Problem 1.1
(a)
Derive an expression for the dimensionless cylinder volume $V(\theta)/V(0) = f(\theta, r, \varepsilon = s/2l)$

Solution:
From Figure 1.1, and using trig relations, we have

The instantaneous stroke is $y = l + a - [(l^2 - a^2 \sin^2\theta)^{1/2} + a \cos\theta]$
The instantaneous volume is $V(\theta) = V(0) + \pi/4\, b^2 y$
The displacement volume is $V_d = \pi/4\, b^2 s$
The stroke is $s = 2a$
The compression ratio is $r = (V(0) + V_d)/V(0) = 1 + V_d/V(0) \Rightarrow V_d = (r-1)\, V(0)$

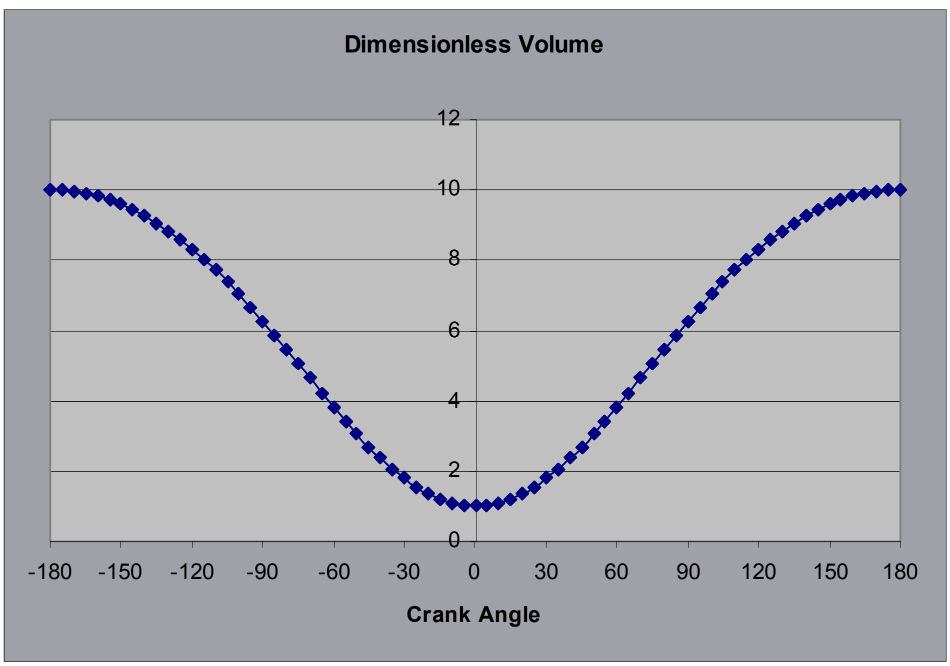
Therefore, $\pi/4\, b^2 = V_d/s = (r-1)\, V(0)/s$
and

$$V(\theta) = V(0) + (r-1)\, V(0) y/s$$

so

$$V(\theta)/V(0) = 1 + (r-1)\,(l + a - [(l^2 - a^2 \sin^2\theta)^{1/2} + a \cos\theta])$$
$$= 1 + (r-1)\, l\, [1 + a/l - (1 - (a/l)^2 \sin^2\theta)^{1/2} - a/l \cos\theta])/s$$
$$= 1 + (r-1)\, l\, [1 + \varepsilon - (1 - \varepsilon^2 \sin^2\theta)^{1/2} - \varepsilon \cos\theta])/2\varepsilon$$

(b) The plot of dimensionless cylinder volume for $r = 10$, $s = 100$ mm and $l = 150$ mm is shown below:



Problem 1.2
(a)
Derive an expression for the dimensionless piston speed.

