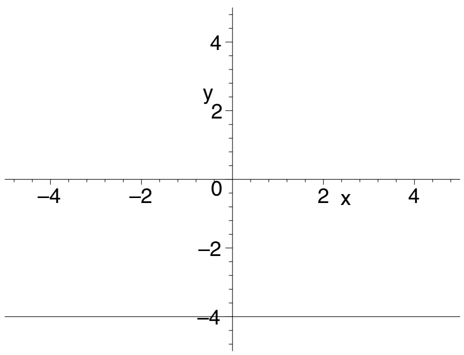
Chapter 1

Functions and Graphs

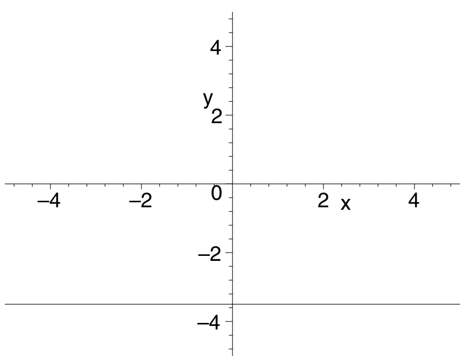
Exercise Set 1.1

1. Graph $y = -4$.

Note that y is constant and therefore any value of x we choose will yield the same value for y, which is -4. Thus, we will have a horizontal line at $y = -4$.

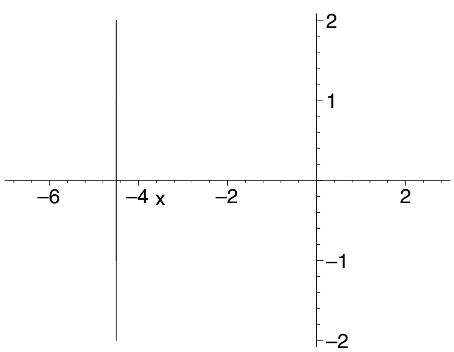


2. Horizontal line at $y = -3.5$

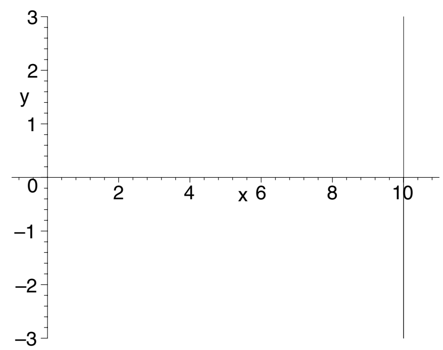


3. Graph $x = -4.5$.

Note that x is constant and therefore any value of y we choose will yield the same value for x, which is 4.5. Thus, we will have a vertical line at $x = -4.5$.



4. A vertical line at $x = 10$



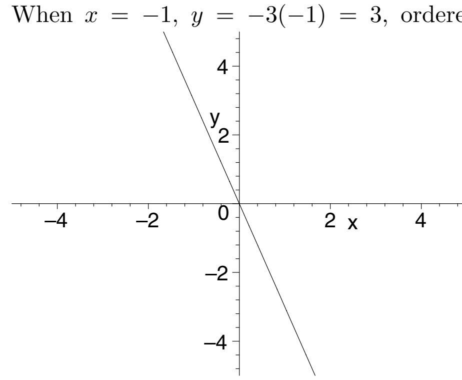
5. Graph. Find the slope and the y-intercept of $y = -3x$.

First, we find some points that satisfy the equation, then we plot the ordered pairs and connect the plotted points to get the graph.

When $x = 0$, $y = -3(0) = 0$, ordered pair $(0, 0)$

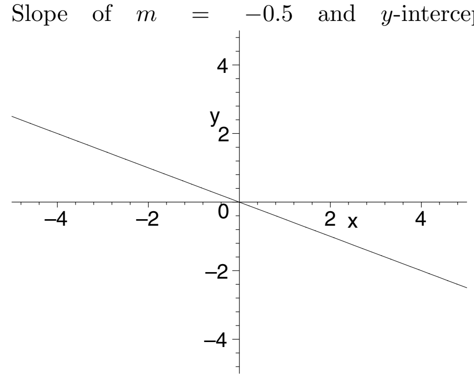
When $x = 1$, $y = -3(1) = -3$, ordered pair $(1, -3)$

When $x = -1$, $y = -3(-1) = 3$, ordered pair $(-1, 3)$



Compare the equation $y = -3x$ to the general linear equation form of $y = mx + b$ to conclude the equation has a slope of $m = -3$ and a y-intercept of $(0, 0)$.

6. Slope of $m = -0.5$ and y-intercept of $(0, 0)$



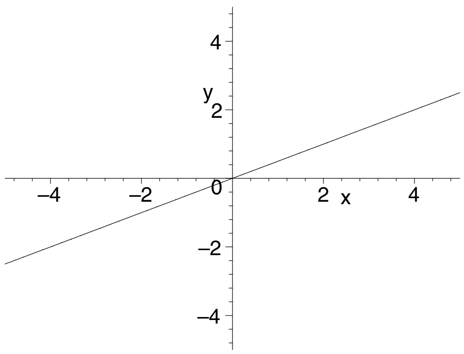
7. Graph. Find the slope and the y-intercept of $y = 0.5x$.

First, we find some points that satisfy the equation, then we plot the ordered pairs and connect the plotted points to get the graph.

When $x = 0$, $y = 0.5(0) = 0$, ordered pair $(0, 0)$

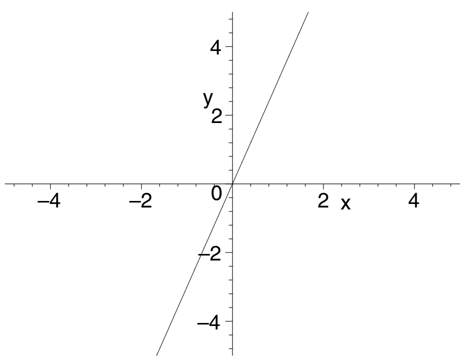
When $x = 6$, $y = 0.5(6) = 3$, ordered pair $(6, 3)$

When $x = -2$, $y = 0.5(-2) = -1$, ordered pair $(-2, -1)$



Compare the equation $y = 0.5x$ to the general linear equation form of $y = mx + b$ to conclude the equation has a slope of $m = 0.5$ and a y-intercept of $(0, 0)$.

8. Slope of $m = 3$ and y-intercept of $(0, 0)$



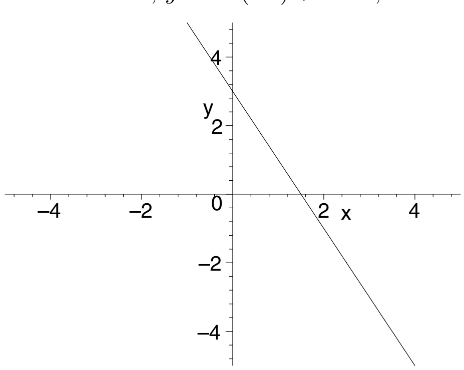
9. Graph. Find the slope and the y-intercept of $y = -2x + 3$.

First, we find some points that satisfy the equation, then we plot the ordered pairs and connect the plotted points to get the graph.

When $x = 0$, $y = -2(0) + 3 = 3$, ordered pair $(0, 3)$

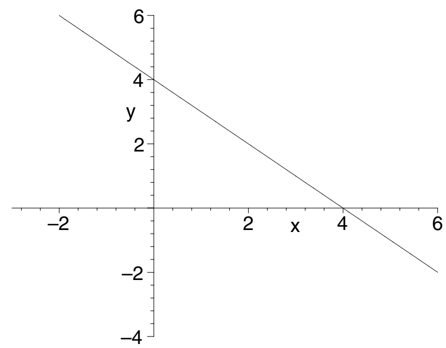
When $x = 2$, $y = -2(2) + 3 = -1$, ordered pair $(2, -1)$

When $x = -2$, $y = -2(-2) + 3 = 7$, ordered pair $(-2, 7)$



Compare the equation $y = -2x + 3$ to the general linear equation form of $y = mx + b$ to conclude the equation has a slope of $m = -2$ and a y-intercept of $(0, 3)$.

10. Slope of $m = -1$ and y-intercept of $(0, 4)$



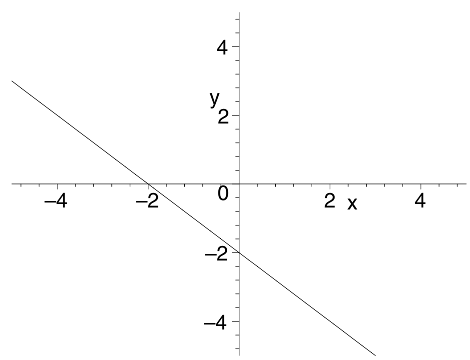
11. Graph. Find the slope and the y-intercept of $y = -x - 2$.

First, we find some points that satisfy the equation, then we plot the ordered pairs and connect the plotted points to get the graph.

When $x = 0$, $y = -(0) - 2 = -2$, ordered pair $(0, -2)$

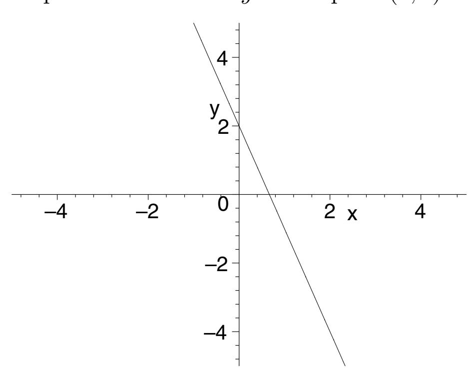
When $x = 3$, $y = -(3) - 2 = -5$, ordered pair $(3, -5)$

When $x = -2$, $y = -(-2) - 2 = 0$, ordered pair $(-2, 0)$



Compare the equation $y = -x - 2$ to the general linear equation form of $y = mx + b$ to conclude the equation has a slope of $m = -1$ and a y-intercept of $(0, -2)$.

12. Slope of $m = -3$ and y-intercept of $(0, 2)$



13. Find the slope and y-intercept of $2x + y - 2 = 0$.

Solve the equation for y.

$$\begin{aligned} 2x + y - 2 &= 0 \\ y &= -2x + 2 \end{aligned}$$

Compare to $y = mx + b$ to conclude the equation has a slope of $m = -2$ and a y-intercept of $(0, 2)$.

14. $y = 2x + 3$, slope of $m = 2$ and y-intercept of $(0, 3)$

15. Find the slope and y-intercept of $2x + 2y + 5 = 0$.

Solve the equation for y.

$$\begin{aligned} 2x + 2y + 5 &= 0 \\ 2y &= -2x - 5 \\ y &= -x - \frac{5}{2} \end{aligned}$$

Compare to $y = mx + b$ to conclude the equation has a slope of $m = -1$ and a y-intercept of $(0, -\frac{5}{2})$.

16. $y = x + 2$, slope of $m = 1$ and y-intercept of $(0, 2)$.

17. Find the slope and y-intercept of $x = 2y + 8$.

Solve the equation for y.

$$\begin{aligned} x &= 2y + 8 \\ x - 8 &= 2y \\ \frac{1}{2}x - 4 &= y \end{aligned}$$

Compare to $y = mx + b$ to conclude the equation has a slope of $m = \frac{1}{2}$ and a y-intercept of $(0, -4)$.

18. $y = -\frac{1}{4}x + \frac{3}{4}$, slope of $m = -\frac{1}{4}$ and y-intercept of $(0, \frac{3}{4})$

19. Find the equation of the line: with $m = -5$, containing $(1, -5)$

Plug the given information into equation $y - y_1 = m(x - x_1)$ and solve for y

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-5) &= -5(x - 1) \\ y + 5 &= -5x + 5 \\ y &= -5x + 5 - 5 \\ y &= -5x \end{aligned}$$

20.

$$\begin{aligned} y - 7 &= 7(x - 1) \\ y - 7 &= 7x - 7 \\ y &= 7x \end{aligned}$$

21. Find the equation of line: with $m = -2$, containing $(2, 3)$

Plug the given information into the equation $y - y_1 = m(x - x_1)$ and solve for y

$$\begin{aligned} y - 3 &= -2(x - 2) \\ y - 3 &= -2x + 4 \\ y &= -2x + 4 + 3 \\ y &= -2x + 7 \end{aligned}$$

22.

$$\begin{aligned} y - (-2) &= -3(x - 5) \\ y + 2 &= -3x + 15 \\ y = -3x + 13 \end{aligned}$$

23. Find the equation of line: with $m = 2$, containing $(3, 0)$

Plug the given information into the equation $y - y_1 = m(x - x_1)$ and solve for y

$$\begin{aligned} y - 0 &= 2(x - 3) \\ y &= 2x - 6 \end{aligned}$$

24.

$$\begin{aligned} y - 0 &= -5(x - 5) \\ y &= -5x + 25 \end{aligned}$$

25. Find the equation of line: with y-intercept $(0, -6)$ and $m = \frac{1}{2}$

Plug the given information into the equation $y = mx + b$

$$\begin{aligned} y &= mx + b \\ y &= \frac{1}{2}x + (-6) \\ y &= \frac{1}{2}x - 6 \end{aligned}$$

26. $y = \frac{4}{3}x + 7$

27. Find the equation of line: with $m = 0$, containing $(2, 3)$

Plug the given information into the equation $y - y_1 = m(x - x_1)$ and solve for y

$$\begin{aligned} y - 3 &= 0(x - 2) \\ y - 3 &= 0 \\ y &= 3 \end{aligned}$$

28.

$$\begin{aligned} y - 8 &= 0(x - 4) \\ y - 8 &= 0 \\ y &= 8 \end{aligned}$$

29. Find the slope given $(-4, -2)$ and $(-2, 1)$

Use the slope equation $m = \frac{y_2 - y_1}{x_2 - x_1}$. **NOTE:** It does not matter which point is chosen as (x_1, y_1) and which is chosen as (x_2, y_2) as long as the order the point coordinates are subtracted in the same order as illustrated below

$$\begin{aligned} m &= \frac{1 - (-2)}{-2 - (-4)} \\ &= \frac{1 + 2}{-2 + 4} \\ &= \frac{3}{2} \end{aligned}$$

$$\begin{aligned} m &= \frac{-2-1)}{-4-(-2)} \\ &= \frac{-3}{-2} \\ &= \frac{3}{2} \end{aligned}$$

30. $m = \frac{3-1}{6-(-2)} = \frac{2}{8} = \frac{1}{4}$

31. Find the slope given $(\frac{2}{5}, \frac{1}{2})$ and $(-3, \frac{4}{5})$

$$\begin{aligned} m &= \frac{\frac{4}{5} - \frac{1}{2}}{-3 - \frac{2}{5}} \\ &= \frac{\frac{8}{10} - \frac{5}{10}}{\frac{-15}{5} - \frac{10}{5}} \\ &= \frac{\frac{3}{10}}{\frac{17}{5}} \\ &= \frac{3}{10} \cdot \frac{5}{17} \\ &= \frac{15}{170} \\ &= \frac{3}{34} \end{aligned}$$

32. $m = \frac{-\frac{3}{16} - \frac{5}{6}}{-\frac{1}{2} - (-\frac{3}{4})} = \frac{-\frac{3}{16}}{\frac{1}{4}} = -\frac{3}{16} \cdot \frac{4}{1} = -\frac{3}{4}$

33. Find the slope given $(3, -7)$ and $(3, -9)$

$$\begin{aligned} m &= \frac{-9-(-7)}{3-3} \\ &= \frac{-2}{0} \quad \text{undefined quantity} \end{aligned}$$

This line has no slope

34. $m = \frac{10-2}{-4-(-4)} = \frac{8}{0}$ This line has no slope

35. Find the slope given $(2, 3)$ and $(-1, 3)$

$$\begin{aligned} m &= \frac{3-3}{-1-2} \\ &= \frac{0}{-3} \\ &= 0 \end{aligned}$$

36. $m = \frac{\frac{1}{2} - \frac{1}{2}}{-7-(-6)} = \frac{0}{-1} = 0$

37. Find the slope given $(x, 3x)$ and $(x+h, 3(x+h))$

$$\begin{aligned} m &= \frac{3(x+h) - 3x}{x+h-x} \\ &= \frac{3x+3h-3x}{h} \\ &= \frac{3h}{h} \\ &= 3 \end{aligned}$$

38. $m = \frac{4(x+h)-4x}{x+h-x} = \frac{4x+4h-4x}{h} = \frac{4h}{h} = 4$

39. Find the slope given $(x, 2x+3)$ and $(x+h, 2(x+h)+3)$

$$\begin{aligned} m &= \frac{[2(x+h)+3] - (2x+3)}{x+h-x} \\ &= \frac{2x+2h+3-2x-3}{h} \\ &= \frac{2h}{h} \\ &= 2 \end{aligned}$$

40. $m = \frac{[3(x+h)-1]-(3x-1)}{x+h-x} = \frac{3x+3h-1-3x+1}{h} = \frac{3h}{h} = 3$

41. Find equation of line containing $(-4, -2)$ and $(-2, 1)$

From Exercise 29, we know that the slope of the line is $\frac{3}{2}$. Using the point$(-2, 1)$ and the value of the slope in the point-slope formula $y - y_1 = m(x - x_1)$ and solving for y we get:

$$\begin{aligned} y-1 &= \frac{3}{2}(x-(-2)) \\ y-1 &= \frac{3}{2}(x+2) \\ y-1 &= \frac{3}{2}x+3 \\ y &= \frac{3}{2}x+3+1 \\ y &= \frac{3}{2}x+4 \end{aligned}$$

NOTE: You could use either of the given points and you would reach the final equation.

42. Using $m = \frac{1}{4}$ and the point $(6, 3)$

$$\begin{aligned} y-3 &= \frac{1}{4}(x-6) \\ y-3 &= \frac{1}{4}x - \frac{6}{4} \\ y &= \frac{1}{4}x - \frac{3}{2} + 3 \\ y &= \frac{1}{4}x + \frac{3}{2} \end{aligned}$$

43. Find equation of line containing $(\frac{2}{5}, \frac{1}{2})$ and $(-3, \frac{4}{5})$

From Exercise 31, we know that the slope of the line is $-\frac{3}{34}$ and using the point $(-3, \frac{4}{5})$

$$\begin{aligned} y - \frac{4}{5} &= -\frac{3}{34}(x-(-3)) \\ y - \frac{4}{5} &= -\frac{3}{34}(x+3) \\ y - \frac{4}{5} &= -\frac{3}{34}x - \frac{9}{34} \\ y &= -\frac{3}{34}x - \frac{9}{34} + \frac{4}{5} \\ y &= -\frac{3}{34}x - \frac{45}{170} + \frac{136}{170} \\ y &= -\frac{3}{34}x + \frac{91}{170} \end{aligned}$$

44. Using $m = -\frac{13}{4}$ and the point $\left(-\frac{3}{4}, \frac{5}{8}\right)$

$$\begin{aligned} y - \frac{5}{8} &= -\frac{13}{4}\left(x - \left(-\frac{3}{4}\right)\right) \\ y - \frac{5}{8} &= -\frac{13}{4}x - \frac{39}{16} \\ y &= -\frac{13}{4}x - \frac{39}{16} + \frac{5}{8} \\ y &= -\frac{13}{4}x - \frac{39}{16} + \frac{10}{16} \\ y &= -\frac{13}{4}x - \frac{29}{16} \end{aligned}$$

45. Find equation of line containing $(3, -7)$ and $(3, -9)$

From Exercise 33, we found that the line containing $(3, -7)$ and $(3, -9)$ has no slope. We notice that the x-coordinate does not change regardless of the y-value. Therefore, the line in vertical and has the equation $x = 3$.

46. Since the line has no slope, it is vertical. The equation of the line is $x = -4$.

47. Find equation of line containing $(2, 3)$ and $(-1, 3)$

From Exercise 35, we found that the line containing $(2, 3)$ and $(-1, 3)$ has a slope of $m = 0$. We notice that the y-coordinate does not change regardless of the x-value. Therefore, the line in horizontal and has the equation $y = 3$.

