-4r)+(-1)(5s)$
$=4r-5s$

125. $2m+9m=(2+9)m$ *Distributive property*
$=11m$

126. $15p^2-7p^2+8p^2$
$=(15-7+8)p^2$ *Distributive property*
$=16p^2$

127. $5p^2-4p+6p+11p^2$
$=(5+11)p^2+(-4+6)p$
Distributive property
$=16p^2+2p$

128. $-2(3k-5)+2(k+1)$
$=-6k+10+2k+2$
Distributive property
$=-4k+12$

129. $7(2m+3)-2(8m-4)$
$=14m+21-16m+8$
Distributive property
$=(14-16)m+29$
$=-2m+29$

130. $-(2k+8)-(3k-7)$
$=-1(2k+8)-1(3k-7)$
Replace $-$ with -1.
$=-2k-8-3k+7$
Distributive property
$=-5k-1$

131. "Seven times a number, subtracted from the product of -2 and three times the number" is written

$-2(3x)-7x=-6x-7x=-13x.$

132. "A number multiplied by 8, added to the sum of 5 and four times the number" is written

$(5+4x)+8x=5+(4x+8x)=5+12x.$

133. **[1.6]** $\frac{6(-4)+2(-12)}{5(-3)+(-3)}=\frac{-24+(-24)}{-15+(-3)}$
$=\frac{-48}{-18}=\frac{8\cdot 6}{3\cdot 6}$
$=\frac{8}{3},$ or $2\frac{2}{3}$

134. **[1.5]** $\frac{3}{8}-\frac{5}{12}=\frac{3\cdot 3}{8\cdot 3}-\frac{5\cdot 2}{12\cdot 2}$
$=\frac{9}{24}-\frac{10}{24}$
$=\frac{9}{24}+\left(-\frac{10}{24}\right)$
$=-\frac{1}{24}$

135. **[1.6]** $\frac{8^2+6^2}{7^2+1^2}=\frac{64+36}{49+1}$
$=\frac{100}{50}=2$

136. **[1.6]** $-\frac{12}{5}\div\frac{9}{7}=-\frac{12}{5}\cdot\frac{7}{9}$
$=-\frac{12\cdot 7}{5\cdot 9}$
$=-\frac{3\cdot 4\cdot 7}{5\cdot 3\cdot 3}$
$=-\frac{28}{15},$ or $-1\frac{13}{15}$

137. **[1.5]** $2\frac{5}{6}-4\frac{1}{3}=\frac{17}{6}-\frac{13}{3}$
$=\frac{17}{6}-\frac{13\cdot 2}{3\cdot 2}$
$=\frac{17}{6}-\frac{26}{6}$
$=\frac{17}{6}+\left(-\frac{26}{6}\right)$
$=-\frac{9}{6}=-\frac{3}{2},$ or $-1\frac{1}{2}$

138. **[1.6]** $\left(-\frac{5}{6}\right)^2=\left(-\frac{5}{6}\right)\left(-\frac{5}{6}\right)$
$=\frac{25}{36}$

139. **[1.5]** $[(-2)+7-(-5)]+[-4-(-10)]$
$=\{[(-2)+7]-(-5)\}+(-4+10)$
$=(5+5)+6$
$=10+6=16$

140. [1.6] $-16(-3.5) - 7.2(-3)$
$= 56 - [(7.2)(-3)]$
$= 56 - (-21.6)$
$= 56 + 21.6$
$= 77.6$

141. [1.5] $-8 + [(-4 + 17) - (-3 - 3)]$
$= -8 + \{(13) - [-3 + (-3)]\}$
$= -8 + [13 - (-6)]$
$= -8 + (13 + 6)$
$= -8 + 19 = 11$

142. [1.8] $-4(2t + 1) - 8(-3t + 4)$
$= -4(2t) - 4(1) - 8(-3t) - 8(4)$
$= -8t - 4 + 24t - 32$
$= 16t - 36$