Solution:
The instantaneous piston speed is $U_p = dy/dt = dy/d\theta * d\theta /dt$

The instantaneous stroke is $y = l + a - [(l^2 - a^2 \sin^2\theta)^{1/2} + a \cos\theta]$

differentiating y with respect to crank angle θ, we have

$$dy/d\theta = - [1/2 (l^2 - a^2 \sin^2\theta)-1/2 (-2a^2 \sin\theta \cos\theta) - a \sin\theta]$$
$$= + a \sin\theta [1 + \cos\theta /[(l/a)^2 - \sin^2\theta]^{1/2}]$$

The angular velocity is $d\theta /dt$ (rad/time) = N (rev/time) 2π (rad/rev)

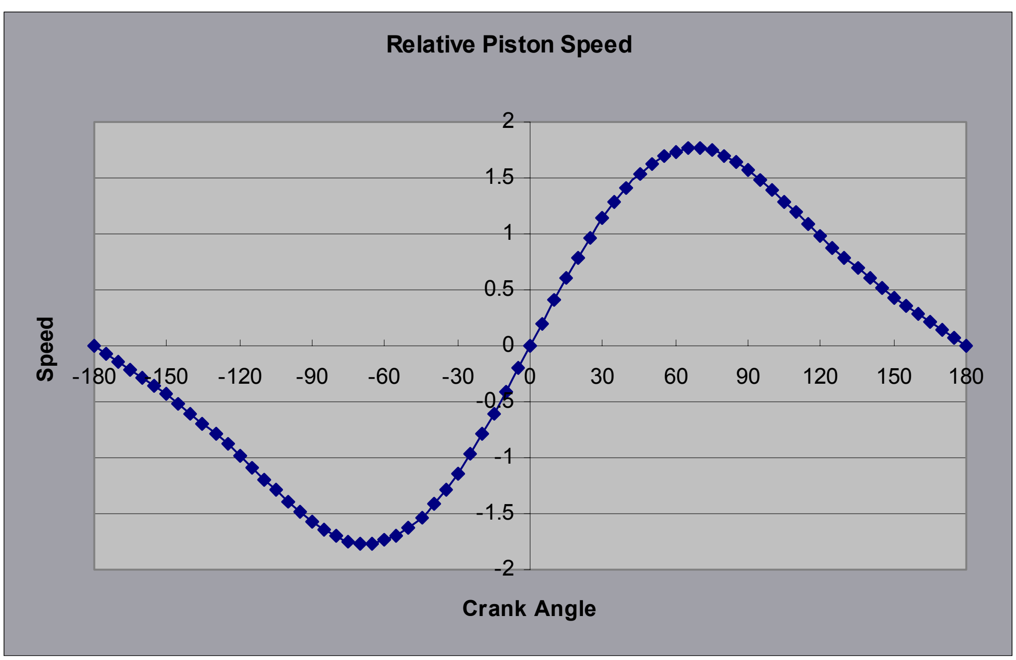
The mean piston speed is $\underline{U}_p = 2Ns = 4Na$

The ratio of the instantaneous piston speed to the mean piston speed is therefore:

$$U_p(\theta)/\underline{U}_p = 2\pi N a \sin\theta [1 + \cos\theta /[(l/a)^2 - \sin^2\theta]^{1/2}]/ 4Na$$

$$= \pi/2 \sin\theta [1 + \cos\theta /[(l/a)^2 - \sin^2\theta]^{1/2}]$$

(b) The plot of dimensionless piston speed for s = 100 mm and l = 150 mm is shown below:



Problem 1-3
Find the ratio of the initial to final swirl speed as f (r, geometry) for a cylinder with a dished piston. The cylinder bore is b, and the diameter of the piston dish is d. The angular velocity distribution is equivalent to solid body rotation.