48. Since the line has a slope of $m = 0$, it is horizontal. The equation of the line is $y = \frac{1}{2}$

49. Find equation of line containing $(x, 3x)$ and $(x+h, 3(x+h))$

From Exercise 37, we found that the line containing $(x, 3x)$ and $(x+h, 3(x+h))$ had a slope of $m = 3$. Using the point $(x, 3x)$ and the value of the slope in the point-slope formula

$$\begin{aligned} y - 3x &= 3(x - x) \\ y - 3x &= 3(0) \\ y - 3x &= = 0 \\ y &= 3x \end{aligned}$$

50. Using $m = 4$ and the point $(x, 4x)$

$$\begin{aligned} y - 4x &= 4(x - x) \\ y - 4x &= 0 \\ y &= 4x \end{aligned}$$

51. Find equation of line containing $(x, 2x+3)$ and $(x+h, 2(x+h)+3)$

From Exercise 37, we found that the line containing $(x, 2x+3)$ and $(x+h, 2(x+h)+3)$ had a slope of $m = 2$. Using the point $(x, 3x)$ and the value of the slope in the point-slope formula

$$\begin{aligned} y - (2x+3) &= 2(x - x) \\ y - (2x+3) &= 2(0) \\ y - (2x+3) &= 0 \\ y &= 2x + 3 \end{aligned}$$

52. Using $m = 3$ and the point $(x, 3x - 1)$

$$\begin{aligned} y - (3x-1) &= 3(x - x) \\ y - (3x-1) &= 0 \\ y &= 3x - 1 \end{aligned}$$

53. Slope $= \frac{0.4}{5} = 0.08$. This means the treadmill has a grade of 8%.

54. The roof has a slope of $\frac{2.6}{6.2} \approx 0.3171$, or 31.71%

55. The slope (or head) of the river is $\frac{43.33}{1238} = 0.035 = 3.5\%$

56. The stairs have a maximum grade of $\frac{8.25}{9} = 0.91\overline{6} \approx 0.9167 = 91.67\%$

57. The average rate of change of life expectancy at birth is computed by finding the slope of the line containing the two points $(1990, 73.7)$ and $(2000, 76.9)$, which is given by

$$\begin{aligned} \text{Rate} &= \frac{\text{Change in Life expectancy}}{\text{Change in Time}} \\ &= \frac{76.9 - 73.7}{2000 - 1990} \\ &= \frac{3.2}{10} \\ &= 0.32 \text{ per year} \end{aligned}$$

58. **a)** $F(-10) = \frac{9}{5} \cdot (-10) + 32 = -18 + 32 = 14^o F$
$F(0) = \frac{9}{5} \cdot (0) + 32 = 0 + 32 = 32^o F$
$F(10) = \frac{9}{5} \cdot (10) + 32 = 18 + 32 = 50^o F$
$F(40) = \frac{9}{5} \cdot (40) + 32 = 72 + 32 = 104^o F$

b) $F(30) = \frac{9}{5} \cdot (30) + 32 = 54 + 32 = 86^o F$

c) Same temperature in both means $F(x) = x$. So

$$\begin{aligned} F(x) &= x \\ \frac{9}{5}x + 32 &= x \\ \frac{9}{5}x - x &= -32 \\ \frac{4}{5}x &= -32 \\ x &= -32 \cdot \frac{5}{4} \\ x &= -40^o \end{aligned}$$

59. **a)** Since R and T are directly proportional we can write that $R = kT$, where k is a constant of proportionality. Using $R = 12.51$ when $T = 3$ we can find k.

$$\begin{aligned} R &= kT \\ 12.51 &= k(3) \\ \frac{12.51}{3} &= k \\ 4.17 &= k \end{aligned}$$

Thus, we can write the equation of variation as $R = 4.17T$

b) This is the same as asking: find R when $T = 6$. So, we use the variation equation

$$\begin{aligned} R &= 4.17T \\ &= 4.17(6) \\ &= 25.02 \end{aligned}$$

60. We need to find t when $D = 6$.

$$\begin{aligned} D &= 293t \\ 6 &= 293t \\ \frac{6}{293} &= t \\ 0.0205 \text{ seconds} &\approx t \end{aligned}$$

61. **a)** Since B s directly proportional to W we can write $B = kW$.

b) When $W = 200$ $B = 5$ means that

$$\begin{aligned} B &= kW \\ 5 &= k(200) \\ \frac{5}{200} &= k \\ 0.025 &= k \\ 2.5\% &= k \end{aligned}$$

This means that the weight of the brain is 2.5% the weight of the person.

c) Find B when $W = 120$

$$\begin{aligned} B &= 0.025W \\ &= 0.025(120\ lbs) \\ &= 3\ lbs \end{aligned}$$

62. **a)**

$$\begin{aligned} M &= kW \\ 80 &= k(200) \\ 0.4 &= k \end{aligned}$$

Thus, the equation of variation is $M = 0.4W$

b) $k = 0.4 = 40\%$ means that 40% of the body weight is the weight of muscles.

c)

$$\begin{aligned} M &= 0.4(120) \\ &= 48 \text{ lb} \end{aligned}$$

63. **a)**

$$\begin{aligned} D(0) &= 2(0) + 115 = 0 + 115\ ft \\ D(-20) &= 2(-20) + 115 = -40 + 115 = 75\ ft \\ D(10) &= 2(10) + 115 = 20 + 115 = 135\ ft \\ D(32) &= 2(32) + 115 = 64 + 115 = 179\ ft \end{aligned}$$

b) The stopping distance has to be a non-negative value. Therefore we need to solve the inequality

$$\begin{aligned} 0 &\leq 2F + 115 \\ -115 &\leq 2F \\ -57.5 &\leq F \end{aligned}$$

The 32^o limit comes from the fact that for any temperature above that there would be no ice. Thus, the domain of the function is restricted in the interval $[-57.5, 32]$.

64. **a)**

$$\begin{aligned} D(5) &= \frac{11 \cdot 0 + 5}{10} = \frac{5}{10} = 0.5\ ft \\ D(10) &= \frac{11 \cdot 10 + 5}{10} = \frac{115}{10} = 11.5\ ft \\ D(20) &= \frac{11 \cdot 20 + 5}{10} = \frac{225}{10} = 22.5\ ft \\ D(50) &= \frac{11 \cdot 50 + 5}{10} = \frac{555}{10} = 55.5\ ft \\ D(65) &= \frac{11 \cdot 65 + 5}{10} = \frac{720}{10} = 72\ ft \end{aligned}$$

b)

c) Since cars cannot have negative speed, and since the car will not need to stop if it has speed of 0 then the domain is any positive real number. **NOTE:** The domain will have an upper bound since cars have a top speed limit, depending on the make and model of the car.

65. **a)**

$$\begin{aligned} M(x) &= 2.89x + 70.64 \\ M(26) &= 2.89(26) + 70.64 \\ &= 75.14 + 70.64 \\ &= 145.78 \end{aligned}$$

The male was 145.78 cm tall.

b)

$$\begin{aligned} F(x) &= 2.75x + 71.48 \\ F(26) &= 2.75(26) + 71.48 \\ &= 71.5 + 71.48 \\ &= 142.98 \end{aligned}$$

The female was 142.98 cm tall.

66. **a)** The equation of variation is given by $N = P + 0.02P = 1.02P$.

b) $N = 1.02(200000) = 204000$

c)

$$\begin{aligned} 367200 &= 1.02P \\ \frac{367200}{1.02} &= P \\ 360000 &= P \end{aligned}$$

67. **a)**

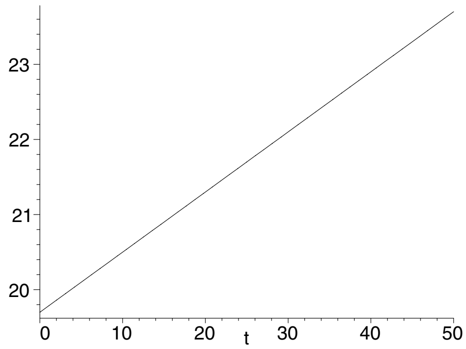
$$\begin{aligned} A(0) &= 0.08(0) + 19.7 = 0 + 19.7 = 19.7 \\ A(1) &= 0.08(1) + 19.7 = 0.08 + 19.7 = 19.78 \\ A(10) &= 0.08(10) + 19.7 = 0.8 + 19.7 = 20.5 \\ A(30) &= 0.08(30) + 19.7 = 2.4 + 19.7 = 22.1 \\ A(50) &= 0.08(50) + 19.7 = 4 + 19.7 = 23.7 \end{aligned}$$

b) First we find the value of $t4$, which is $2003 - 1950 = 53$. So, we have to find $A(53)$.

$$A(53) = 0.08(53) + 19.7 = 4.24 + 19.8 = 23.94$$

The median age of women at first marriage in the year 2003 is 23.94 years.

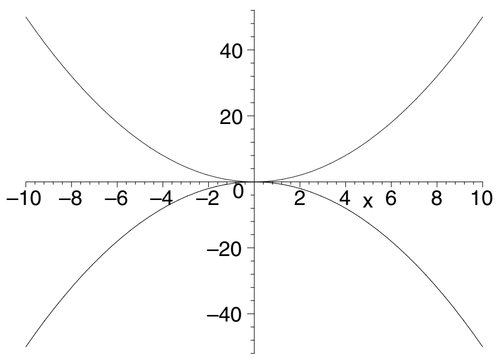
c) $A(t) = 0.08t + 19.7$



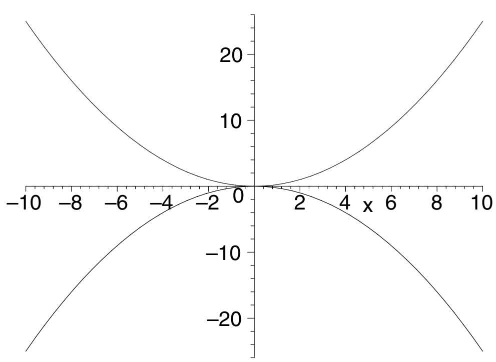
68. The use of the slope-intercept equation or the point-slope equation depends on the problem. If the problem gives the slope and the y-intercept then one should use the slope-intercept equation. If the problem gives the slope and a point that falls on the line, or two points that fall on the line then the point-slope equation should be used.

Exercise Set 1.2

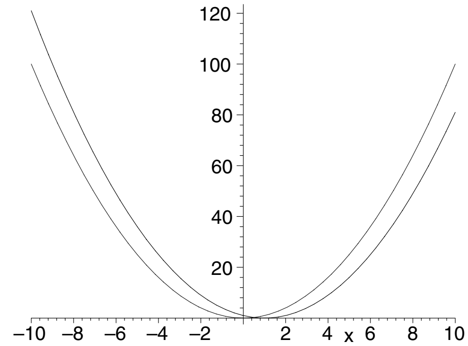
1. $y = \frac{1}{2}x^2$ and $y = -\frac{1}{2}x^2$



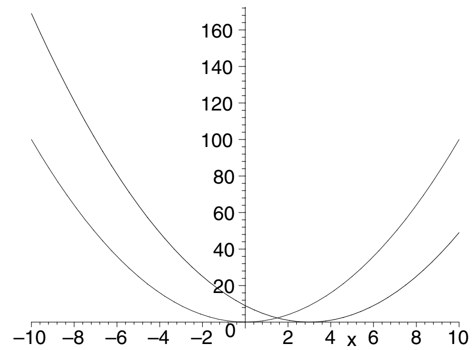
2. $y = \frac{1}{4}x^2$ and $y = -\frac{1}{4}x^2$



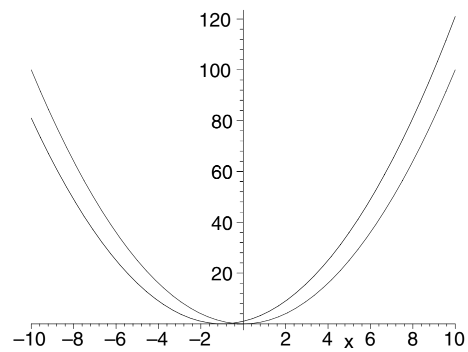
3. $y = x^2$ and $y = (x-1)^2$



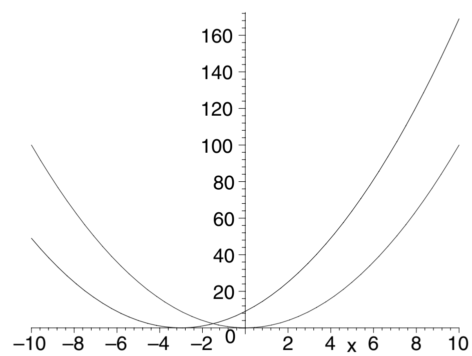
4. $y = x^2$ and $y = (x-3)^2$



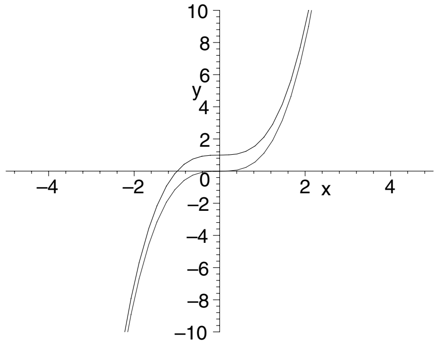
5. $y = x^2$ and $y = (x+1)^2$



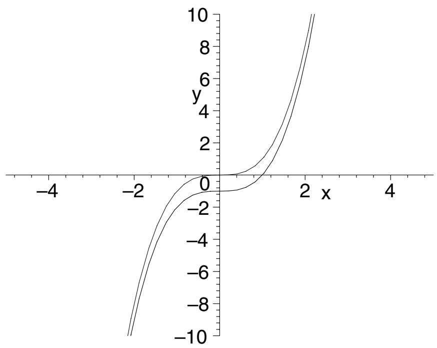
6. $y = x^2$ and $y = (x+3)^2$



7. $y = x^3$ and $y = x^3 + 1$



8. $y = x^3$ and $y = x^3 - 1$



9. Since the equation has the form $ax^2 + bx + c$, with $a \neq 0$, the graph of the function is a parabola. The x-value of the vertex is given by

$$x = -\frac{b}{2a} = -\frac{4}{2(1)} = -2$$

The y-value of the vertex is given by

$$\begin{aligned} y &= (-2)^2 + 4(-2) - 7 \\ &= 4 - 8 - 7 \\ &= -11 \end{aligned}$$

Therefore, the vertex is $(-2, 11)$.

10. Since the equation is not in the form of $ax^2 + bx + c$, the graph of the function is not a parabola.

11. Since the equation is not in the form of $ax^2 + bx + c$, the graph of the function is not a parabola.

12. Since the equation has the form $ax^2 + bx + c$, with $a \neq 0$, the graph of the function is a parabola. The x-value of the vertex is given by

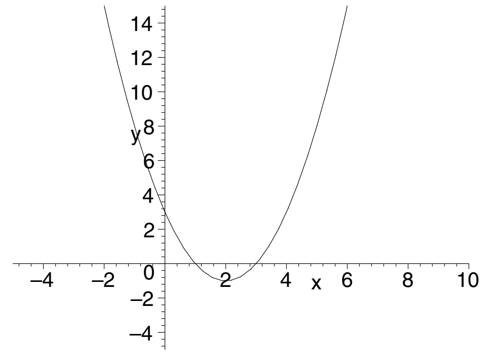
$$x = -\frac{b}{2a} = -\frac{-6}{2(3)} = 1$$

The y-value of the vertex is given by

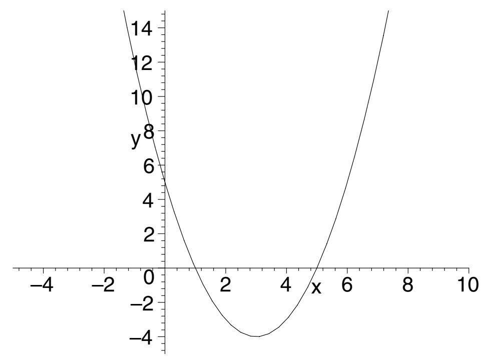
$$\begin{aligned} y &= 3(1)^2 - 6(1) \\ &= 3 - 6 \\ &= -3 \end{aligned}$$

Therefore, the vertex is $(1, -3)$.

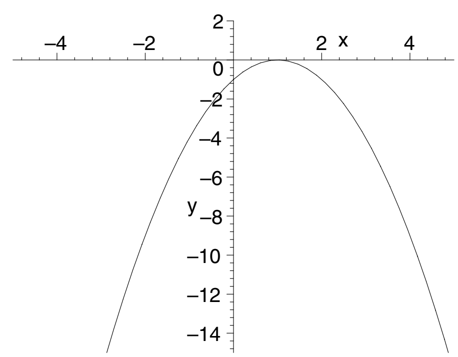
13. $y = x^2 - 4x + 3$



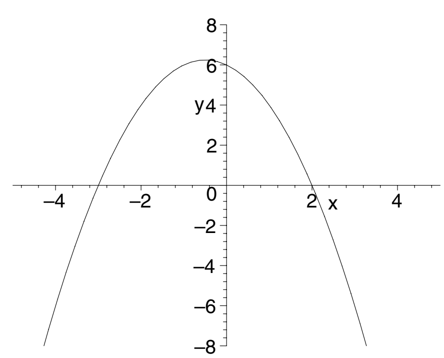
14. $y = x^2 - 6x + 5$



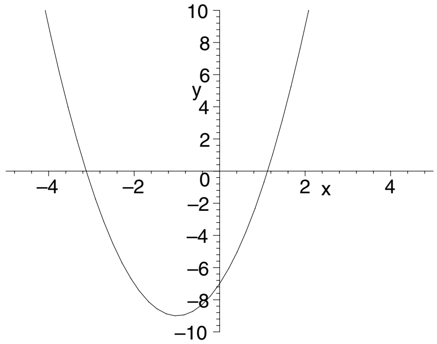
15. $y = -x^2 + 2x - 1$



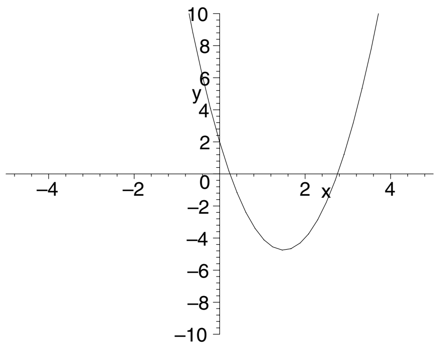
16. $y = -x^2 - x + 6$



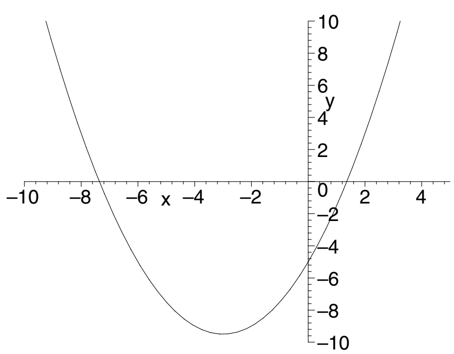
17. $y = 2x^2 + 4x - 7$



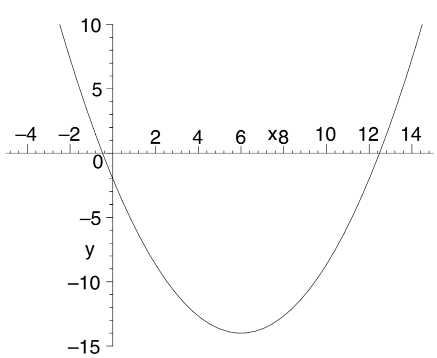
18. $y = 3x^2 - 9x + 2$



19. $y = \frac{1}{2}x^2 + 3x - 5$



20. $y = \frac{1}{3}x^2 + 4x - 2$



21. Solve $x^2 - 2x = 2$
Write the equation so that one side equals zero, that is $x^2 - 2x - 2 = 0$, then use the quadratic formula, with $a = 1$, $b = -2$, and $c = -2$, to solve for x.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)} \\ &= \frac{2 \pm \sqrt{4+8}}{2} \\ &= \frac{2 \pm \sqrt{12}}{2} \\ &= \frac{2 \pm 2\sqrt{3}}{2} \\ &= \frac{2(1 \pm \sqrt{3})}{2} \\ &= 1 \pm \sqrt{3} \end{aligned}$$