143. [1.8] $5x^2 - 12y^2 + 3x^2 - 9y^2$
$= (5x^2 + 3x^2) + (-12y^2 - 9y^2)$
$= (5 + 3)x^2 + (-12 - 9)y^2$
$= 8x^2 - 21y^2$

144. [1.6] $(-8 - 3) - 5(2 - 9)$
$= [-8 + (-3)] - 5[2 + (-9)]$
$= -11 - 5(-7)$
$= -11 - (-35)$
$= -11 + 35 = 24$

145. [1.6] Dividing 0 *by* a nonzero number gives a quotient of 0. However, dividing a number *by* 0 is undefined.

146. [1.5] $118 - 165 = 118 + (-165)$
$= -47$

The lowest temperature ever recorded in Iowa was -47°F.

147. [1.5] The change in enrollment from 1980 to 1985 was $12.39 - 13.23 = -0.84$ million students. Expressed as an integer, this number is $-840{,}000$.

148. [1.5] The change in enrollment from 1985 to 1990 was $11.34 - 12.39 = -1.05$ million students. Expressed as an integer, this number is $-1{,}050{,}000$.

149. [1.5] The change in enrollment from 1995 to 2000 was $13.52 - 12.50 = 1.02$ million students. Expressed as an integer, this number is 1,020,000.

150. [1.5] The change in enrollment from 2000 to 2005 was $14.91 - 13.52 = 1.39$ million students. Expressed as an integer, this number is 1,390,000.

Chapter 1 Test

1. $\frac{63}{99} = \frac{7 \cdot 9}{11 \cdot 9} = \frac{7}{11}$

2. The denominators are $8,\ 12,$ and 15; or equivalently, $2^3, 2^2 \cdot 3,$ and $3 \cdot 5$. So the LCD is $2^3 \cdot 3 \cdot 5 = 120$.

$\frac{5}{8} + \frac{11}{12} + \frac{7}{15}$
$= \frac{5 \cdot 15}{8 \cdot 15} + \frac{11 \cdot 10}{12 \cdot 10} + \frac{7 \cdot 8}{15 \cdot 8}$
$= \frac{75}{120} + \frac{110}{120} + \frac{56}{120}$
$= \frac{241}{120}$, or $2\frac{1}{120}$

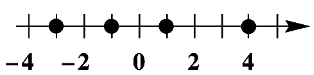
3. $\frac{19}{15} \div \frac{6}{5} = \frac{19}{15} \cdot \frac{5}{6} = \frac{19 \cdot 5}{3 \cdot 5 \cdot 6} = \frac{19}{18}$, or $1\frac{1}{18}$

4. $4[-20 + 7(-2)] = 4[-20 + (-14)]$
$= 4(-34) = -136$

Since $-136 \leq 135$, the statement "$4[-20 + 7(-2)] \leq 135$" is true.

5. $-1, -3, |-4|, |-1|$

Recall that $|-4| = 4$ and $|-1| = 1$. From smallest to largest, the numbers are $-3, -1, |-1|, |-4|$.



6. The number $-\frac{2}{3}$ can be written as a quotient of two integers with denominator not 0, so it is a *rational number*. Since all rational numbers are real numbers, it is also a *real number*.

7. If -8 and -1 are both graphed on a number line, we see that the point for -8 is to the *left* of the point for -1. This indicates that -8 is *less than* -1.

8. "The quotient of -6 and the sum of 2 and -8" is written $\frac{-6}{2 + (-8)}$,

and $\frac{-6}{2 + (-8)} = \frac{-6}{-6} = 1$.