Solution:
At bottom dead center (1), the moment of inertia is $I_1 = \pi \rho_1 (b^4 s + d^4 h)/32$
At top dead center (o), the moment of inertia is $I_o = \pi \rho_o d^4 h/32$

If angular momentum is conserved, then $I_1 \omega_1 = I_o \omega_o$
so the ratio of the initial to final swirl speeds is $\omega_o / \omega_1 = I_1/I_o = \rho_1/\rho_o [(b^4 s + d^4 h)/ d^4 h]$

We now need to express the density ratio as a function of cylinder geometry.
The gas density increases with cylinder compression. Since the amount of gas in the cylinder is constant,

$$\rho_1/\rho_o = V_o/V_1 = 1/r$$

so

$$\omega_o / \omega_1 = 1/r(b^4 s + d^4 h)/ d^4 h]$$
$$= 1/r [(b/d)^4 s/h + 1]$$

we can simplify this by expressing the stroke in terms of the other variables

Since $r = V_1/V_o = (b^2 s + d^2 h)/ d^2 h$

then $s/h = (r-1)d^2/b^2$

and in final form, the swirl speed ratio is

$$\omega_o / \omega_1 = (b/d)^2 (r-1)/r + 1/r$$

For r >> 1, the swirl increases quadratically with diameter ratio.

Problem 1-4
Determine how the power/weight = f(number of cylinders keeping V_d constant) assuming mep, mean piston speed, mass/ displacement volume, and Power/area are not a function of V_d, and equal bore and stroke.

Solution:
The power/weight can be expanded in terms of the power/area and mass/ displacement volume as

$$\frac{\dot{W}}{m} = \frac{\frac{\dot{W}}{A}\frac{\pi}{4}b^2 n_c}{\frac{m}{V_d}\frac{\pi}{4}b^2 s\, n_c}$$

If $b = s$, then $V_d = \pi/4\ b^3 n_c$, and

$$b = \left(\frac{4\,V_d}{\pi\ nc}\right)^{1/3}$$

and

$$\frac{\dot{W}}{m} = \frac{\frac{\dot{W}}{A}}{\frac{\dot{m}}{V_d}}\left(\frac{\pi\ n_c}{4\,V_d}\right)^{1/3} \approx n_c^{\ 1/3}$$

The power per unit weight scales as the number of cylinders to the 1/3 power.

Problem 1-5
Compute the mean piston speed, bmep, torque, and power/area for three engines, 1: Marine, 2: Dragster, and 3: Formula One.

Solution:

a) Mean Piston Speed

$$\overline{U}_p = 2\ N\ s$$

$$1:\overline{U}_p = 2 \cdot 2600\ \frac{rev}{\min} \cdot \frac{\min}{60s} \cdot 0.127 = 11.0\ m/s$$

$$2:\overline{U}_p = 2 \cdot 6400\ \frac{rev}{\min} \cdot \frac{\min}{60s} \cdot 0.095 = 20.3\ m/s$$

$$3:\overline{U}_p = 2 \cdot 10500\ \frac{rev}{\min} \cdot \frac{\min}{60s} \cdot 0.057 = 19.95\ m/s$$

b) bmep (assuming all are four stroke engines)

$$bmep = \frac{2\ \dot{W}_b}{V_d\ N}$$

$$1: bmep = \frac{2 \cdot 1118}{12 \cdot \frac{\pi}{4} \cdot 0.136^2 \cdot 0.127 \cdot \frac{2600}{60}} = 2.33\ x10^3\ kPa = 23.3\ bar$$

$$2: bmep = \frac{2 \cdot 447}{8 \cdot \frac{\pi}{4} \cdot 0.108^2 \cdot 0.095 \cdot \frac{6400}{60}} = 1.20\ x10^3\ kPa = 12.0\ bar$$

$$3: bmep = \frac{2 \cdot 522}{8 \cdot \frac{\pi}{4} \cdot 0.086^2 \cdot 0.057 \cdot \frac{10500}{60}} = 2.25\ x10^3\ kPa = 22.5\ bar$$