The solutions are $1 + \sqrt{3}$ and $1 - \sqrt{3}$

22. $x^2 - 2x + 1 = 5$ can be rewritten as $x^2 - 2x - 4 = 0$

$$\begin{aligned} x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)} \\ &= \frac{2 \pm \sqrt{4+16}}{2} \\ &= \frac{2 \pm \sqrt{20}}{2} = \frac{2(1 \pm \sqrt{5})}{2} \\ &= 1 \pm \sqrt{5} \end{aligned}$$

The solutions are $1 + \sqrt{5}$ and $1 - \sqrt{5}$

23. Solve $3y^2 + 8y + 2 = 0$
Use the quadratic formula, with $a = 3$, $b = 8$, and $c = 2$, to solve for y.

$$\begin{aligned} y &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ y &= \frac{-8 \pm \sqrt{(8)^2 - 4(3)(2)}}{2(3)} \\ &= \frac{-8 \pm \sqrt{64 - 24}}{6} \\ &= \frac{-8 \pm \sqrt{40}}{6} \\ &= \frac{-8 \pm 2\sqrt{10}}{6} \\ &= \frac{2(-4 \pm \sqrt{10})}{6} \\ &= \frac{-4 \pm \sqrt{10}}{3} \end{aligned}$$

The solutions are $\frac{-4+\sqrt{10}}{3}$ and $\frac{-4-\sqrt{10}}{3}$

24. $2p^2 - 5p = 1$ can be rewritten as $2p^2 - 5p - 1$

$$\begin{aligned} p &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-1)}}{2(2)} \\ &= \frac{5 \pm \sqrt{25+8}}{4} \\ &= \frac{5 \pm \sqrt{33}}{4} \end{aligned}$$

The solutions are $\frac{5+\sqrt{33}}{4}$ and $\frac{5-\sqrt{33}}{4}$

25. Solve $x^2 - 2x + 10 = 0$
Using the quadratic formula with $a = 1$, $b = -2$, and $c = 10$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(10)}}{2(1)} \\ &= \frac{2 \pm \sqrt{4 - 40}}{2} \\ &= \frac{2 \pm \sqrt{-36}}{2} \\ &= \frac{2 \pm 6i}{2} \\ &= \frac{2(1 \pm 3i)}{2} \\ &= 1 \pm 3i \end{aligned}$$

The solutions are $1 + 3i$ and $1 - 3i$

26.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-6 \pm \sqrt{(6)^2 - 4(1)(10)}}{2(1)} \\ &= \frac{-6 \pm \sqrt{36 - 40}}{2} \\ &= \frac{-6 \pm \sqrt{-4}}{2} \\ &= \frac{-6 \pm 2i}{2} \\ &= \frac{2(-3 \pm i)}{2} \\ &= -3 \pm i \end{aligned}$$

The solutions are $-3 + i$ and $-3 - i$

27. Solve $x^2 + 6x = 1$
Write the equation so that one side equals zero, that is $x^2 + 6x - 1 = 0$, then use the quadratic formula, with $a = 1$, $b = 6$, and $c = -1$, to solve for x.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-6 \pm \sqrt{(6)^2 - 4(1)(-1)}}{2(1)} \\ &= \frac{-6 \pm \sqrt{36 + 4}}{2} \\ &= \frac{-6 \pm \sqrt{40}}{2} \\ &= \frac{-6 \pm 2\sqrt{10}}{2} \\ &= \frac{2(-3 \pm \sqrt{10})}{2} \\ &= -3 \pm \sqrt{10} \end{aligned}$$

The solutions are $-3 + \sqrt{10}$ and $-3 - \sqrt{10}$

28. $x^2 + 4x = 3$ can be rewritten as $x^2 + 4x - 3 = 0$

$$\begin{aligned} x &= \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-3)}}{2(1)} \\ &= \frac{-4 \pm \sqrt{16 + 12}}{2} \\ &= \frac{-4 \pm \sqrt{28}}{2} = \frac{2(-2 \pm \sqrt{7})}{2} \\ &= -2 \pm \sqrt{7} \end{aligned}$$

The solutions are $-2 + \sqrt{7}$ and $-2 - \sqrt{7}$

29. Solve $x^2 + 4x + 8 = 0$
Using the quadratic formula with $a = 1$, $b = 4$, and $c = 8$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-4 \pm \sqrt{(4)^2 - 4(1)(8)}}{2(1)} \\ &= \frac{-4 \pm \sqrt{16 - 32}}{2} \\ &= \frac{-4 \pm \sqrt{-16}}{2} \\ &= \frac{-4 \pm 4i}{2} \\ &= \frac{4(1 \pm i)}{2} \\ &= 2(1 \pm i) = 2 \pm 2i \end{aligned}$$

The solutions are $2 + 2i$ and $2 - 2i$

30.

$$\begin{aligned} x &= \frac{-10 \pm \sqrt{(10)^2 - 4(1)(27)}}{2(1)} \\ &= \frac{-10 \pm \sqrt{100 - 108}}{2} \\ &= \frac{-10 \pm \sqrt{-8}}{2} \\ &= \frac{-10 \pm 2i\sqrt{2}}{2} \\ &= \frac{2(-5 \pm i\sqrt{2})}{2} \\ &= -5 \pm i\sqrt{2} \end{aligned}$$

The solutions are $-5 + i\sqrt{2}$ and $-5 + i\sqrt{2}$

31. Solve $4x^2 = 4x - 1$
Write the equation so that one side equals zero, that is $4x^2 - 4x - 1 = 0$, then use the quadratic formula, with $a = 4$, $b = -4$, and $c = -1$, to solve for x.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(-1)}}{2(4)} \\ &= \frac{4 \pm \sqrt{16 + 16}}{8} \\ &= \frac{4 \pm \sqrt{32}}{8} \end{aligned}$$

$$\begin{aligned} &= \frac{4 \pm 4\sqrt{2}}{8} \\ &= \frac{4(1 \pm \sqrt{2})}{8} \\ &= \frac{1 \pm \sqrt{2}}{2} \end{aligned}$$

The solutions are $\frac{1+\sqrt{2}}{2}$ and $\frac{1-\sqrt{2}}{2}$

32. $-4x^2 = 4x - 1$ can be rewritten as $0 = 4x^2 + 4x - 1$

$$\begin{aligned} x &= \frac{-4 \pm \sqrt{(4)^2 - 4(4)(-1)}}{2(4)} \\ &= \frac{-4 \pm \sqrt{16+16}}{8} \\ &= \frac{-4 \pm \sqrt{32}}{8} = \frac{4(-1 \pm \sqrt{4})}{8} \\ &= \frac{-1 \pm \sqrt{2}}{2} \end{aligned}$$

The solutions are $\frac{-1+\sqrt{2}}{2}$ and $\frac{-1-\sqrt{2}}{2}$

33. Find $f(7)$, $f(10)$, and $f(12)$

$$\begin{aligned} f(7) &= \frac{1}{6}(7)^3 + \frac{1}{2}(7)^2 + \frac{1}{2}(7) \\ &= \frac{343}{6} + \frac{49}{2} + \frac{7}{2} \\ &= \frac{343}{6} + \frac{147}{6} + \frac{21}{6} \\ &= \frac{511}{6} \approx 85.1\overline{6} \approx 85 \text{ oranges} \\ f(10) &= \frac{1}{6}(10)^3 + \frac{1}{2}(10)^2 + \frac{1}{2}(10) \\ &= \frac{1000}{6} + 50 + 5 \\ &= \frac{500}{3} + \frac{150}{3} + \frac{15}{3} \\ &= \frac{665}{3} \approx 221.\overline{6} \approx 222 \text{ oranges} \\ f(12) &= \frac{1}{6}(12)^3 + \frac{1}{2}(12)^2 + \frac{1}{2}(12) \\ &= 288 + 72 + 6 \\ &= 366 \text{ oranges} \end{aligned}$$

34. **a)** $x = 2009 - 1985 = 24$

$$\begin{aligned} f(24) &= 4.8565 + 0.2841(24) + 0.1784(24)^2 \\ &= 4.8565 + 6.8184 + 102.7584 \\ &= 114.4333 \end{aligned}$$

The average payroll for 2009-10 is \$114.4333 million

b) Solve $100 = 4.8565 + 0.2841x + 0.1784x^2$. First, let us rewrite the equation as $0 = -95.1435 + 0.2841x + 0.1784x^2$ then we can use the quadratic formula to solve for x

$$\begin{aligned} x &= \frac{-0.2841 \pm \sqrt{0.2841^2 - 4(0.1784)(-95.1435)}}{2(0.1784)} \\ &= \frac{-0.2841 \pm \sqrt{0.0807 + 67.8944}}{0.3568} = \frac{-0.2841 \pm \sqrt{67.9751}}{0.3568} \\ &= \frac{-0.2841 \pm 8.2447}{0.3568} \\ &= \frac{-0.2841 + 8.2447}{0.3568} = 22.3111 \end{aligned}$$

Therefore, the average payroll will be \$100 million is the, $1985 + 23.3111 = 2007.3111$, 2007-08 season. **NOTE:** We could not choose the negative option of the quadratic formula since it would result in the result that is negative which corresponds to a year before 1985 and that does not make sense.

35. Solve $50 = 9.41 - 0.19x + 0.09x^2$. First, let us rewrite the equation as $0 = -40.59 - 0.19x + 0.09x^2$ then we can use the quadratic formula to solve for x

$$\begin{aligned} x &= \frac{-(-0.19) \pm \sqrt{(-0.19)^2 - 4(0.09)(-40.59)}}{2(0.09)} \\ &= \frac{0.19 \pm \sqrt{0.0361 + 14.6124}}{0.18} = \frac{0.19 \pm \sqrt{14.6485}}{0.18} \\ &= \frac{0.19 \pm 3.8273}{0.18} \\ &= \frac{0.19 + 3.8273}{0.18} = 22.3183 \end{aligned}$$

Therefore, the average price of a ticket will be \$50 will happen during the, $1990 + 22.3183 = 2012.3183$ 2012-13 season. **NOTE:** We could not choose the negative option of the quadratic formula since it would result in the result that is negative which corresponds to a year before 1990 and that does not make physical sense.

36. **a)**

$$\begin{aligned} w(72) &= 0.0728(72)^2 - 6.986(72) + 289 \\ &= 163.4032 \text{ pounds} \end{aligned}$$

b) Solve $170 = 0.0728h^2 - 6.986h + 289$, which can be written as $0.0728h^2 - 6.986h + 119 = 0$

$$\begin{aligned} h &= \frac{-(-6.986) \pm \sqrt{(-6.986)^2 - 4(0.0728)(119)}}{2(0.0728)} \\ &= \frac{6.986 \pm \sqrt{14.1514}}{0.1456} \\ &= \frac{6.986 \pm 3.7618)}{0.1456} \end{aligned}$$

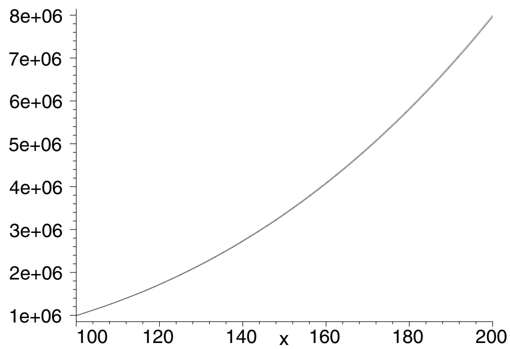
The possible two answers are $\frac{6.986-3.7618}{0.1456} = 22.1440$ *in*, which is out side of the domain of the function, and $\frac{6.986+3.7618}{0.1456} = 73.8173$ *in*, which is in the domain interval of the function w. Therefore, the man is about 73.8 inches tall.

37. $f(x) = x^3 - x^2$

a) For large values of x, x^3 would be larger than x^2. $x^3 = x \cdot x \cdot x$ and $x^2 = x \cdot x$ so for very large values of x there is an extra factor of x in x^3 which causes x^3 to be larger than x^2.

b) As x gets very large the values of x^3 become much larger than those of x^2 and therefore we can "ignore" the effect of x^2 in the expression $x^3 - x^2$. Thus, we can approximate the function to look like x^3 for very large values of x.

c) Below is a graph of $x^3 - x^2$ and x^3 for $100 \leq x \leq 200$. It is hard to distinguish between the two graphs confirming the conclusion reached in part b).

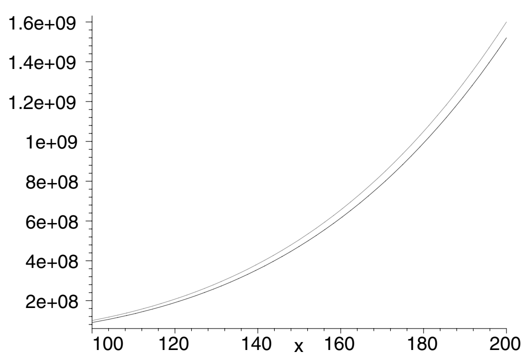


38. $f(x) = x^4 - 10x^3 + 3x^2 - 2x + 7$

a) For large values of x, x^4 will be larger than $|-10x^3 + 3x^2 - 2x + 7|$ since the second term is a third degree polynomial (compared to a fourth degree polynomial) and has terms being subtracted.

b) Since the values of x^4 "dominate" the function for very large values of x the function will look like x^4 for very large values of x.

c) Below is a graph of $x^4 - 10x^3 + 3x^2 - 2x + 7$ and x^4 for $100 \leq x \leq 200$. The graphs are close to each other confirming our conclusion from part b).

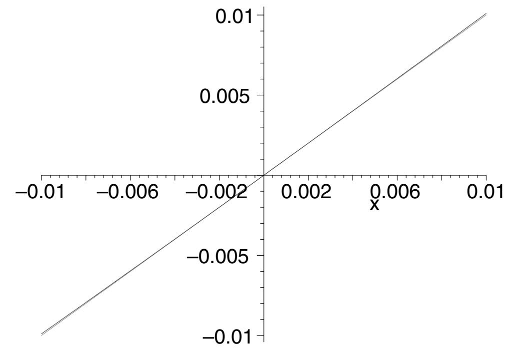


39. $f(x) = x^2 + x$

a) For values very close to 0, x is larger than x^2 since for values of x less than 1 $x^2 < x$.

b) For values of x very close to 0 $f(x)$ looks like x since the x^2 can be "ignored".

c) Below is a graph of $x^2 + x$ and x for $-0.01 \leq x \leq 0.01$. It is very hard to distinguish between the two graphs confirming our conclusion from part b).

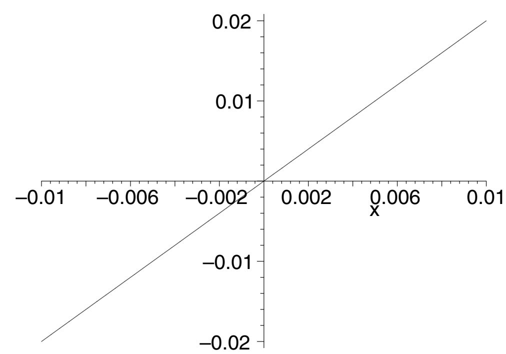


40. $f(x) = x^3 + 2x$

a) For x values very close to 0, $2x$ is larger than x^3 since for x values less than 1 the higher the degree the smaller the values of the term.

b) For x values very close to 0, the function will looks like $2x$ since the x^3 term may be "ignored".

c) Below is a graph of $x^3 + 2x$ and $2x$ for $-0.01 \leq x \leq 0.01$. It is very difficult to distinguish between the two graphs confirming our conclusion in part b).



41. $f(x) = x^3 - x$

$$\begin{aligned}
f(x) &= 0 \\
x^3 - x &= 0 \\
x(x^2 - 1) &= 0 \\
x(x-1)(x+1) &= 0 \\
x &= 0 \\
x &= 1 \\
x &= -1
\end{aligned}$$

42. $x = 2.359$

43. $x = -1.831$, $x = -0.856$, and $x = 3.188$

44. $x = 2.039$, and $x = 3.594$

45. $x = -10.153$, $x = -1.871$, $x = -0.821$, $x = -0.303$, $x = 0.098$, $x = 0.535$, $x = 1.219$, and $x = 3.297$

46. $y = 8.254x - 5.457$

47. $y = -0.279x + 4.036$

48. $y = 1.004x^2 + 1.904x - 0.601$

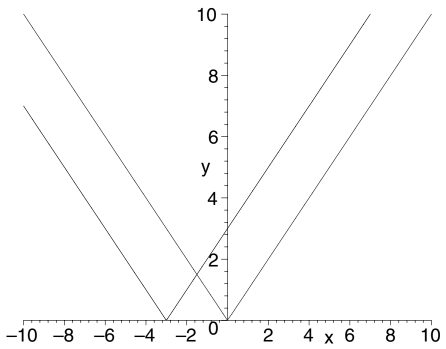
49. $y = 0.942x^2 - 2.651x - 27.943$

50. $y = 0.218x^3 + 0.188x^2 - 29.643x + 57.067$

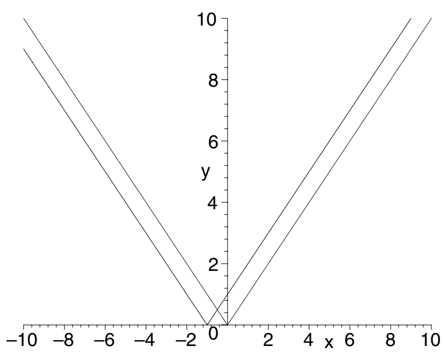
51. $y = 0.237x^4 - 0.885x^3 - 29.224x^2 + 165.166x - 210.135$