9. $-2 - (5 - 17) + (-6)$
$= -2 - [5 + (-17)] + (-6)$
$= -2 - (-12) + (-6)$
$= (-2 + 12) + (-6)$
$= 10 + (-6) = 4$

10. $-5\frac{1}{2}+2\frac{2}{3}=-\frac{11}{2}+\frac{8}{3}$

$=-\frac{11\cdot 3}{2\cdot 3}+\frac{8\cdot 2}{3\cdot 2}$

$=-\frac{33}{6}+\frac{16}{6}$

$=-\frac{17}{6}$, or $-2\frac{5}{6}$

11. $-6-[-7+(2-3)]$
$=-6-[-7+(-1)]$
$=-6-(-8)$
$=-6+8=2$

12. $4^2+(-8)-(2^3-6)$
$=16+(-8)-(8-6)$
$=[16+(-8)]-2$
$=8-2=6$

13. $(-5)(-12)+4(-4)+(-8)^2$
$=(-5)(-12)+4(-4)+64$
$=[60+(-16)]+64$
$=44+64=108$

14. $\frac{30(-1-2)}{-9[3-(-2)]-12(-2)}$

$=\frac{30(-3)}{-9(5)-(-24)}$

$=\frac{-90}{-45+24}$

$=\frac{-90}{-21}$

$=\frac{30\cdot 3}{7\cdot 3}=\frac{30}{7}$, or $4\frac{2}{7}$

15. $-x+3=-3$

If $x=6$,

$$-6+3=-3.$$

Therefore, the solution is 6.

16. $-3x=-12$

If $x=4$,

$$-3x=-3(4)=-12.$$

Therefore, the solution is 4.

17. $3x-4y^2$
$=3(-2)-4(4^2)$ *Let x = –2, y = 4.*
$=3(-2)-4(16)$
$=-6-64=-70$

18. $\frac{5x+7y}{3(x+y)}$

$=\frac{5(-2)+7(4)}{3(-2+4)}$ *Let x = –2, y = 4.*

$=\frac{-10+28}{3(2)}$

$=\frac{18}{6}=3$

19. The difference between the highest and lowest elevations is

$6960-(-40)=6960+40=7000$ meters.

20. 4 saves (3 points per save)

+ 3 wins (3 points per win)

+ 2 losses (-2 points per loss)

+ 1 blown save (-2 points per blown save)

$=4(3)+3(3)+2(-2)+1(-2)$
$=12+9-4-2$
$=15$ points

He has a total of 15 points.

21. $2.10-3.52=2.10+(-3.52)=-1.42$

As a signed number, the federal budget deficit is $-\$1.42$ trillion.

22. Commutative property

$$(5+2)+8=8+(5+2)$$

illustrates a commutative property because the order of the numbers is changed, but not the grouping. The correct response is **B**.

23. Associative property

$$-5+(3+2)=(-5+3)+2$$

illustrates an associative property because the grouping of the numbers is changed, but not the order. The correct response is **D**.

24. Inverse property

$$-\frac{5}{3}\left(-\frac{3}{5}\right)=1$$

illustrates an inverse property. The correct response is **E**.

25. Identity property

$$3x+0=3x$$

illustrates an identity property. The correct response is **A**.

26. Distributive property

$$-3(x+y)=-3x+(-3y)$$

illustrates the distributive property. The correct response is **C**.

27. $3(x+1)=3\cdot x+3\cdot 1$
$=3x+3$

The distributive property is used to rewrite $3(x+1)$ as $3x+3$.

28. **(a)** $-6[5+(-2)]=-6(3)=-18$

(b) $-6[5+(-2)]=-6(5)+(-6)(-2)$
$=-30+12=-18$

(c) The distributive property assures us that the answers must be the same, because $a(b+c)=ab+ac$ for all a, b, c.

29. $8x+4x-6x+x+14x$
$=(8+4-6+1+14)x$
$=21x$

30. $5(2x-1)-(x-12)+2(3x-5)$
$=5(2x-1)-1(x-12)+2(3x-5)$
$=10x-5-x+12+6x-10$
$=(10-1+6)x+(-5+12-10)$
$=15x-3$