Problem 1-5 (cont)
c) torque

$$\tau = \frac{\dot{W}}{2\pi N}$$

$$1: \tau = \frac{(1118\,x10^3)}{2\pi \cdot \frac{2600}{60}} \frac{W}{\frac{rev}{s}} = 4106\,Nm$$

$$2: \tau = \frac{(447\,x10^3)}{2\pi \cdot \frac{6400}{60}} = 667\,Nm$$

$$3: \tau = \frac{(522\,x10^3)}{2\pi \cdot \frac{10500}{60}} = 475\,Nm$$

d) power/piston area

$$\frac{\dot{W}}{A_p} = \frac{\dot{W}}{\frac{\pi}{4} b^2 n_c}$$

$$1: \frac{\dot{W}}{A_p} = \frac{1118}{12 \cdot \frac{\pi}{4} \cdot 0.136^2} = 6413\,kW/m^2$$

$$2: \frac{\dot{W}}{A_p} = \frac{447}{8 \cdot \frac{\pi}{4} \cdot 0.108^2} = 6099\,kW/m^2$$

$$3: \frac{\dot{W}}{A_p} = \frac{522}{8 \cdot \frac{\pi}{4} \cdot 0.086^2} = 11{,}233\,kW/m^2$$

Problem 1-6
Compute the air flow rate, given e_v, ρ_i, V_d, N, for the marine engine given in Problem 1-5.

Solution:
Assuming that the engine is a four stroke fuel injected engine, then

$$\begin{aligned}\dot{m}_a &= \frac{1}{2} e_v \rho_i V_d\ N \\ &= \frac{1}{2} \cdot 0.8 \cdot 1.5 \cdot 1.17 \cdot 0.0221 \cdot \frac{2600}{60} \\ &= 0.67\ kg/s\end{aligned}$$

Problem 1-7

Find the bmep, efficiency, and air/fuel ratio of an engine, given V_d, power, N, e_v, bsfc, and q_c.

Solution:

At 298 K and 1 bar, assume the gas density is 1.17 kg/m^3

$$bmep = \frac{2\dot{W}}{V_d N} = \frac{2 \cdot 88}{3.8x10^{-3} \cdot \frac{4000}{60}} = 694\,kPa$$

$$\eta = \frac{1}{bsfc \cdot q_c} = \frac{1}{0.35 \cdot \frac{42000}{3600}} = 0.24$$

$$\dot{m}_f = \dot{W} \cdot bsfc = 88 \cdot \frac{0.35}{3600} = 8.55x10^{-3}\ kg/s$$

$$\dot{m}_a = \frac{1}{2} e_v \rho V_d\ N = \frac{1}{2} 0.85 \cdot 1.17 \cdot 3.8x10^{-3} \cdot \frac{4000}{60} = 0.125\,kg/s$$

$$A = \frac{\dot{m}_a}{\dot{m}_f} = \frac{0.125}{8.55x10^{-3}} = 14.7$$

Problem 1-8
Find the bmep and mean piston speed of an engine, given n_c, τ, N, b, s

Solution:

$$bmep = \frac{2\pi\tau}{V_d} = \frac{2\pi \cdot 1100}{6 \cdot \frac{\pi}{4} \cdot 0.123^2 \cdot 0.127} = 7.63 x 10^5 \ N/m^2 = 7.63 \, bar$$

$$\overline{U}_p = 2\,N\,s = 2 \cdot 2100 \cdot 0.127 \cdot \frac{1}{60} = 8.89 \, m/s$$

Problem 1-9
Compute the fuel/air ratio and the air flow rate for an engine given V_d, N, b, s, bmep, bsfc, and D_r.

Solution:

Assume $\rho_{amb} = 1.17$ kg/m^3

$$F = \frac{\dot{m}_f}{\dot{m}_a}$$

$$bmep = \frac{\dot{W}_b}{V_d\ N}$$

$$bsfc = \frac{\dot{m}_f}{\dot{W}_b}$$

$$\therefore \quad \dot{m}_f = bmep \cdot bsfc \cdot V_d \cdot N$$

$$= 6.81x10^5 \cdot \frac{0.49}{3600} \cdot \frac{\pi}{4} \cdot 0.082^2 \cdot 0.072 \cdot \frac{5500}{60} = 3.23\, g/s$$

$$\dot{m}_a = D_r \cdot \rho_{amb} \cdot V_d\ N$$

$$= 0.748 \cdot 1.17 \cdot 3.8x10^{-4} \cdot \frac{5500}{60} = 3.048x10^{-2}\ kg/s = 30.48\ g/s$$

$$F = \frac{3.23}{30.48} = 0.106$$