Exercise Set 1.3

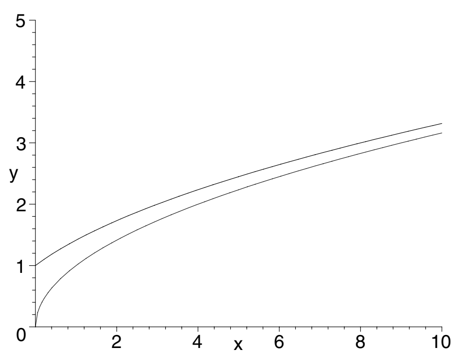
1. $y = |\ x\ |$ and $y = |\ x + 3\ |$



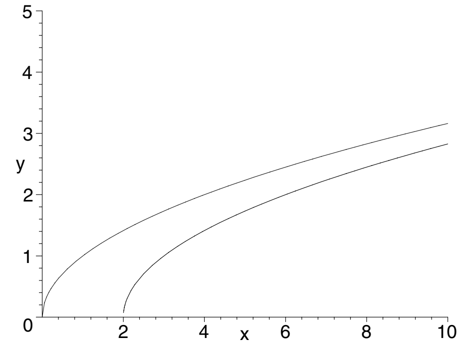
2. $y = |\ x\ |$ and $y = |\ x + 1\ |$



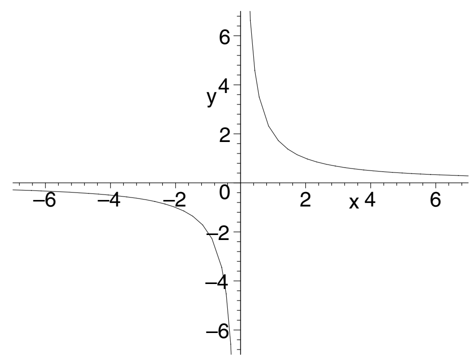
3. $y = \sqrt{x}$ and $y = \sqrt{x+1}$



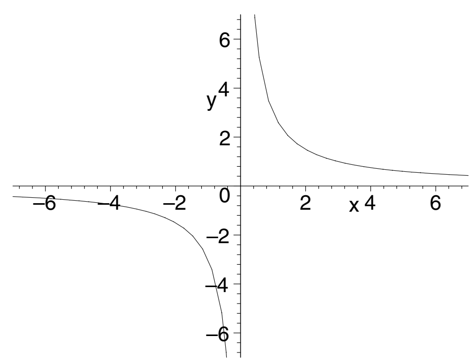
4. $y = \sqrt{x}$ and $y = \sqrt{x-2}$



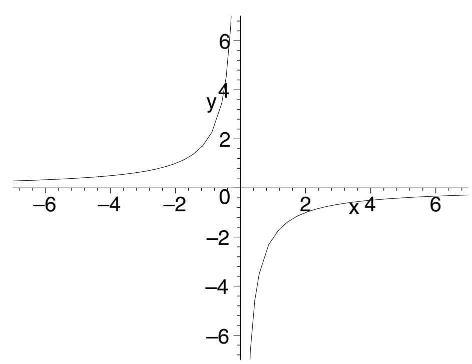
5. $y = \frac{2}{x}$



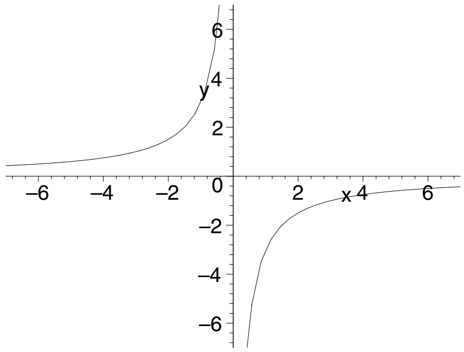
6. $y = \frac{3}{x}$



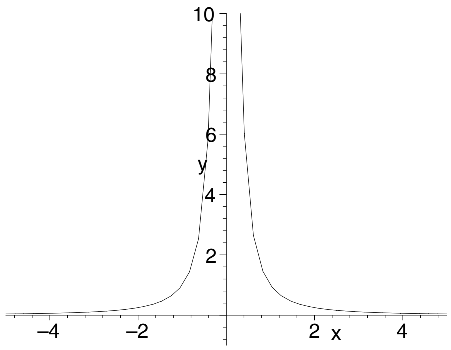
7. $y = \frac{-2}{x}$



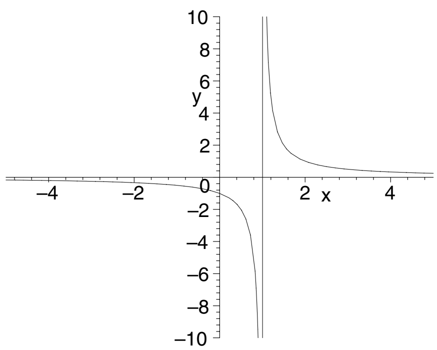
8. $y = \frac{-3}{x}$



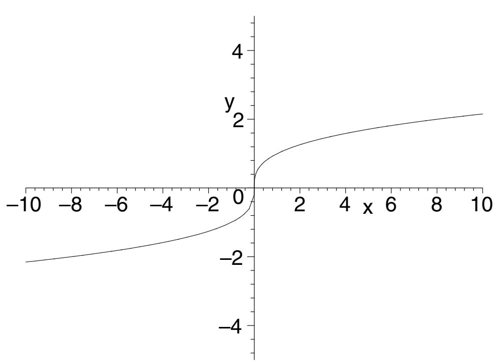
9. $y = \frac{1}{x^2}$



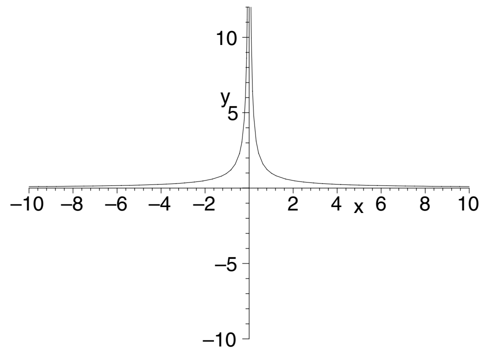
10. $y = \frac{1}{x-1}$



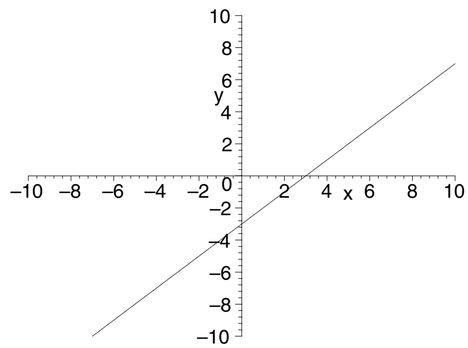
11. $y = \sqrt[3]{x}$



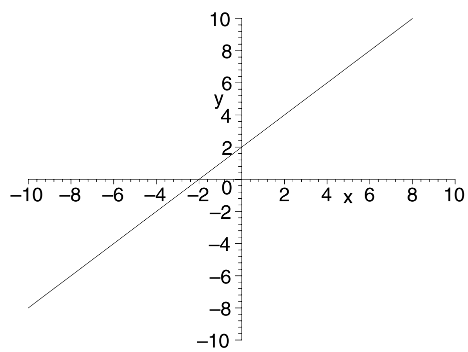
12. $y = \frac{1}{|x|}$



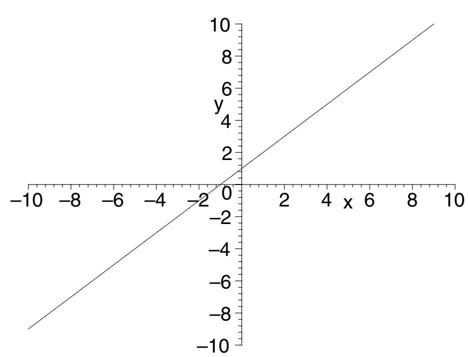
13. $y = \frac{x^2-9}{x+3}$. It is important to note here that $x = -3$ is not in the domain of the plotted function.



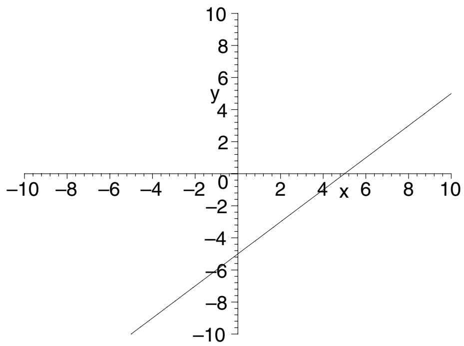
14. $y = \frac{x^2-4}{x-2}$. Note: $x = 2$ is not in the domain of the plotted function.



15. $y = \frac{x^1-1}{x-1}$. It is important to note here that $x = 1$ is not in the domain of the plotted function.



16. $y = \frac{x^2-25}{x+5}$. Note: $x = -5$ is not in the domain of the plotted function.



17. $\sqrt{x^3} = x^{\left(\frac{3}{2}\right)}$

18. $\sqrt{x^5} = x^{\left(\frac{5}{2}\right)}$

19. $\sqrt[5]{a^3} = a^{\left(\frac{3}{5}\right)}$

20. $\sqrt[4]{b^2} = b^{\left(\frac{2}{4}\right)} = b^{\left(\frac{1}{2}\right)}$

21. $\sqrt[7]{t} = t^{\left(\frac{1}{7}\right)}$

22. $\sqrt[8]{c} = c^{\left(\frac{1}{8}\right)}$

23. $\frac{1}{\sqrt[3]{t^4}} = \frac{1}{t^{\left(\frac{4}{3}\right)}} = t^{-\left(\frac{4}{3}\right)}$

24. $\frac{1}{\sqrt[5]{t^6}} = \frac{1}{t^{\left(\frac{6}{5}\right)}} = t^{-\left(\frac{6}{5}\right)}$

25. $\frac{1}{\sqrt{t}} = \frac{1}{t^{\left(\frac{1}{2}\right)}} = t^{-\left(\frac{1}{2}\right)}$

26. $\frac{1}{\sqrt{m}} = \frac{1}{m^{\left(\frac{1}{2}\right)}} = m^{-\left(\frac{1}{2}\right)}$

27. $\frac{1}{\sqrt{x^2+7}} = \frac{1}{(x^2+7)^{\left(\frac{1}{2}\right)}} = (x^2+7)^{-\left(\frac{1}{2}\right)}$

28. $\frac{1}{\sqrt{x^3+4}} = \frac{1}{(x^3+4)^{\left(\frac{1}{2}\right)}} = (x^3+4)^{-\left(\frac{1}{2}\right)}$

29. $x^{\frac{1}{5}} = \sqrt[5]{x}$

30. $t^{\frac{1}{7}} = \sqrt[7]{t}$

31. $y^{\frac{2}{3}} = \sqrt[3]{y^2}$

32. $t^{\frac{2}{5}} = \sqrt[5]{t^2}$

33. $t^{\frac{-2}{5}} = \frac{1}{t^{\frac{2}{5}}} = \frac{1}{\sqrt[5]{t^2}}$

34. $y^{\frac{-2}{3}} = \frac{1}{y^{\frac{2}{3}}} = \frac{1}{\sqrt[3]{y^2}}$

35. $b^{\frac{-1}{3}} = \frac{1}{b^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{b^2}}$

36. $b^{\frac{-1}{5}} = \frac{1}{b^{\frac{1}{5}}} = \frac{1}{\sqrt[5]{b^2}}$

37. $e^{\frac{-17}{6}} = \frac{1}{e^{\frac{17}{6}}} = \frac{1}{\sqrt[6]{e^{17}}}$

38. $m^{\frac{-19}{6}} = \frac{1}{m^{\frac{19}{6}}} = \frac{1}{\sqrt[6]{m^{19}}}$

39. $(x-3)^{\frac{-1}{2}} = \frac{1}{(x-3)^{\frac{1}{2}}} = \frac{1}{\sqrt{x-3}}$

40. $(y+7)^{\frac{-1}{4}} = \frac{1}{(y+7)^{\frac{1}{4}}} = \frac{1}{\sqrt[4]{y+7}}$

41. $\frac{1}{t^2 3} = \frac{1}{\sqrt[3]{t^2}}$

42. $\frac{1}{w^{-\frac{4}{5}}} = w^{\frac{4}{5}} = \sqrt[5]{w^4}$

43. $9^{3/2} = (\sqrt{9})^3 = (3)^3 = 27$

44. $16^{5/2} = (\sqrt{16})^5 = (4)^5 = 1024$

45. $64^{2/3} = (\sqrt[3]{64})^3 = (4)^2 = 16$

46. $8^{2/3} = (\sqrt[3]{8})^2 = (2)^2 = 4$

47. $16^{3/4} = (\sqrt[4]{16})^3 = (2)^3 = 8$

48. $25^{5/2} = (\sqrt{25})^5 = (5)^5 = 3125$

49. The domain consists of all x-values such that the denominator does not equal 0, that is $x - 5 \neq 0$, which leads to $x \neq 5$. Therefore, the domain is $\{x|x \neq 5\}$

50. $x + 2 \neq 0$ leads to $x \neq -2$. Therefore, the domain is $(-\infty, -2) \cup (-2, \infty)$.

51. Solving for the values of the x in the denominator that make it 0.

$$\begin{aligned} x^2 - 5x + 6 &= 0 \\ (x-3)(x-2) &= 0 \\ &\text{So} \\ x &= 3 \text{ and} \\ x &= 2 \end{aligned}$$

Which means that the domain is the set of all x -values such that $x \neq 3$ or $x \neq 2$

52. Solving $x^2 + 6x + 5 = 0$ leads to $(x+3)(x+2) = 0$ which means the domain consists of all real numbers such that $x \neq -3$ and $x \neq -2$

53. The domain of a square root function is restricted by the value where the radicant is positive. Thus, the domain of $f(x) = \sqrt{5x+4}$ can be found by finding the solution to the inequality $5x + 4 \geq 0$.

$$\begin{aligned} 5x + 4 &\geq 0 \\ 5x &\geq -4 \\ x &\geq \frac{-4}{5} \end{aligned}$$

54. The domain is the solution to $2x - 6 \geq 0$.

$$\begin{aligned} 2x - 6 &\geq 0 \\ 2x &\geq 6 \\ x &\geq 3 \end{aligned}$$

55. To complete the table we will plug the given W values into the equation

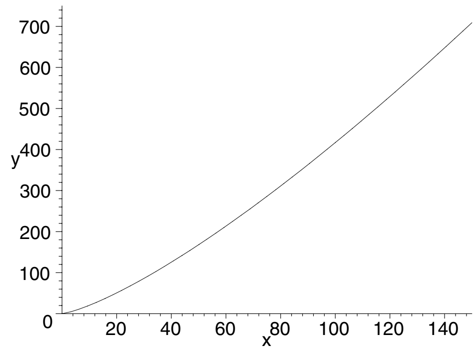
$$\begin{aligned} T(20) &= (20)^{1.31} = 50.623 \approx 51 \\ T(30) &= (30)^{1.31} = 86.105 \approx 86 \\ T(40) &= (40)^{1.31} = 125.516 \approx 126 \end{aligned}$$

$$\begin{aligned} T(50) &= (50)^{1.31} = 168.132 \approx 168 \\ T(100) &= (100)^{1.31} = 416.869 \approx 417 \\ T(150) &= (150)^{1.31} = 709.054 \approx 709 \end{aligned}$$

Therefore the table is given by

W	0	10	20	30	40	50	100	150
T	0	20	51	86	126	168	417	709

Now the graph



56. First find the constant of the variation. Let N represent the number of cities with a population greater than S.

$$\begin{aligned} N &= \frac{k}{S} \\ 48 &= \frac{k}{350000} \\ (48)(350000) &= k \\ 16800000 &= k \end{aligned}$$

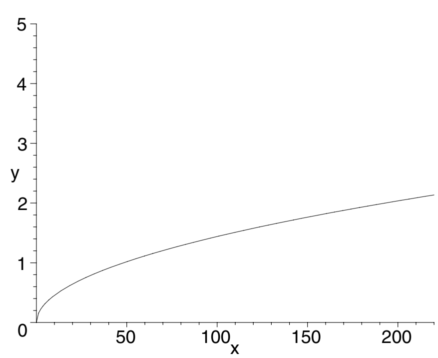
So the variation equation is $N = \frac{16800000}{S}$. Now, we have to find N when $S = 200000$.

$$\begin{aligned} N &= \frac{16800000}{200000} \\ &= 84 \end{aligned}$$

57. **a)** $f(180) = 0.144(180)^{1/2} = 0.144(13.41640786) \approx 1.932\ m^2$.

b) $f(170) = 0.144(170)^{1/2} = 0.144(13.03840481) \approx 1.878\ m^2$.

c) The graph



58. **a)** $y(2.7) = 0.73(2.7)^{3.63} \approx 26.864\ kg$.

b) $y(2.7) = 0.73(7)^{3.63} \approx 853.156\ kg$.

c)

$$\begin{aligned} 5000 &= 0.73(x)^{3.63} \\ \frac{5000}{0.73} &= x^{3.63} \\ \left(\frac{5000}{0.73}\right)^{\frac{1}{3.63}} &= x \\ 11.393\ m &\approx x \end{aligned}$$

59. Let V be the velocity of the blood, and let A be the cross sectional area of the blood vessel. Then

$$V = \frac{k}{A}$$

Using $V = 30$ when $A = 3$ we can find k.

$$\begin{aligned} 30 &= \frac{k}{3} \\ (30)(3) &= k \\ 90 &= k \end{aligned}$$

Now we can write the proportial equation

$$V = \frac{90}{A}$$

we need to find A when $V = 0.026$

$$\begin{aligned} 0.026 &= \frac{90}{A} \\ 0.026A &= 90 \\ A &= \frac{90}{0.026} \\ &= 3461.538\ m^2 \end{aligned}$$

60. Let V be the velocity of the blood, and let A be the cross sectional area of the blood vessel. Then

$$V = \frac{k}{A}$$

Using $V = 28$ when $A = 2.8$ we can find k.

$$\begin{aligned} 28 &= \frac{k}{2.8} \\ (28)(2.8) &= k \\ 78.4 &= k \end{aligned}$$

Now we can write the proportial equation

$$V = \frac{78.4}{A}$$

we need to find A when $V = 0.025$

$$\begin{aligned} 0.025 &= \frac{78.4}{A} \\ 0.025A &= 78.4 \\ A &= \frac{78.4}{0.025} \\ &= 3136\ m^2 \end{aligned}$$

61.

$$\begin{aligned} x+7+\frac{9}{x} &= 0 \\ x(x+7+\frac{9}{x}) &= x(0) \\ x^2+7x+9 &= 0 \\ x &= \frac{-7\pm\sqrt{49-4(1)(9)}}{2} \\ &= \frac{-7\pm\sqrt{13}}{2} \\ x &= \frac{-7-\sqrt{13}}{2} \\ \text{and} & \\ x &= \frac{-7+\sqrt{13}}{2} \end{aligned}$$

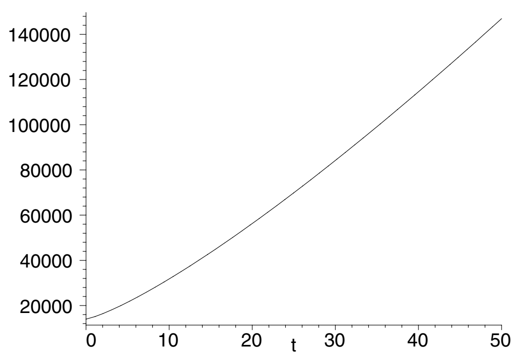
62.

$$\begin{aligned} 1-\frac{1}{w} &= \frac{1}{w^2} \\ w^2-w &= 1 \\ w^2-w-1 &= 0 \\ w &= \frac{1\pm\sqrt{1+4}}{2} \\ &= \frac{1\pm\sqrt{5}}{2} \end{aligned}$$

63. $P = 1000t^{5/4} + 14000$

a) $t = 37$, $P = 1000(37)^{5/4} + 14000 = 105254.0514$.
$t = 40$, $P = 1000(40)^{5/4} + 14000 = 114594.6744$
$t = 50$, $P = 1000(50)^{5/4} + 14000 = 146957.3974$

b) Below is the graph of P for $0 \le t \le 50$.



64. At most a function of degree n can have n y-intercepts. A polynomial of degree n can be factored into at most n linear terms and each of those linear terms leads to a y-intercept. This is sometimes called the Fundamental Theorem of Algebra

65. A rational function is a function given by the quotient of two polynomial functions while a polynomial function is a function that has the form $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$. Since every polynomial function can be written as a quotient of two other polynomial function then every polynomial function is a rational function.

66. $x = 1.5$ and $x = 9.5$

67. $x = 2.6458$ and $x = -2.6458$

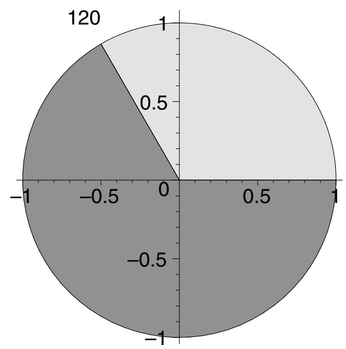
68. $x = -2$ and $x = 3$

69. The function has no zeros

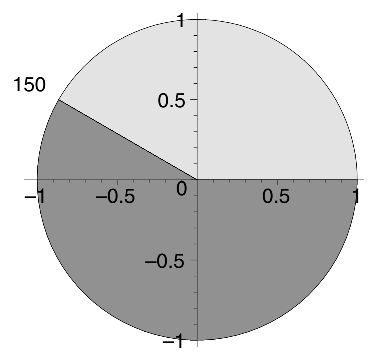
70. $x = 1$ and $x = 2$

Exercise Set 1.4

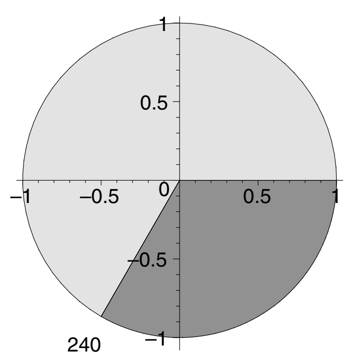
1. $(120^o)(\frac{\pi\ rad}{180^o}) = \frac{2\pi}{3}\ rad$



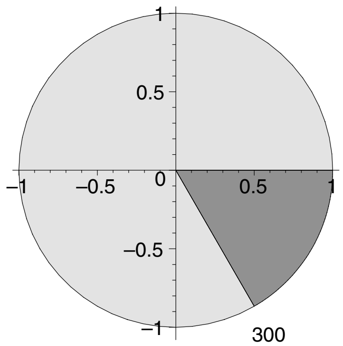
2. $(150^o)(\frac{\pi\ rad}{180^o}) = \frac{5\pi}{6}\ rad$



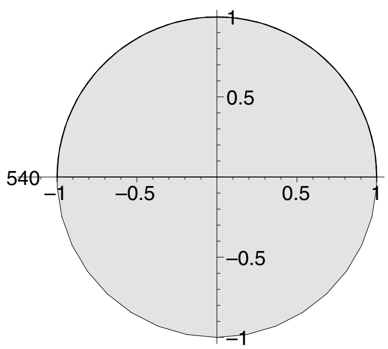
3. $(240^o)(\frac{\pi\ rad}{180^o}) = \frac{4\pi}{3}\ rad$



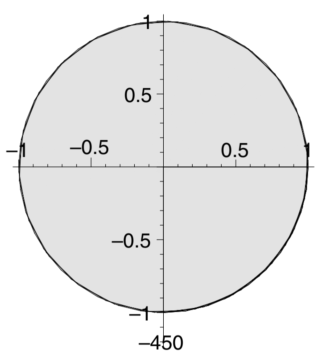
4. $(300^o)(\frac{\pi\ rad}{180^o}) = \frac{5\pi}{3}\ rad$



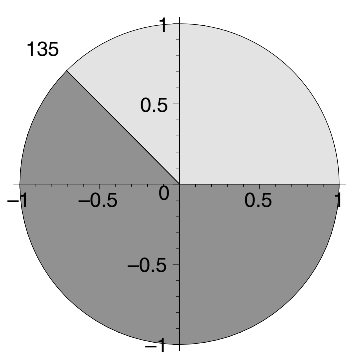
5. $(540^o)(\frac{\pi\ rad}{180^o}) = 3\pi\ rad$



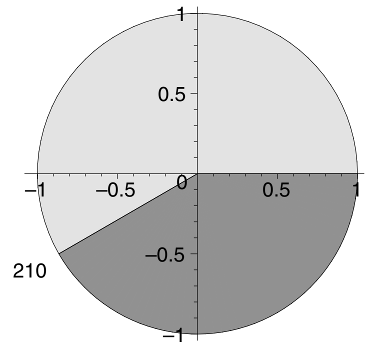
6. $(-450^o)(\frac{\pi\ rad}{180^o}) = \frac{-5\pi}{2}\ rad$



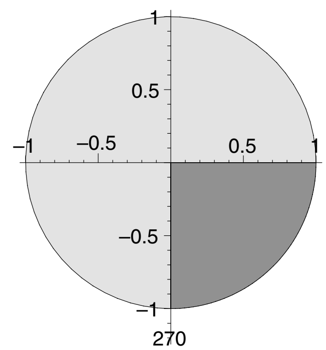
7. $(\frac{3\pi}{4})(\frac{180^o}{\pi\ rad}) = 135^o$



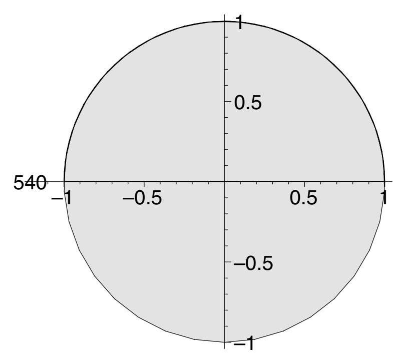
8. $(\frac{7\pi}{6})(\frac{180^o}{\pi\ rad}) = 210^o$



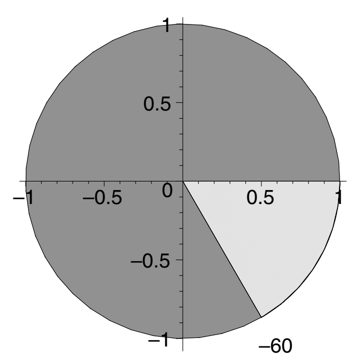
9. $(\frac{3\pi}{2})(\frac{180^o}{\pi\ rad}) = 270^o$



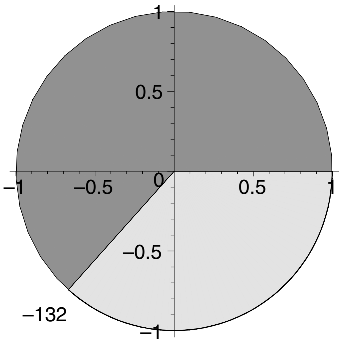
10. $(3\pi)(\frac{180^o}{\pi\ rad}) = 540^o$



11. $(\frac{-\pi}{3})(\frac{180^o}{\pi\ rad}) = -60^o$



12. $(\frac{-11\pi}{15})(\frac{180^o}{\pi\ rad}) = -132^o$



13. We need to solve $\theta_1 = \theta_2 + 360(k)$ for k. If the solution is an integer then the angles are coterminal otherwise they are not coterminal.

$$\begin{aligned} 395 &= 15 + 360(k) \\ 380 &= 360(k) \\ \frac{380}{360} &= k \\ 1.0\overline{5} &= k \end{aligned}$$

Since k is not an integer, we conclude that 15^o and 395^o are not coterminal.

14.

$$\begin{aligned} 225 &= -135 + 360(k) \\ 360 &= 360(k) \\ \frac{360}{360} &= k \\ 1 &= k \end{aligned}$$

Since k is an integer, we conclude that 225^o and -135^o are coterminal.

15. We need to solve $\theta_1 = \theta_2 + 360(k)$ for k. If the solution is an integer then the angles are coterminal otherwise they are not coterminal.

$$\begin{aligned} 107 &= -107 + 360(k) \\ 214 &= 360(k) \\ \frac{214}{360} &= k \\ 0.59\overline{4} &= k \end{aligned}$$

Since k is not an integer, we conclude that 15^o and 395^o are not coterminal.

16.

$$\begin{aligned} 140 &= 440 + 360(k) \\ -300 &= 360(k) \\ \frac{-300}{360} &= k \\ 1.6\overline{1} &= k \end{aligned}$$

Since k is not an integer, we conclude that 140^o and 440^o are not coterminal.

17. We need to solve $\theta_1 = \theta_2 + 2\pi(k)$ for k. If the solution is an integer then the angles are coterminal otherwise they are not coterminal.

$$\begin{aligned} \frac{\pi}{2} &= \frac{3\pi}{2} + 2\pi(k) \\ -\pi &= 2\pi(k) \\ \frac{-\pi}{2\pi} &= k \\ \frac{-1}{2} &= k \end{aligned}$$

Since k is not an integer, we conclude that $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ are not coterminal.

18.

$$\begin{aligned} \frac{\pi}{2} &= -\frac{3\pi}{2} + 2\pi(k) \\ 2\pi &= 2\pi(k) \\ \frac{2\pi}{2\pi} &= k \\ 1 &= k \end{aligned}$$

Since k is an integer, we conclude that $\frac{\pi}{2}$ and $\frac{-3\pi}{2}$ are coterminal.

19. We need to solve $\theta_1 = \theta_2 + 2\pi(k)$ for k. If the solution is an integer then the angles are coterminal otherwise they are not coterminal.

$$\begin{aligned} \frac{7\pi}{6} &= \frac{-5\pi}{6} + 2\pi(k) \\ 2\pi &= 2\pi(k) \\ \frac{2\pi}{2\pi} &= k \\ 1 &= k \end{aligned}$$

Since k is an integer, we conclude that $\frac{7\pi}{6}$ and $\frac{-5\pi}{6}$ are coterminal.

20. We need to solve $\theta_1 = \theta_2 + 2\pi(k)$ for k. If the solution is an integer then the angles are coterminal otherwise they are not coterminal.

$$\begin{aligned} \frac{3\pi}{4} &= \frac{-\pi}{4} + 2\pi(k) \\ \pi &= 2\pi(k) \\ \frac{\pi}{2\pi} &= k \\ \frac{1}{2} &= k \end{aligned}$$

Since k is not an integer, we conclude that $\frac{3\pi}{4}$ and $\frac{-\pi}{4}$ are not coterminal.

21. $sin\ 34^o = 0.5592$

22. $sin\ 82^o = 0.9903$

23. $cos\ 12^o = 0.9781$

24. $cos\ 41^o = 0.7547$

25. $tan\ 5^o = 0.0875$

26. $tan\ 68^o = 2.4751$

27. $cot\ 34^o = \frac{1}{tan\ 34^o} = 1.4826$

28. $cot\ 56^o = \frac{1}{tan\ 56^o} = 0.6745$

29. $sec\ 23^o = \frac{1}{cos\ 23^o} = 1.0864$

30. $csc\ 72^o = \frac{1}{sin\ 72^o} = 1.0515$

31. $sin(\frac{\pi}{5}) = 0.5878$

32. $cos(\frac{2\pi}{5}) = 0.3090$

33. $tan(\frac{\pi}{7}) = 0.4816$

34. $cot(\frac{3\pi}{11}) = \frac{1}{tan(\frac{3\pi}{11})} = 0.8665$

35. $sec(\frac{3\pi}{8}) = \frac{1}{cos(\frac{3\pi}{8})} = 2.6131$

36. $csc(\frac{4\pi}{13}) = \frac{1}{sin(\frac{4\pi}{13})} = 1.2151$

37. $sin(2.3) = 0.7457$

38. $cos(0.81) = 0.6895$

39. $t = sin^{-1}(0.45) = 26.7437^o$

40. $t = sin^{-1}(0.87) = 60.4586^o$

41. $t = cos^{-1}(0.34) = 70.1231^o$

42. $t = cos^{-1}(0.72) = 43.9455^o$

43. $t = tan^{-1}(2.34) = 66.8605^o$

44. $t = tan^{-1}(0.84) = 40.0302^o$

45. $t = sin^{-1}(0.59) = 0.6311$

46. $t = sin^{-1}(0.26) = 0.2630$

47. $t = cos^{-1}(0.60) = 0.9273$

48. $t = cos^{-1}(0.78) = 0.6761$

49. $t = tan^{-1}(0.11) = 0.1096$

50. $t = tan^{-1}(1.26) = 0.8999$

51.

$$\begin{aligned} sin\ 57^o &= \frac{x}{40} \\ x &= 40 sin\ 57^o \\ x &= 33.5468 \end{aligned}$$

52.

$$\begin{aligned} tan\ 20^o &= \frac{15}{x} \\ x &= \frac{15}{tan\ 20^o} \\ x &= 41.2122 \end{aligned}$$

53.

$$\begin{aligned} cos\ 50^o &= \frac{15}{x} \\ x &= \frac{15}{cos\ 50^o} \\ x &= 23.3359 \end{aligned}$$

54.

$$\begin{aligned} sin\ 25^o &= \frac{1.4}{x} \\ x &= \frac{1.4}{sin\ 25^o} \\ x &= 3.3127 \end{aligned}$$

55.

$$\begin{aligned} cos\ t &= \frac{40}{60} \\ t &= cos^{-1}(\frac{40}{60}) \\ t &= 48.1897^o \end{aligned}$$

56.

$$\begin{aligned} tan\ t &= \frac{20}{25} \\ t &= tan^{-1}(\frac{20}{25}) \\ t &= 38.6598^o \end{aligned}$$

57.

$$\begin{aligned} tan\ t &= \frac{18}{9.3} \\ t &= tan^{-1}(\frac{18}{9.3}) \\ t &= 62.6761^o \end{aligned}$$

58.

$$\begin{aligned} sin\ t &= \frac{30}{50} \\ t &= sin^{-1}\frac{30}{50}) \\ t &= 36.8699^o \end{aligned}$$

59. We can rewrite $75^o = 30^o + 45^o$ then use a sum identity

$$\begin{aligned} cos(A+B) &= cos\ Acos\ B - sin\ Asin\ B \\ cos\ 75^o &= cos(30^o + 45^o) \\ &= cos\ 30^o cos\ 45^o - sin\ 30^o sin\ 45^o \\ &= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}} \\ &= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} \\ &= \frac{-1+\sqrt{3}}{2\sqrt{2}} \end{aligned}$$

60. The x coordinate can be found as follows

$$\begin{aligned} cos\ 20^o &= \frac{x}{200} \\ x &= 200c0s\ 20^o \\ &= 187.939 \end{aligned}$$

The y coordinate

$$\begin{aligned} sin\ 20^o &= \frac{y}{200} \\ y &= 200 sin\ 20^o \\ &= 68.404 \end{aligned}$$

61. Five miles is the same as $5 \cdot 5280\ ft = 26400\ ft$. The difference in elevation, y, is

$$\begin{aligned} sin\ 4^o &= \frac{y}{26400} \\ y &= 26400 sin\ 4^o \\ &= 1841.57\ ft \end{aligned}$$

62. First a grade of 5% means that the ratio of the y coordinate to the x coordinate is 0.05 since $tan\ t = \frac{y}{x}$. This means that $x = \frac{y}{0.05} = 20y$ The distance from the base to the top is $6 \cdot 5280\ ft = 31680\ ft$. Using the pythagorean theorem

$$\begin{aligned} x^2 + y^2 &= 31680^2 \\ (20y)^2 + y^2 &= 1003622400 \\ 401y^2 &= 1003622400 \\ y &= \sqrt{\frac{1003622400}{401}} \\ &= 1582.02\ ft \end{aligned}$$

63. **a)**

$$\begin{aligned} cos\ 40^o &= \frac{x}{150} \\ x &= 150 cos\ 40^o \\ &= 114.907 \end{aligned}$$

b)

$$\begin{aligned} sin\ 40^o &= \frac{y}{150} \\ y &= 150 sin\ 40^o \\ &= 96.4181 \end{aligned}$$

c)

$$\begin{aligned} z^2 &= (x+180)^2 + y^2 \\ &= (114.907+180)^2 + (96.4181)^2 \\ z^2 &= 96266.58866 \\ z &= \sqrt{96266.58866} \\ &= 310.268 \end{aligned}$$

64.

$$\begin{aligned} v &= \frac{77000 \cdot 200 \cdot sec\ 60^o}{5000000} \\ &= \frac{15400000}{5000000 cos\ 60^o} \\ &= 6.16\ cm/sec \end{aligned}$$

65.

$$\begin{aligned} v &= \frac{77000 \cdot 100 \cdot sec\ 65^o}{4000000} \\ &= \frac{7700000}{4000000 cos\ 65^o} \\ &= 4.55494\ cm/sec \end{aligned}$$

66. **a)** $tan(67^o) = \frac{h}{x}$ so, $x = \frac{h}{tan(67^o)}$

b)

$$\begin{aligned} tan(24^o) &= \frac{h}{1012+x} \\ h &= tan(24^o)(1012+x) \\ h &= (1012)tan(24^o) + x\ tan(24^o) \\ h &= (1012)tan(24^o) + \frac{h}{tan(67^o)} tan(24^o) \\ h(1 - \frac{tan(24^o)}{tan(67^o)} &= (1012)tan(24^o) \\ h &= \frac{(1012)tan(24^o)}{1 - \frac{tan(24^o)}{tan(67^o)}} \\ &= 555.567\ \ ft \end{aligned}$$

67. **a)** When we consider the two triangles we have a new triangle that has three equal angles which is the definition of an equilateral triangle.

b) The short leg of each triangle is given by $2sin(30) = 2(\frac{1}{2}) = 1$

c) The long leg (L) is given by

$$\begin{aligned} 2^2 &= L^2 + 1^2 \\ 4 - 1 &= L^2 \\ \sqrt{3} &= L \end{aligned}$$

d) By considering all possible ratios between the long, short and hypotenuse of small triangles we obtain the trigonometric functions of $\frac{\pi}{6} = 30^o$ and $\frac{\pi}{3} = 60^o$

68. **a)** Since the triangle has two angles equal in magnitude it should have two sides that are equal as well. (It is an isosceles triangle)

b)

$$\begin{aligned} h^2 &= 1^1 + 1^1 \\ h^2 &= 2 \\ h &= \sqrt{2} \end{aligned}$$

c) Since the hypotenuse is known then we can use the figure to find the trigonometric functions of $\frac{\pi}{4} = 45^o$ using the ratios of the sides of the triangle.

69. **a)** The tangent of an angle is equal to the ratio of the opposite side to the adjacent side (of a right triangle), and for the small triangle that ratio is $\frac{5}{7}$.

b) For the large right triangle, the opposite side is 10 and the adjacent side is $7+7 = 14$. Thus the tangent is $\frac{10}{14}$

c) Because the trigonometric functions depend on the ratios of the sides and not the size of triangle. Note that the answer in part b) is equivalent to that in part a) even though the triangle in part b) was larger that that used in part a)

70. Let (x, y) be a non-origin point that defines the terminal side of an angle, t, and let $r = \sqrt{x^2 + y^2}$ be the distance from the origin to the point (x, y). Then the trigonometric function are defined as follows:

$sin\ t = \frac{y}{r}$ and $csc\ t = \frac{r}{y}$ $(y \neq 0)$
$cos\ t = \frac{x}{r}$ and $sec\ t = \frac{r}{x}$ $(x \neq 0)$ and
$tan\ t = \frac{y}{x}$ $(x \neq 0)$ and $cot\ t = \frac{x}{y}$ $(y \neq 0)$
From the above definitions and recalling that the reciprocal of a non zero number x is given by $\frac{1}{x}$ we show that
$sin\ t = \frac{1}{csc\ t}$

$cos\ t = \frac{1}{sec\ t}$ and

$tan\ t = \frac{1}{cot\ t}$

71.

$$\begin{aligned} \frac{sin\ t}{cos\ t} &= \frac{y/r}{x/r} \\ &= \frac{y}{r} \div \frac{x}{r} \\ &= \frac{y}{r} \cdot \frac{r}{x} \\ &= \frac{y}{x} \\ &= tan\ t \end{aligned}$$

$$\begin{aligned} &Thus \\ \frac{sin\ t}{cos\ t} &= tan\ t \end{aligned}$$

and

$$\begin{aligned} \frac{cos\ t}{sin\ t} &= \frac{x/r}{y/r} \\ &= \frac{x}{r} \div \frac{y}{r} \\ &= \frac{x}{r} \cdot \frac{r}{y} \\ &= \frac{x}{y} \\ &= cot\ t \end{aligned}$$

$$\begin{aligned} &Thus \\ \frac{cos\ t}{sin\ t} &= cot\ t \end{aligned}$$

72. **a)** $sin(t) = \frac{u}{1} = u$

b) Consider the triangle made by the sides v, w, and y. The angle vw has a value of $90 - r$ (completes a straight angle). The sum of angles in any triangle is 180. Therefore

$$\begin{aligned} s + 90 + (90 - r) &= 180 \\ s + 180 - r &= 180 \\ s - r &= 0 \\ s &= r \end{aligned}$$

c) $sin(s) = \frac{w}{v}$ which means that $w = sin(s)v$. $cos(t) = \frac{v}{1} = v$
Thus, $w = sin(s)v = sin(s)\ cos(t)$

d) $sin(t) = \frac{u}{1} = u$ and $cos(r) = \frac{x}{u}$ which means that $x = u\ cos(r)$.
In part b) we showed that $r = s$ therefore $cos(r) = cos(s)$.
So, $x = u\ cos(r) = sin(t)cos(s)$

e) $sin(s+t) = \frac{(w+x)}{1} = w+x$. Using the results we have obtained from previous parts we can conclude
$sin(s+t) = w + x = sin(s)cos(t) + cos(s)sin(t)$

73. **a)** $sin(t) = \frac{u}{1} = u$, and $cos(t) = \frac{v}{1} = v/$

b) Consider the triangle made by the sides v, w, and y. The angle vw has a value of $90 - r$ (completes a straight angle). The sum of angles in any triangle is 180. Therefore

$$\begin{aligned} s + 90 + (90 - r) &= 180 \\ s + 180 - r &= 180 \\ s - r &= 0 \\ s &= r \end{aligned}$$

c) $cos(s) = \frac{y}{v}$, which means $y = cos(s)v$.
But from part a) $v = cos(t)$, therefore $y = cos(s)cos(t)$

d) $sin(r) = \frac{z}{u}$, which means $z = sin(r)u$.
Using results from part a) and part b) we get $sin(r) = sin(s)$ and $u = sin(t)$, therefore $z = sin(s)sin(t)$

e) $cos(s+t) = \frac{(y-z)}{1} = y - z$. Replacing ur results for y and z we get $cos(s+t) = cos(s)cos(t) - sin(s)sin(t)$

74. **a)** $cos(\frac{\pi}{2} - t) = \frac{u}{1} = u = sin(t)$

b) $sin(\frac{\pi}{2} - t) = \frac{v}{1} = v = cos(t)$

75. Use $cos^2t + sin^2t = 1$ as follows

$$\begin{aligned} cos^2t + sin^2t &= 1 \\ \frac{cos^2t}{cos^2t} + \frac{sin^2t}{cos^2t} &= \frac{1}{cos^2t} \\ 1 + tan^t &= sec^2t \end{aligned}$$

76.

$$\begin{aligned} cos^2t + sin^2t &= 1 \\ \frac{cos^2t}{sin^2t} + \frac{sin^2t}{sin^2} &= \frac{1}{sin^2t} \\ cot^2t + 1 &= csc^2t \end{aligned}$$

77. Let $2t = t + t$

$$\begin{aligned} sin(a+b) &= sin(a)cos(b) + cos(a)sin(b) \\ sin(2t) &= sin(t+t) \\ &= sin(t)cos(t) + cos(t)sin(t) \\ &= 2sin(t)cos(t) \end{aligned}$$

78. **a)**

$$\begin{aligned} cos(2t) &= cos(t+t) \\ &= cos(t)cos(t) - sin(t)sin(t) \\ &= cos^2(t) - sin^2(t) \end{aligned}$$

b)

$$\begin{aligned} cos(2t) &= cos^2(t) - sin^2(t) \\ &= cos^2(t) - (1 - cos^2(t)) \\ &= 2cos^2(t) - 1 \end{aligned}$$

c)

$$\begin{aligned} cos(2t) &= cos^2(t) - sin^2(t) \\ &= (1 - sin^2(t)) - sin^2(t) \\ &= 1 - 2sin^2(t) \end{aligned}$$

79. Using the result from Exercise 78 part (c)

$$\begin{aligned} cos(2t) &= 1 - 2sin^2(t) \\ cos(2t) - 1 &= -2sin^2(t) \\ \frac{cos(2t) - 1}{-2} &= sin^2(t) \\ \frac{1 - cos(2t)}{2} &= sin^2(t) \end{aligned}$$

80.

$$\begin{aligned} cos(2t) &= 2cos^2 - 1 \\ cos(2t) + 1 &= 2cos^2(t) \\ \frac{cos(2t) + 1}{2} &= cos^2(t) \end{aligned}$$

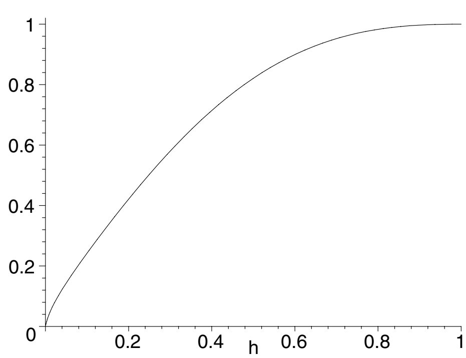
81. **a)** $V(0) = sin^p(0)sin^q(0)sin^r(0)sin^s(0) = 0$
$V(1) = sin^p(\frac{\pi}{2})sin^q(\frac{\pi}{2})sin^r(\frac{\pi}{2})sin^s(\frac{\pi}{2}) = 1$

b) When $h = 0$ the volume of the tree is zero since there is no height and therefore the proportion of volume under that height is zero. While at the top of the tree, $h = 1$, the proportion of volume under the tree is 1 since the entire tree volume falls below its height.

82. **a)**

$$\begin{aligned} V(0.5) &= sin^{-3.728}(\frac{\pi}{4})sin^{48.646}(\frac{\pi}{2\sqrt{2}}) \\ &\quad \times sin^{-123.208}(\frac{\pi}{2\sqrt[3]{2}})sin^{86.629}(\frac{\pi}{2\sqrt[4]{2}}) \\ &= 0.8208 \end{aligned}$$

b) $V(h)$



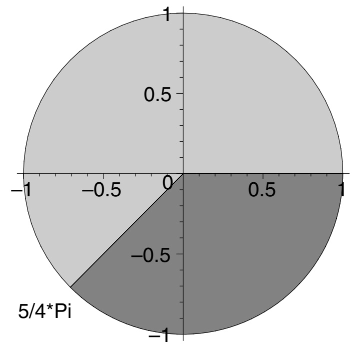
c) The result from part b) agrees with the definition of $V(h)$ since the values of $V(h)$ are limited between 0 and 1.

83.

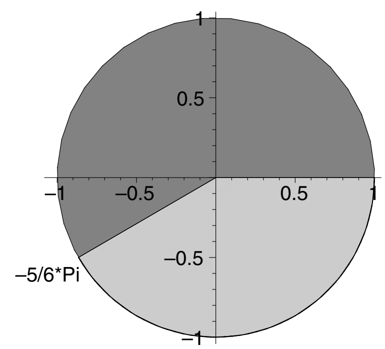
$$\begin{aligned} V(\frac{1}{2}) &= sin^{-5.621}(\frac{\pi}{4})sin^{74.831}(\frac{\pi}{2\sqrt{2}}) \\ &\quad \times sin^{-195.644}(\frac{\pi}{2\sqrt[3]{2}})sin^{138.959}(\frac{\pi}{2\sqrt[4]{2}}) \\ &= 0.8219 \end{aligned}$$

Exercise Set 1.5

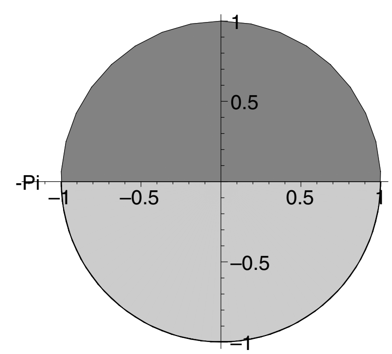
1. $5\pi/4$



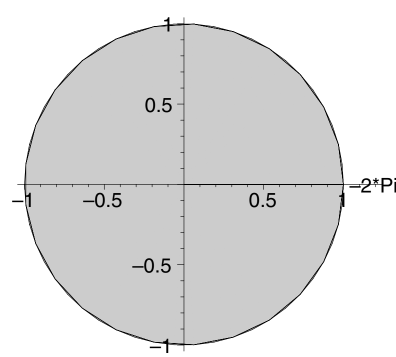
2. $-5\pi/6$



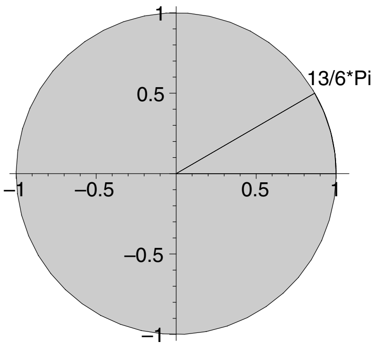
3. $-\pi$



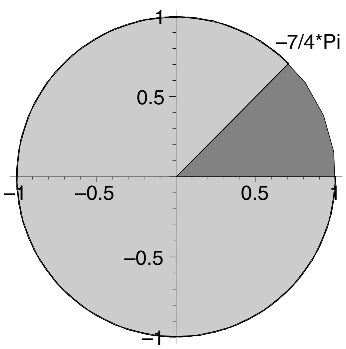
4. 2π



5. $13\pi/6$



6. $-7\pi/4$



7. $cos(9\pi/2) = 0$

8. $sin(5\pi/4) = \frac{-1}{\sqrt{2}}$

9. $sin(-5\pi/6) = \frac{-1}{2}$

10. $cos(-5\pi/4) = \frac{-1}{\sqrt{2}}$

11. $cos(5\pi) = -1$

12. $sin(6\pi) = 0$

13. $tan(-4\pi/3) = -\sqrt{3}$

14. $tan(-7\pi/3) = -\sqrt{3}$

15. $cos\ 125^o = -0.5736$

16. $sin\ 164^o = 0.2756$

17. $tan(-220^o) = -0.8391$

18. $cos(-253^o) = -0.2924$

19. $sec\ 286^o = \frac{1}{cos\ 286^o} = 3.62796$

20. $csc\ 312^o = \frac{1}{sin\ 312^o} = -1.34563$

21. $sin(1.2\pi) = -0.587785$

22. $tan(-2.3\pi) = -1.37638$

23. $cos(-1.91) = -0.332736$

24. $sin(-2.04) = -0.891929$

25. $t = sin^{-1}(1/2) = \frac{\pi}{6} + 2n\pi$ and $\frac{5\pi}{6} + 2n\pi$

26. $t = sin^{-1}(-1) = \frac{3\pi}{2} + 2n\pi$

27. $2t = sin^{-1}(0) = n\pi$ so $t = \frac{n\pi}{2}$

28.

$$\begin{aligned} 2sin(t+\frac{\pi}{3}) &= -\sqrt{3} \\ sin(t+\frac{\pi}{3}) &= \frac{-\sqrt{3}}{2} \\ t+\frac{\pi}{3} &= sin^{-1}(\frac{-\sqrt{3}}{2}) \\ t &= \frac{-\pi}{3} - \frac{\pi}{3} + 2n\pi \\ &= \frac{-2\pi}{3} + 2n\pi \\ \text{and} \\ t &= \frac{-\pi}{3} + \frac{4\pi}{3} + 2n\pi \\ &= \pi + 2n\pi \end{aligned}$$

29.

$$\begin{aligned} cos(3t+\frac{\pi}{4}) &= -\frac{1}{2} \\ 3t+\frac{\pi}{4} &= cos^{-1}(-\frac{1}{2}) \\ 3t &= -\frac{\pi}{4} + \frac{2\pi}{3} + 2n\pi \\ 3t &= \frac{5\pi}{12}2n\pi \\ t &= \frac{5\pi}{36}\frac{2}{3}n\pi \\ \text{and} \\ 3t &= -\frac{\pi}{4} + \frac{4\pi}{3} + 2n\pi \\ 3t &= \frac{13\pi}{12} + 2n\pi \\ t &= \frac{13}{36}\pi + \frac{2}{3}n\pi \end{aligned}$$

30.

$$\begin{aligned} cos(2t) &= 0 \\ 2t &= cos^{-1}(0) \\ 2t &= \frac{\pi}{2} + 2n\pi \\ t &= \frac{\pi}{4} + n\pi \\ \text{and} \\ 2t &= \frac{3\pi}{2} + 2n\pi \\ t &= \frac{3\pi}{4} + n\pi \end{aligned}$$

31.

$$\begin{aligned} cos(3t) &= 1 \\ 3t &= cos^{-1}(1) \\ 3t &= 2n\pi \\ t &= \frac{2}{3}n\pi \end{aligned}$$

32.

$$\begin{aligned} 2cos(\frac{t}{2}) &= -\sqrt{3} \\ cos(\frac{t}{2}) &= \frac{-\sqrt{3}}{2} \\ \frac{t}{2} &= cos^{-1}(\frac{-\sqrt{3}}{2}) \\ \frac{t}{2} &= \frac{5\pi}{6} + 2n\pi \\ t &= \frac{5\pi}{3} + 4n\pi \\ \text{and} & \\ \frac{t}{2} &= \frac{7\pi}{6} + 2n\pi \\ t &= \frac{7\pi}{3} + 4n\pi \end{aligned}$$

33.

$$\begin{aligned} 2sin^2 t - 5sin\ t - 3 &= 0 \\ (2sin\ t + 1)(sin\ t - 3) &= 0 \\ \text{The only solution comes from} & \\ (2sin\ t + 1) &= 0 \\ sin\ t &= -\frac{1}{2} \\ t &= sin^{-1}(-\frac{1}{2}) \\ t &= \frac{7\pi}{6} + 2n\pi \\ \text{and} & \\ t &= \frac{11\pi}{6} + 2n\pi \end{aligned}$$

34.

$$\begin{aligned} cos^2 x + 5cos\ x &= 6 \\ cos^2 x + 5cos\ x - 6 &= 0 \\ (cos\ x + 6)(cos\ x - 1) &= 0 \\ \text{The only solution comes from} & \\ cos\ x - 1 &= 0 \\ x &= cos^{-1}(1) \\ x &= 2n\pi \end{aligned}$$

35.

$$\begin{aligned} cos^2 x + 5cos\ x &= -6 \\ cos^2 x + 5cos\ x + 6 &= 0 \\ cos\ x &= \frac{-5 \pm \sqrt{25 - 4(1)(6)}}{2} \\ &= \frac{-5 \pm 1}{2} \\ &= \frac{-5 - 1}{2} = -3 \\ \text{and} & \\ &= \frac{-5 + 1}{2} = -2 \end{aligned}$$

Since both values are larger than one, then the equation has no solutions.

36.

$$\begin{aligned} sin^2 t - 2sin\ t - 3 &= 0 \\ (sin\ t - 3)(sin\ t + 1) &= 0 \\ \text{The only solution comes from} & \\ sin\ t + 1 &= 0 \\ t &= sin^{-1}(-1) \\ &= \frac{3\pi}{2} + 2n\pi \end{aligned}$$

37. $y = 2sin\ 2t + 4$
amplitude $= 2$, period $= \frac{2\pi}{2} = \pi$, mid-line $y = 4$
maximum $= 4 + 2 = 6$, minimum $= 4 - 2 = 2$

38. $y = 3cos\ 2t - 3$
amplitude $= 3$, period $= \frac{2\pi}{2} = \pi$, mid-line $y = -3$
maximum $= -3 + 3 = 0$, minimum $= -3 - 3 = -6$

39. $y = 5cos(t/2) + 1$
amplitude $= 5$, period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$, mid-line $y = 1$
maximum $= 1 + 5 = 6$, minimum $= 1 - 5 = -4$

40. $y = 3sin(t/3) + 2$
amplitude $= 3$, period $= \frac{2\pi}{\frac{1}{3}} = 6$, mid-line $y = 2$
maximum $= 2 + 3 = 5$, minimum $= 2 - 3 = -1$

41. $y = \frac{1}{2}sin(3t) - 3$
amplitude $= \frac{1}{2}$, period $= \frac{2\pi}{3}$, mid-line $y = -3$
maximum $-3 + \frac{1}{2} = \frac{-5}{2}$, minimum $= -3 - \frac{1}{2} = \frac{-7}{2}$

42. $y = \frac{1}{2}cos(4t) + 2$
amplitude $= \frac{1}{2}$, period $= \frac{2\pi}{4} = \frac{\pi}{2}$, mid-line $y = 2$
maximum $= 2 + \frac{1}{2} = \frac{5}{2}$, minimum $= 2 - \frac{1}{2} = \frac{3}{2}$

43. $y = 4sin(\pi t) + 2$
amplitude $= 4$, period $= \frac{2\pi}{\pi} = 2$, mid-line $y = 2$
maximum $= 2 + 4 = 6$, minimum $= 2 - 4 = -2$

44. $y = 3cos(3\pi t) - 2$
amplitude $= 3$, period $= \frac{2\pi}{3\pi} = \frac{2}{3}$, mid-line $y = -2$
maximum $= -2 + 3 = 1$, minimum $= -2 - 3 = -5$

45. The maximum is 10 and the minimum is -4 so the amplitude is $\frac{10-(-4)}{2} = 7$. The mid-line is $y = 10 - 7 = 3$, and the period is 2π (the distance from one peak to the next one) which means that $b = \frac{2\pi}{2\pi} = 1$. From the information above, and the graph, we conclude that the function is

$$y = 7sin\ t + 3$$

46. The maximum is 4 and the minimum is -1 so the amplitude is $\frac{4-(-1)}{2} = \frac{5}{2}$. The mid-line is $y = 4 - \frac{5}{2} = \frac{3}{2}$, and the period is 4π which means $b = \frac{\pi}{2}$. From the information above, and the graph, we conclude that the function is

$$y = \frac{5}{2}cos(t/2) + \frac{3}{2}$$

47. The maximum is 1 and the minimum is -3 so the amplitude is $\frac{1-(-3)}{2} = 2$. The mid-line is $y = 1 - 2 = -1$, and the period is 4π which means $b = \frac{\pi}{2}$. From the information above, and the graph, we conclude that the function is

$$y = 2cos(t/2) - 1$$

48. The maximum is -0.5 and the minimum is -1.5 so the amplitude is $\frac{-0.5-(-1.5)}{2} = \frac{1}{2}$. The mid-line is $y = -0.5 - \frac{-1}{2} = -1$, and the period is 1 which means that $b = \frac{2\pi}{1} = 2\pi$. From the information above, and the graph, we conclude that the function is

$$y = \frac{1}{2}sin(2\pi t) - 1$$

49.

$$\begin{aligned} R &= 0.339 + 0.808cos\ 40^o\ cos\ 30^o \\ &\quad -0.196sin\ 40^o\ sin\ 30^o - 0.482cos\ 0^o\ cos\ 30^o \\ &= 0.571045\ \ megajoules/m^2 \end{aligned}$$

50.

$$\begin{aligned} R &= 0.339 + 0.808cos\ 30^o\ cos\ 20^o \\ &\quad -0.196sin\ 30^o\ sin\ 20^o - 0.482cos\ 180^o\ cos\ 20^o \\ &= 1.12788\ \ megajoules/m^2 \end{aligned}$$

51.

$$\begin{aligned} R &= 0.339 + 0.808cos\ 50^o\ cos\ 55^o \\ &\quad -0.196sin\ 50^o\ sin\ 55^o - 0.482cos\ 45^o\ cos\ 55^o \\ &= 0.234721\ \ megajoules/m^2 \end{aligned}$$

52.

$$\begin{aligned} R &= 0.339 + 0.808cos\ 50^o\ cos\ 0^o \\ &\quad -0.196sin\ 50^o\ sin\ 0^o - 0.482cos\ 0^o\ cos\ 0^o \\ &= 0.858372\ \ megajoules/m^2 \end{aligned}$$

53. Period is 5 so $b = \frac{2\pi}{5}$, $k = 2500$, $a = 250$. Therefore, the function is

$$V(t) = 250cos\ \frac{2\pi t}{5} + 2500$$

54. Period is 2 sp $b = \frac{2\pi}{2} = \pi$, $a = \frac{3400}{2} = 1700$, $k = 1700 + 1100 = 2800$. Therefore, the function is

$$V(t) = 1700cos\ \pi t + 2800$$

55. Since our lungs increase and decrease as we breathe then there is a maximum and minimum volume for the air capacity in our lungs. We have a regular period of time at which we breathe (inhale and exhale). These facotrs are reasons why the cosine model is appropriate for describing lung capacity.

56. The minimum is 35.33 and the maximum is 36.87 so the amplitude is $\frac{36.87-35.33}{2} = 0.77$. The period is 24 so $b = \frac{2\pi}{24} = \frac{\pi}{12}$, $k = 36.87 - 0.77 = 36.1$. Thus, the function is

$$T(t) = 0.77cos\ \frac{\pi}{12} + 36.1$$

57. The frequency is the reciprocal of the period. Therefore, $f = \frac{b}{2\pi} = \frac{880\pi}{2\pi} = 440\ Hz$

58. $f = \frac{440\pi}{2\pi} = 220\ Hz$

59. The amplitude is given as 5.3. $b = f \cdot 2\pi$ where f is the frequency, $b = 0.172 \cdot 2\pi = 1.08071$, $k = 143$. Therefore, the function is

$$p(t) = 5.3cos(1.08071t) + 143$$

60. $p(t) = 6.7cos(0.496372t) + 137$

61. $x = cos(140^o)$, $y = sin(140^o)$, $(-0.76604, 0.64279)$

62. $(-0.17365, -0.98481)$

63. $x = cos(\frac{9\pi}{5})$, $y = sin(\frac{9\pi}{5})$, $(0.80902, -0.58779)$

64. $(-0.22252, -0.97493)$

65. Rewrite $105^o = 45^o + 60^o$ and use a sum identity.

$$\begin{aligned} sin\ 105^o &= sin(45^o + 60^o) \\ &= sin\ 45^o\ cos\ 60^o + cos\ 45^o\ sin\ 60^o \\ &= \frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} \\ &= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} \\ &= \frac{1+\sqrt{3}}{2\sqrt{2}} \end{aligned}$$

66.

$$\begin{aligned} cos\ 165^o &= cos(120^o + 45^o) \\ &= cos\ 120^o\ cos\ 45^o - sin\ 120^o\ sin\ 45^o \\ &= \frac{-1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} \\ &= -\frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} \\ &= -\frac{1+\sqrt{3}}{2\sqrt{2}} \end{aligned}$$

67. **a)** From the graph we can see that the point with angle t has an opposite x and y coordinate than the point with angle $t + \pi$. Since the x coordinate corresponds to the cos of the angle which the point makes and the y coordinate corresponds to the sin of the angle which the point makes it follows that $sin(t + \pi) = -sin(t)$ and $cos(t + \pi) = -cos(t)$.

b)

$$\begin{aligned} sin(t + \pi) &= sin\ tcos\ \pi + cos\ tsin\ \pi \\ &= sin\ t \cdot -1 + cos\ t \cdot 0 \\ &= -sin\ t \end{aligned}$$

and

$$\begin{aligned} cos(t + \pi) &= cos\ tcos\ \pi - sin\ tsin\ \pi \\ &= cos\ t \cdot -1 - sin\ t \cdot 0 \\ &= -cos\ t \end{aligned}$$

c)

$$tan(t + \pi) = \frac{sin(t + \pi)}{cos(t + \pi)}$$

$$\begin{aligned} &= \frac{-sin\ t}{-cos\ t} \\ &= \frac{sin\ t}{cos\ t} \\ &= tan\ t \end{aligned}$$

68. **a)** The amplitude could be thought of as half the difference between the maximum and minimum, $a = \frac{max-min}{2}$, which implies that $2a = max - min$. k is the average mean of the maximum and the minimum, $k = \frac{max+\text{min}}{2}$, which implies that $2k = max + min$. Solving the system of equations above for max and min gives the desired results.

b) The average mean of the maximum and minimum, using the results from part a), implies that the midline equation is $y = \frac{(k+a)+(k-a)}{2} = \frac{2k}{2} = k$.

c) Half the difference between the maximum and minimum, using the results from part a), implies that the amplitude is $\frac{(k+a)-(k-a)}{2} = \frac{2a}{2} = a$.

69. **a)** Since the radius of a unit circle is 1, the circumference of the unit circle is 2π. Therefore any point $t + 2\pi$ will have exactly the same terminal side as the point t, that is to say that the points t and $t+2\pi$ are coterminal on the unit circle. Therefore, $sin\ t = sin(t + 2\pi)$ for all numbers t.

b)

$$\begin{aligned} g(t + 2\pi/b) &= asin[b(t + 2\pi/b)] + k \\ &= asin(bt + 2\pi) + k \end{aligned}$$

from part a)

$$= asin(bt) + k$$

by definition

$$g(t + 2\pi/b) = g(t)$$

c) Since the function evaluated at $t+2\pi/b$ has the same value as the function evaluated at t and $2\pi/b \neq 0$ then $t + 2\pi/b$ is evaluated after t. Since we have a periodic function in $g(t)$ it follows that the period of the function is implied to be $2\pi/b$.

70. Since at the apex, L is large, T is small, and d is small, then the basilar membrane is affected mostly by low frequency sounds.

71. Since at the base, L is small, T is large, and d is large, then the basilar membrane is affected mostly by high frequency sounds.

72. $f = \frac{880\pi}{2\pi} = 440$

73. $f = \frac{880\cdot 2^{-9/12}\pi}{2\pi} = 261.626$

74. From the equation, n has to be 12 in order for $\frac{880(2^{n/12})\pi}{2\pi}$ to equal 880. There are 12 notes above A above middle C.

75.

$$\begin{aligned} \frac{880(2^{n/12})\pi}{2\pi} &= 1760 \\ 2^{n/12-1} &= \frac{1760}{880} \\ 2^{n/12-1} &= 2 \end{aligned}$$

Comparing exponents we can conclude that

$$\begin{aligned} \frac{n}{12} - 1 &= 1 \\ \frac{n}{12} &= 2 \\ n &= 24 \end{aligned}$$

There are 24 notes above A above middle C.

76.

$$\begin{aligned} \frac{880(2^{n/12})\pi}{2\pi} &= 1320 \\ 2^{n/12-1} &= \frac{1320}{880} \\ 2^{n/12-1} &= 1.5 \\ (\frac{n}{12} - 1)ln(2) &= ln(1.5) \\ \frac{n}{12} - 1 &= \frac{ln(1.5)}{ln(2)} \\ \frac{n}{12} &= \frac{ln(1.5)}{ln(2)} + 1 \\ n &= 12\left(\frac{ln(1.5)}{ln(2)} + 1\right) \\ n &= 19.01955 \end{aligned}$$

There are 19 notes above A above middle C.

77.

$$\begin{aligned} \frac{880(2^{n/12})\pi}{2\pi} &= 2200 \\ 2^{n/12-1} &= \frac{2200}{880} \\ 2^{n/12-1} &= 2.5 \\ (\frac{n}{12} - 1)ln(2) &= ln(2.5) \\ \frac{n}{12} - 1 &= \frac{ln(2.5)}{ln(2)} \\ \frac{n}{12} &= \frac{ln(2.5)}{ln(2)} + 1 \\ n &= 12\left(\frac{ln(2.5)}{ln(2)} + 1\right) \\ n &= 27.86314 \end{aligned}$$

There are 28 notes above A above middle C

78. **a)** Left to the student

b) $y = \frac{1}{2}cos(2t) - \frac{1}{2}$

c) We use the double angle identity obtained in Exercise 79 of Section 1.4 and solve for $-sin^2(t)$ to obtain the model in part b).

79. **a)** Left to the student

b) $y = \frac{1}{2}cos(2t) + \frac{1}{2}$

c) We use the double angle identity obtained in Exercise 79 of Section 1.4 and solve for $cos^2(t)$ to obtain the model in part b).

80. **a)** Left to the student
b) $y = sin(2t)$
c) We use the double angle identity for $sin(2t)$ obtained in Exercise 77 of Section 1.4 to obtain the model in part b).

81. **a)** Left to the student
b) Left to the student
c) The horizontal shift moves every point of the original graph $\frac{\pi}{4}$ units to the right.

82. **a)** Left to the student
b) Left to the student
c) The horizontal shift moves every point of the original graph $\frac{\pi}{3}$ units to the left.

83. Left to the student

84. Left to the student

85. Left to the student

Chapter Review Exercises

1. **a)** 100 live births per 1000 women
b) 20 years old and 30 years old

2. $f(-2) = 2(-2)^2 - (-2) + 3 = 13$

3.

$$\begin{aligned} f(1+h) &= 2(1+h)^2 - (1+h) + 3 \\ &= 2(1+2h+h^2) - 1 - h + 3 \\ &= 2 + 4h + 2h^2 - 1 - h + 3 \\ &= 2h^2 + 3h + 4 \end{aligned}$$

4. $f(0) = 2(0)^2 - (0) + 3 = 3$

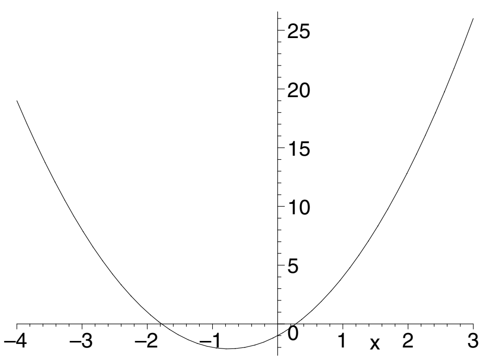
5. $f(-5) = (1-(-5))^2 = (1+5)^2 = 6^2 = 36$

6.

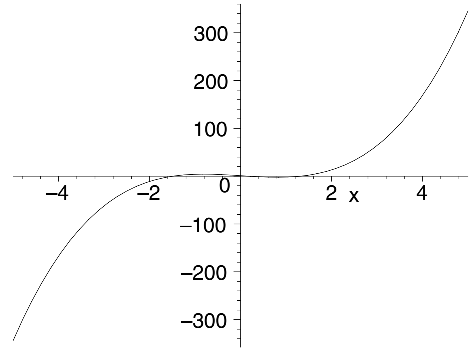
$$\begin{aligned} f(2-h) &= (1-(2-h))^2 \\ &= (h-1)^2 \\ &= h^2 - 2h + 1 \end{aligned}$$

7. $f(4) = (1-4)^2 = (-3)^2 = 9$

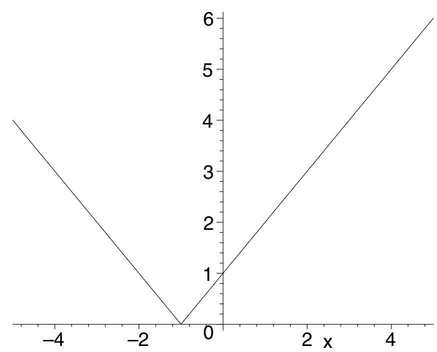
8. $f(x) = 2x^2 + 3x - 1$



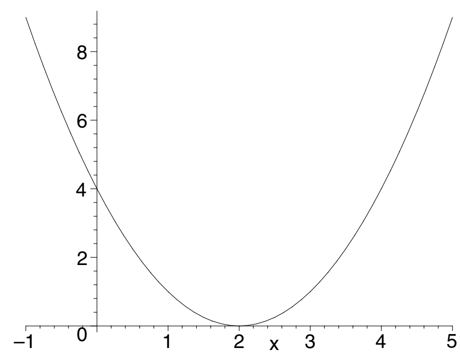
9. $y = 3x^3 - 6x + 1$



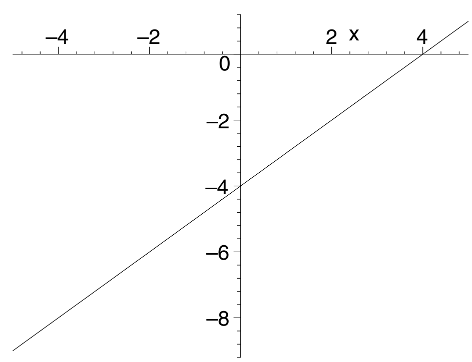
10. $y = \mid x+1 \mid$



11. $f(x) = (x-2)^2$

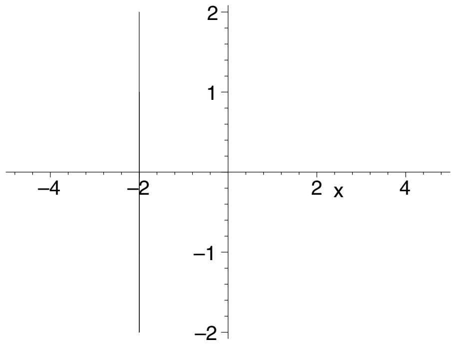


12. $f(x) = \frac{x^2-16}{x+4}$. It is important to note that $x = -4$ does not belong to the domain of the plotted function.

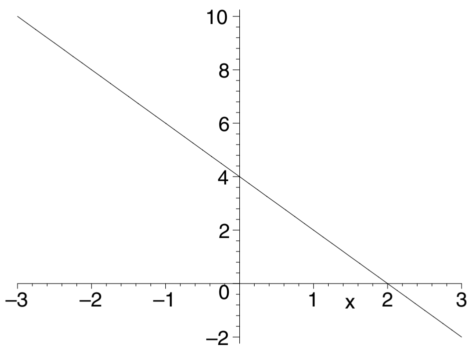


13. **a)** $f(2) = 1.2$
b) $x = -3$

14. $x = -2$



15. $y = 4 - 2x$



16. $m = \frac{-2-5}{4-(-7)} = \frac{-7}{11}$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-2) &= \frac{-7}{11}(x - 4) \\ y &= -\frac{7x}{11} + \frac{28}{11} - 2 \\ y &= -\frac{7x}{11} + \frac{6}{11} \end{aligned}$$

17. Use the slope-point equation

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 11 &= 8(x - \frac{1}{2}) \\ y &= 8x - 4 + 11 \\ y &= 8x + 7 \end{aligned}$$

18. Slope $= -\frac{1}{6}$, y-intercept $(0, 3)$

19.

$$\begin{aligned} x^2 + 5x + 4 &= 0 \\ (x+1)(x+4) &= 0 \\ x + 1 &= 0 \\ x &= -1 \\ &\mathit{Or} \\ x + 4 &= 0 \\ x &= -4 \end{aligned}$$

20.

$$\begin{aligned} x^2 - 7x + 12 &= 0 \\ (x-3)(x-4) &= 0 \\ x - 3 &= 0 \\ x &= 3 \\ &\mathit{Or} \\ x - 4 &= 0 \\ x &= 4 \end{aligned}$$

21. $x^2 + 2x = 8$

$$\begin{aligned} x^2 + 2x - 8 &= 0 \\ (x+4)(x-2) &= 0 \\ x + 4 &= 0 \\ x &= -4 \\ &\mathit{Or} \\ x - 2 &= 0 \\ x &= 2 \end{aligned}$$

22. $x^2 + 6x = 20$

$$\begin{aligned} x^2 + 6x - 20 &= 0 \\ x &= \frac{-6 \pm \sqrt{36 + 80}}{2} \\ &= \frac{-6 \pm 2\sqrt{29}}{2} \\ &= -3 \pm \sqrt{29} \end{aligned}$$

23.

$$\begin{aligned} x^3 + 3x^2 - x - 3 &= 0 \\ x^2(x+3) - (x+3) &= 0 \\ (x+3)(x^2-1) &= 0 \\ (x+3)(x-1)(x+1) &= 0 \\ x + 3 &= 0 \\ x &= -3 \\ &\mathit{Or} \\ x - 1 &= 0 \\ x &= 1 \\ &\mathit{Or} \\ x + 1 &= 0 \\ x &= -1 \end{aligned}$$

24.

$$\begin{aligned} x^4 + 2x^3 - x - 2 &= 0 \\ x^3(x+2) - (x+2) &= 0 \\ (x+2)(x^3-1) &= 0 \\ x &= -2 \\ x &= -1 \end{aligned}$$

25. Using the points (one could use any two points on the line) (0, 50, and (4, 350) the rate of change is

$$\frac{350 - 50}{4 - 0} = \frac{300}{4} = 75 \text{ pages per day}$$

26. The rate of change is

$$\frac{20-100}{12-0} = \frac{-80}{12} = \frac{-20}{3} \text{ meters per second}$$

27. The variation equation is $M = kW$, with k constant. When $W = 150$, $M = 60$ means

$$\begin{aligned} 60 &= k(150) \\ \frac{60}{150} &= k \\ \frac{2}{5} &= k \end{aligned}$$

Find M when $W = 210$

$$\begin{aligned} M &= \frac{2}{5}W \\ &= \frac{2}{5}(210) \\ &= 84\ lbs \end{aligned}$$

28. $5x^2 - x - 7 = 0$

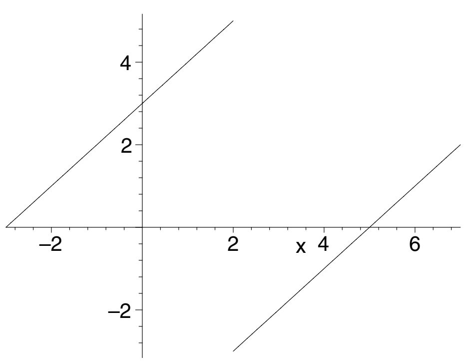
$$\begin{aligned} x &= \frac{1 \pm \sqrt{1+140}}{10} \\ &= \frac{1 \pm \sqrt{141}}{10} \end{aligned}$$

29. $y^{1/6} = \sqrt[6]{y}$

30. $\sqrt[20]{x^3} = x^{3/20}$

31. $27^{2/3} = (\sqrt[3]{27})^2 = 3^2 = 9$

32.



33. **a)** $m = \frac{92-74}{23-9} = \frac{18}{14} = \frac{9}{7}$

$$\begin{aligned} G - 74 &= \frac{9}{7}(x-9) \\ G &= \frac{9}{7}x - \frac{81}{7} + 74 \\ G &= \frac{9}{7}x + \frac{437}{7} \end{aligned}$$

b) $G(18) = \frac{9}{7}(18) + \frac{437}{7} = 85.6$
$G(25) = \frac{9}{7}(25) + \frac{437}{7} = 94.6$

34. $sin(2\pi/3) = \frac{\sqrt{3}}{2}$

35. $cos(-\pi) = -1$

36. $tan(7\pi/4) = -1$

37.

$$\begin{aligned} sin(70^o) &= \frac{x}{127} \\ x &= 127\ sin(70^o) \\ &= 119.341 \end{aligned}$$

38. $t = sin^{-1}(1) = \frac{\pi}{2} + 2n\pi$

39. $t = tan^{-1}(\sqrt{3}) = \frac{\pi}{3} + n\pi$

40. $2t = cos^{-1}(2)$, No solution

41.

$$\begin{aligned} 12cos^2(2t - \frac{\pi}{4}) &= 9 \\ cos^2(2t - \frac{\pi}{4}) &= \frac{9}{12} \\ cos^2(2t - \frac{\pi}{4}) &= \frac{3}{4} \\ cos(2t - \frac{\pi}{4}) &= \pm\sqrt{3/4} \end{aligned}$$

Two solutions

$$\begin{aligned} 2t - \frac{\pi}{4} &= cos^{-1}(\sqrt{3/4}) \\ 2t - \frac{\pi}{4} &= \frac{\pi}{6} + 2n\pi \\ 2t &= \frac{\pi}{6} + \frac{\pi}{4} + 2n\pi \\ 2t &= \frac{5\pi}{12} + 2n\pi \\ t &= \frac{5pi}{24} + n\pi \end{aligned}$$

and

$$\begin{aligned} 2t - \frac{\pi}{4} &= cos^{-1}(-\sqrt{3/4}) \\ 2t - \frac{\pi}{4} &= \frac{-\pi}{6} + 2n\pi \\ 2t &= \frac{-\pi}{6} + \frac{\pi}{4} + 2n\pi \\ 2t &= \frac{\pi}{12} + 2n\pi \\ t &= \frac{pi}{24} + n\pi \end{aligned}$$

42.

$$\begin{aligned} (2\ sin(t) - 1)(sin(t) + 4) &= 0 \\ sin(t) + 4 &= 0 \\ sin(t) &= -4 \text{ No solution} \\ 2\ sin(t) - 1 &= 0 \\ sin(t) &= \frac{1}{2} \\ t &= sin^{-1}(1/2) \\ t &= \frac{\pi}{6} + 2n\pi \\ t &= \frac{5\pi}{6} + 2n\pi \end{aligned}$$

43. $y = 2\ sin(t/3) - 4$
amplitude $= 2$, period $= \frac{2\pi}{(1/3)} = 6\pi$
mid-line $y = -4$, max $= -4 + 2 = 2$, min $= -4 - 2 = -6$

44. $y = \frac{1}{2}cos(2\pi t) + 3$
amplitude $= \frac{1}{2}$, period $= \frac{2\pi}{2\pi} = 1$
mid-line $y = 3$, max $= 3 + \frac{1}{2} = \frac{7}{2}$, min $= 3 - \frac{1}{2} = \frac{5}{2}$

45. Amplitude $= \frac{5-1}{2} = 2$, period $= \pi$, mid-line value $= 3$
$y = 2sin(2t) + 3$

46. Amplitude $= \frac{1-(-5)}{2} = 3$, period $= 2$, mid-line value $= -2$
$y = 3cos(\pi t) - 2$

47. **a)** Amplitude $= \frac{135-1}{2} = 67$, period $= 1/2$ means that $b = \frac{2\pi}{(1/2)} = 4\pi$, mid-line value $= 1 + 67 = 68$
Since the heel begins on the top of the eye we will use a cosine model
$h(t) = 67\ cos(4\pi t) + 68$

b) $h(10) = 67\ cos(40\pi) + 68 = 101.5\ m$

48. $(64^{5/3})^{-1/2} = 64^{-5/6} = \frac{1}{(\sqrt[6]{64})^5} = \frac{1}{32}$

49. $x = 0$, $x = -2$, and $x = 2$

50. $x = \pm\sqrt{10}$ and $x = \pm 2\sqrt{2}$

51. $(-1.8981, 0.7541)$, $(-0.2737, 1.0743)$, and $(2.0793, 0.6723)$

52. **a)** $G(x) = 0.6255x + 75.4766$

b) $G(18) = 0.6255(18) + 75.4766 = 86.7356$
$G(25) = 0.6255(25) + 75.4766 = 91.1141$

c) In Exercise 33, $G(18) = 85.6$ and $G(25) = 94.6$. The results obtained with the regression line are close to those obtained in Exercise 33.

53. **a)** $w(h) = 0.003968x^2 + 3.269048x - 76.428571$

b)

$$\begin{aligned} w(67) &= 0.003968(67)^2 + 3.269048(67) - 76.428571 \\ &= 160.415\ \ lbs \end{aligned}$$

Chapter 1 Test

1. **a)** Approximately 1150 minutes per month

b) About 62 years old

2. $f(x) = x^2 + 2$

a) $f(-3) = (-3)^2 + 2 = 11$

b) $f(x+h) = (x+h)^2 + 2 = x^2 + 2xh + h^2 + 2$

3. $f(x) = 2x^2 + 3$

a) $f(-2) = 2(-2)^2 + 3 = 8 + 3 = 11$

b) $f(x+h) = 2(x+h)^2 + 3 = 2(x^2 + 2xh + h^2) + 3 = 2x^2 + 4xh + 2h^2 + 3$

4. Slope $= -3$, y-intercept $(0, 2)$

5.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-5) &= \frac{1}{4}(x - 8) \\ y &= \frac{1}{4}x - 2 - 5 \\ y &= \frac{1}{4}x - 7 \end{aligned}$$

6. $m = \frac{10-(-5)}{-3-2} = 3$

7. Use the points $(0, 30)$ and $(3, 9)$
Average rate of change $= \frac{9-30}{3-0} = \frac{-21}{3} = -7$
The computer loses \$700 of its value each year.

8. Rate of change $= \frac{3-0}{6-0} = \frac{1}{2}$

9. Variation equation $F = kW$. Use $F = 120$ when $W = 180$ to find k

$$\begin{aligned} 120 &= k(180) \\ \frac{120}{180} &= k \\ \frac{2}{3} &= k \end{aligned}$$

The equation of variation is $F = \frac{2}{3}W$

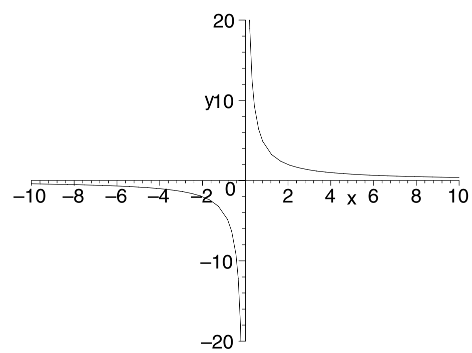
10. **a)** $f(1) = -4$

b) $x = -3$ and $x = 3$

11.

$$\begin{aligned} x^2 + 4x - 2 &= 0 \\ x &= \frac{-4 \pm \sqrt{16+8}}{2} \\ &= \frac{-4 \pm \sqrt{24}}{2} \\ &= \frac{-4 \pm 2\sqrt{6}}{2} \\ &= -2 \pm \sqrt{6} \end{aligned}$$

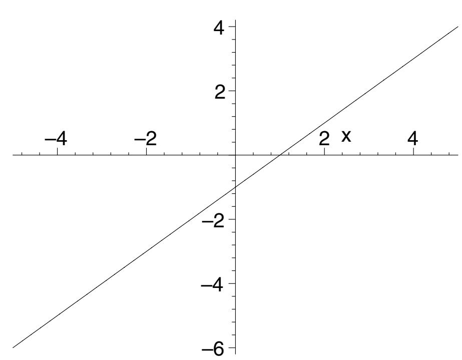
12.



13. $1/\sqrt{t} = 1/t^{1/2} = t^{-1/2}$

14. $t^{-3/5} = 1/t^{3/5} = 1/\sqrt[5]{t^3}$

15. $f(x) = \frac{x^2-1}{x+1}$. It is important to note that $x = -1$ is not in the domain of the plotted function



16. $sin(11\pi/6) = -\frac{1}{2}$

17. $cos(-3\pi/4) = \frac{sqrt2}{2}$

18. $tan(\pi) = 0$

19.

$$\begin{aligned} tan(40^o) &= \frac{3.28}{x} \\ x &= \frac{3.28}{tan(40^o)} \\ &= 3.909 \end{aligned}$$

20.

$$\begin{aligned} tan(t) &= \pm\sqrt{3} \\ t &= tan^{-1}(\sqrt{3}) \\ t &= \frac{\pi}{3} + 2n\pi \\ \text{and} & \\ t &= tan^{-1}(-\sqrt{3}) \\ t &= -\frac{\pi}{3} + 2n\pi \end{aligned}$$

21.

$$\begin{aligned} cos^2(t) &= 2 \\ cos(t) &= \pm\sqrt{2} \\ cos(t) &= 1.414 \end{aligned}$$

No solution, $cos(t)$ cannot have values larger than 1.

22.

$$\begin{aligned} 2sin^3(2t) - 3sin^2(2t) - 2sin(2t) &= 0 \\ sin(2t)(2sin(2t) - 1)(sin(2t) + 2) &= 0 \\ t &= \frac{n\pi}{2} \\ Or & \\ t &= \frac{-\pi}{12} + n\pi \\ Or & \\ t &= \frac{7\pi}{12} + n\pi \end{aligned}$$

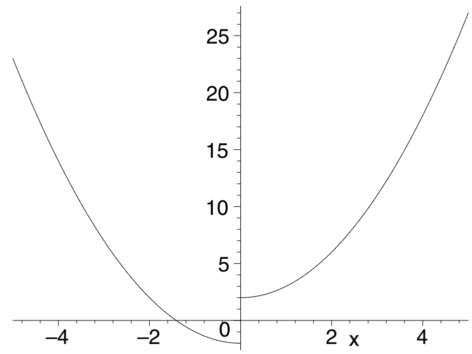
23. Amplitude = 4, period = $\frac{2\pi}{2} = \pi$, mid-line $y = 4$
max $= 4 + 4 = 8$, min $= 4 - 4 = 0$

24. Amplitude = 6, period = $\frac{2\pi}{(1/3)} = 6\pi$, mid-line $y = -10$
max $= -10 + 6 = -4$, min $= -10 - 6 = -16$

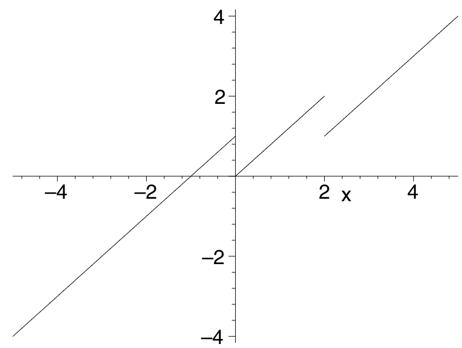
25. Amplitude = $\frac{-0.5-(-1.5)}{2} = \frac{1}{2}$, period = $\frac{2\pi}{3}$,
$b = \frac{2\pi}{(2\pi/3)} = 3$, mid-line value is -1
Thus, equation of the line is $y = \frac{1}{2}cos(3t) - 1$

26. Amplitude = $\frac{4-1}{2} = \frac{3}{2}$, period = 1,
$b = \frac{2\pi}{1} = 2\pi$, mid-line value is 2.5
Thus, equation of the line is $y = \frac{3}{2}sin(2\pi t) + \frac{5}{2}$

27.



28.



29. **a)** Find the slope $m = \frac{176-170}{80-50} = \frac{1}{5}$
Use slope-point equation

$$\begin{aligned} M - M_1 &= m(r - r_1) \\ M - 170 &= \frac{1}{5}(r - 50) \\ M &= \frac{1}{5}r - 10 + 170 \\ M &= \frac{1}{5}r + 160 \end{aligned}$$

b) $M(62) = \frac{1}{5}(62) + 160 = 172.4$
$M(75) = \frac{1}{5}(75) + 160 = 175$

30.

$$\begin{aligned} 3x + \frac{8}{x} - 1 &= 0 \\ 3x^2 - x + 8 &= 0 \\ x &= \frac{1 \pm \sqrt{1 - 96}}{6} \\ &= \frac{1 \pm i\sqrt{95}}{6} \\ &= \frac{1}{6} \pm \frac{i\sqrt{95}}{6} \end{aligned}$$

31. $x = --1.2543$

32. There are no real zeros for this function.

33. $(-1.21034, 2.36346)$

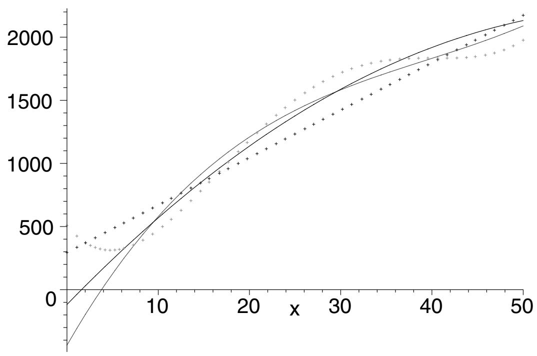
34. **a)** $M(r) = 0.2r + 160$

b) $M(62) = 172.4$
$M(75) = 174$

c) The results from the regression model are exactly the same as the result obtained in Exercise 29.

35. **a)** Linear Model: $y = 37.57614x + 294.47744$
Quadratic Model: $y = -0.59246x^2 + 74.60681x - 117.72472$
Cubic Model: $y = 0.02203x^3 - 2.60421x^2 + 125.71434x - 439.64751$
Quartic Model: $y = 0.00284x^4 - 0.32399x^3 + 11.45714x^2 - 88.51211x + 507.83874$

b)



c) By consider the graph in part b) and the scatter plot of the data points, it seems like quartic model best fits the data. The reason for this conclusion is because the scatter plot and the quartic model have the least amount of deviation (sometimes called residue) between them compared to the other models.

d) Left to the student (answers vary).

Technology Connection

- **Page 5:**
Left to the student

- **Page 7:**
  1. The line will look like a vertical line.
  2. The line will look like a horizontal line.
  3. The line will look like a vertical line.
  4. The line will look like a horizontal line.

- **Page 10:**
  1. Graphs are parallel
  2. The function values differ by the constant value added.
  3. Graphs are parallel

- **Page 19:**
  1. $f(-5) = 6$, $f(-4.7) = 3.99$, $f(11) = 150$, $f(2/3) = -1.556$
  2. $f(-5) = -21.3$, $f(-4.7) = -12.3$, $f(11) = -117.3$, $f(2/3) = 3.2556$
  3. $f(-5) = -75$, $f(-4.7) = -45.6$, $f(11) = 420.6$, $f(2/3) = 1.6889$

- **Page 21:**

1.

-1	0	1	2	3	4	5	6	7
11	4	-1	-4	-5	-4	-1	4	11

2.

-3	-2	-1	0	1	2	3	4	5
-29	-15	-5	1	3	1	-5	-15	-29

- **Page 23:**
  1. $x = 4.4149$
  2. $x = -0.618034$ and $x = 1.618034$

- **Page 27:**
  1. $y = -0.37393x + 1.02464$
  2. $y = 0.46786x^2 - 3.36786x + 5.26429$
  3. $y = 0.975x^3 - 6.031x^2 + 8.625x - 3.055$

- **Page 28:**
  1. $x = 2$ and $x = -5$
  2. $x = -4$ and $x = 6$
  3. $x = -2$ and $x = 1$
  4. $x = -1.414214$, $x = 0$, and $x == 1.414214$
  5. $x = 0$ and $x = 700$
  6. $x = -2.079356$, $x = 0.46295543$, and $x = 3.1164004$
  7. $x = -3.095574$, $x = -0.6460838$, $x = 0.6460838$, and $x = 3.095574$
  8. $x = -1$ and $x = 1$
  9. $x = -2$, $x = 1.414214$, $x = 1$, and $x = 1.414214$
  10. $x = -3$, $x = -1$, $x = 2$, and $x = 3$
  11. $x = -0.3874259$ and $x = 1.7207592$
  12. $x = 6.1329332$

- **Page 37:**
  1. $[0, \infty)$
  2. $[-2, \infty)$
  3. $(-\infty, \infty)$
  4. $(-\infty, \infty)$
  5. $[1, \infty)$
  6. $(-\infty, \infty)$
  7. $[-3, \infty)$
  8. $(-\infty, \infty)$
  9. $(-\infty, \infty)$
  10. $(-\infty, \infty)$
  11. Not correct
  12. Correct

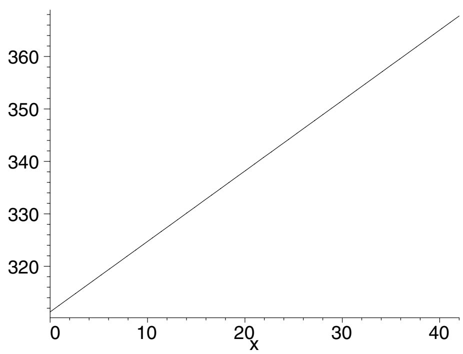
- **Page 46:**
  1. $t = 6.89210^o$
  2. $t = 46.88639^o$
  3. No solution
  4. $t = 1.01599$
  5. $t = 0.66874$
  6. 0.46677

- **Page 56:**
  Number 2 equation: shifts the $cos(\pi x)$ graph up by 1 unit
  Number 3 equation: shifts the $cos(\pi x)$ graph up by 1 unit and shrinks the period by a factor of 2
  Number 4 equation: shifts the $cos(\pi x)$ graph up by 1 unit, shrinks the period by a factor of 2, and increases the amplitude by a factor of 3
  Number 5 equation: shifts the $cos(\pi x)$ graph up by 1 unit, shrinks the period by a factor of 2, increases the amplitude by a factor of 3, and shifts the graph to the right by 0.5 units

Extended Life Science Connection

1. **a)** $y = 1.343450619x + 311.3019556$

b)



c) January 1990 corresponds to $t = 31$
$y = 1.343450619(31) + 311.3019556 = 352.95$
January 2000 corresponds to $t = 41$
$y = 1.343450619(41) + 311.3019556 = 366.38$
The estimates seem to be reasonable when compared to the data.

d) January 2010 corresponds to $t = 51$
$y = 1.343450619(51) + 311.3019556 = 379.82$
January 2050 corresponds to $t = 91$
$y = 1.343450619(91) + 311.3019556 = 433.56$
The estimates seem to be reasonable when compared to the data.

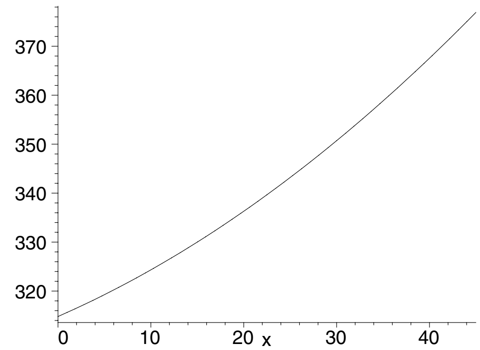
e) Find x when $y = 500$

$$\begin{aligned} y &= 1.34345x + 311.30196 \\ 500 &= 1.34345x + 311.30196 \\ 500 - 311.30196 &= 1.34345x \\ \frac{500 - 311.30196}{1.34345} &= x \\ 140.5 &\approx x \end{aligned}$$

The carbon dioxide concentration will reach 500 parts per million sometime in the year 2099.

2. **a)** $y = 0.0122244281x^2 + 0.8300246407x + 314.8103665$

b)



c) January 1990 corresponds to $t = 31$
$y = 0.0122(31)^2 + 0.8300(31) + 314.8104 = 352.29$
January 2000 corresponds to $t = 41$
$y = 0.0122(41)^2 + 0.8300(41) + 314.8104 = 369.39$
The estimates seem to be reasonable when compared to the data.

d) January 2010 corresponds to $t = 51$
$y = 0.0122(51)^2 + 0.8300(51) + 314.8104 = 388.94$
January 2050 corresponds to $t = 91$
$y = 0.0122(91)^2 + 0.8300(91) + 314.8104 = 491.57$

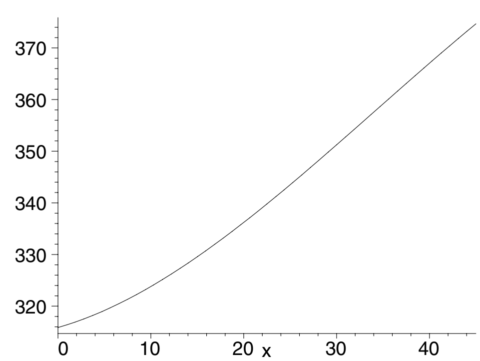
e) Find x when $y = 500$

$$\begin{aligned} 500 &= 0.0122x^2 + 0.8300x + \\ & \quad 314.8104 \\ 0 &= 0.0122x^2 + 0.8300x - \\ & \quad 185.1896 \\ x &= \frac{0.8300}{2(0.0122)} + \\ & \quad \frac{\sqrt{(0.8300)^2 - 4(0.0122)(-185.1896)}}{2(0.0122)} \\ x &\approx 161.63 \end{aligned}$$

The carbon dioxide concentration will reach 500 parts per million sometime in the year 2120.

3. **a)** $y = -0.000307x^3 + 0.031536x^2 + 0.509387x + 315.8660781$

b)



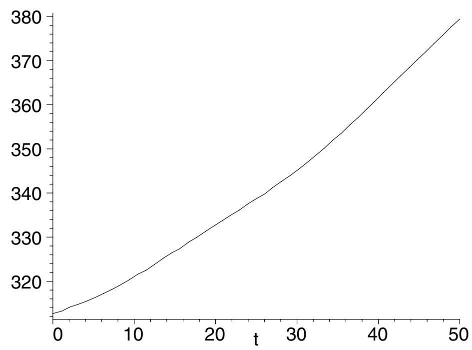
c) January 1990 corresponds to $t = 31$
$y = -0.000307x^3 + 0.031536x^2 + 0.509387x + 315.8660781 = 352.82$
January 2000 corresponds to $t = 41$
$y = -0.000307x^3 + 0.031536x^2 + 0.509387x + 315.8660781 = 368.60$

The estimates seem to be reasonable when compared to the data

d) January 2010 corresponds to $t = 51$
$y = -0.000307x^3 + 0.031536x^2 + 0.509387x + 315.8660781 = 383.15$
January 2050 corresponds to $t = 91$
$y = -0.000307x^3 + 0.031536x^2 + 0.509387x + 315.8660781 = 392.02$

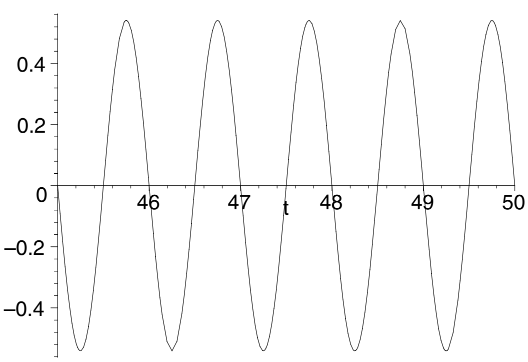
e) Find x when $y = 500$. The maximum of the cubic function does not intersect the line $y = 500$ therefore under this model the carbon dioxide concentration will never reach 500 parts per million.

4. **a)**



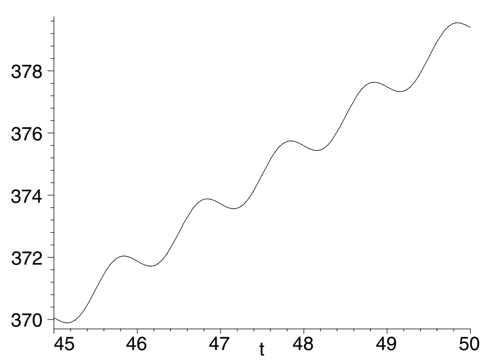
b) The graph represents a steady increase in the concentration of carbon dioxide.

c)



d) The graph shows an oscillating behavior for the concentration of carbon dioxide.

e)



f) The graph behavior shows that there is a periodic fluctuation in the concentration of carbon dioxide.

5.

- **LINEAR MODEL:** This model is the easiest mathematically to compute and explain. It does resemble the scatter plot of the original data sets. Under this model, the concentration levels of carbon dioxide will increase with time indefinitely.
- **QUADRATIC MODEL:** This model also resembles the original data set's scatter plot. At relatively small values of t it allows for a longer time for the increase in the concentration of carbon dioxide since it is a parabola. As time increases though the level at which the concentration of carbon dioxide will increase will be quicker than the linear model.
- **CUBIC MODEL:** This modelalso resembled the original data set's scattor plot indicates that there is a level after which the concentration of carbon dioxide will not increase. It is the only model that did not allow the concentration level of carbon dioxide to reach 500 parts per million. This model suggests that as time increased the concentration of carbon dioxide will begin to decrease indefinitely.
- **PERIODIC MODEL:** This model, as the other, modeled the data set to a very good degree of accuracy. It was the only model that allowed for oscillating behavior in the future, which is more likely to happen than what the other models suggested.