CHAPTER THIRTEEN

HYPOTHESIS TESTING:
BASIC CONCEPTS AND TESTS OF ASSOCIATIONS

Outline of the Chapter

Understand the logic behind hypothesis testing.
Become familiar with the concepts basic to the hypothesis-testing procedure.
Describe the steps involved in testing of hypothesis.
Interpret the significance level of a test.
Understand the difference between Type I and Type II errors.
Describe the chi-square test of independence and the chi-square goodness-of-fit test
Discuss the purpose of measuring strength of association
Be exposed to the more commonly used hypothesis tests in marketing research - tests of means and proportions.
Understand the relationship between confidence interval and hypothesis testing.
Describe the effect of sample size on hypothesis testing.
Discuss the use of the analysis of variance technique.
Describe one-way and *n*-way analysis of variance.

Teaching Suggestions

The emphasis in this chapter (as in the whole data analysis section of the book) is not upon calculation. The computer or a statistical consultant can do the calculating. The emphasis is, rather, upon asking the right questions and making proper interpretation of the results. Thus, the chapter seeks to get the student to:

1. Ask the hypothesis test question. Maybe these empirical findings simply represent sampling variation. What is the probability that such (or even more impressive) results would have emerged if the null hypothesis was true? A low p-value means the results are impressive and that their implications are worth considering. A high p-value means that the results should be disregarded or discounted.

2. How to interpret the significance level: a significance level of 0.10 simply means that the p-value was less than 0.10. The four steps in Figire 13-1summarize the logic.

There are two things that this book is not. It is not a reference book for the hundreds of statistical tests that could be used. We do not feel a marketing research book should provide that function or that a student should be burdened with sorting out all the available tests. Second, this book does not attempt to provide the ability to perform calculations. Rather, it emphasizes inputs, outputs, assumptions, and interpretations. The inclusion of the formulas behind chi-square tests in cross tabulations is the exception to the rule. The computer will perform the calculations. The task is again to ask the right questions and to interpret the results appropriately.

The cross-tabulation example is used as a vehicle to explain conceptually what independence (the null hypothesis) is. The experiment in Table 13-2 is the primary vehicle. The use of chi-square as an association measure is discussed but it is the appendix that provides a more detailed discussion of association measures for nominally scaled variables.

ANOVA is introduced in the context of a small numerical example with actual (though contrived) data. The use of "actual data" is intended to make the discussion more understandable and less abstract and will be the rule followed in the later chapters.

The chapter has a minimum of symbols and concepts but it still does include some concepts that are normally taught in a statistics course. Further, the basic idea is exposed in Chapter 12.

This chapter, like other technical chapters, should usually be supported by a lecture and discussion which follows the text fairly closely. The students should understand the various figures and tables. Make sure that they see the link between the difference between means discussion in Chapters 12 and this chapter including the ANOVA table. The interaction discussion is also worth reviewing.

Questions and Problems

1.

Part 1

				Raw Total
	E_1 = 17.8	E_4 = 46.8	E_7 = 24.5	22.3% (89)
	E_2 = 19.8	E_5 = 52.1	E_8 = 27.3	24.8% (99)
	E_3 = 42.4	E_6 = 111.3	E_9 = 58.3	53.0% (212)
Column Total	(80)	(210)	(110)	

Part 2

E_1 means that if the rows and columns were independent (a knowledge of one provides no information about the other-like flipping a coin or drawing a card), then a total of 17.8 people would be "expected" to be in cell 1. If the experiment were repeated many times, on the average 17.8 would be in cell 1.

Part 3

Cell	**$(O-E_i)^2$**	**$(O_i-E_i)^2/E_i$**
1	17.6	.99
2	38.4	1.94
3	73.9	1.78
4	33.6	.72
5	198.8	3.82
6	445.2	4.08
7	6.3	.25
8	59.3	2.17
9	84.6	1.48
		Total 17.3 = chi square

Part 4

With four degrees of freedom, (r- 1) (c- 1), the critical value given in the table at the end of the book is 18.5 at the 0.001 level and 14.9 at the 0.005 level.

Thus, the chi-square statistic is significant at the 0.005 significance level and we would reject the independence null hypothesis.

Part 5

False. It just shows that if usage differs by age, then the probability of getting a chi-square value this large or larger would be very small. Thus, the evidence points to the conclusion that usage differs by age.

2. H_o: Preferences and brands are not related.
   H_a Preferences and brands are related.

Purchaser	A	B	C	D	Total
Buys the brand	45(50)	50(50)	45(50)	60(50)	200
Doesn't buy the brand	55(50)	50(50)	55(50)	40(50)	200
Total	100	100	100	100	400

Expected value (represented in brackets) = $\frac{\text{Row total x Column total}}{\text{Grand total}}$

$$E_{II} = \frac{200 \times 100}{400}$$

$$E_{II} = 50$$

$$\chi^2_{cal} = \frac{(O - E)^2}{E} = \frac{(45 - 50)^2}{50} + \frac{(50 - 50)^2}{50} + ... + \frac{(40 - 50)^2}{50}$$

$$= 0.5 + 0 + ... + 2 = 6$$

χ^2 test statistic at (4-1)(2-1) = (3)(1) = 3 degrees of freedom

a. α = 0.05 = 7.815
χ^2_{cal}, χ^2_{table} therefore H_o cannot be rejected
Preferences and brands are not related.

3. H_o: The observed distribution attending the concert fits with the on campus distribution (statistically equivalent)
   H_a: The observed distribution attending the concert with the on campus distribution

Observed value		Expected value	
Juniors =	74 % (59)	Juniors =	62 % (50)
Seniors =	17 % (14)	Seniors =	23 % (18)
Freshmen & Sophomores =	9 % (7)	Freshmen & Sophomores =	15 % (12)

degrees of freedom = (3 - 1) = 2

χ^2_{tab} at α = 0.05 = 5.991

$$\chi^2_{cal} = \frac{(O - E)^2}{E} + \frac{(59 - 50)^2}{50} + \frac{(14 - 18)^2}{18} + \frac{(7\ 12)^2}{7}$$

$= 1.62 + 0.889 + 3.571$

$= 6.08$

$\chi^2_{cal} > \chi^2_{tab}$

Therefore, reject H_o and conclude that the observed distribution does not fit with the on campus distribution.

4. H_o: The observed application pool coincides with the historical pattern.
   H_a: The observed application pool does not coincide with the historical pattern.

   at α = 0-05.

Observed pattern		Expected pattern	
In-state =	75	In-state =	70
Neighboring states =	15	Neighboring States =	20
Other states =	10	Other states =	10

$\chi^2_{(df = (3-1) = 2)}$ at α = 0.05 = 5.991

$$\chi^2_{cal} = \frac{(75 - 70)^2}{70} + \frac{(15 - 20)^2}{20} + \frac{(10 - 10)^2}{10}$$

$= 0.357 + 1.25$

$= 1.607$

$\chi^2_{cal} < \chi^2_{tab}$

Therefore, do not reject H_o and conclude that the observed application pool coincides with the historical pattern.

5. α=0.1
   H_o: There is no association between a child's sex and the hours of play.
   H_a: There is an association between a child's sex and the hours of play.

	Less than 2.5	2.5 or more	Total
Boys	18 (16.9)	10 (11.1)	28
Girls	17 (18.1)	13 (11.9)	30
	35	23	58

Expected value is given in the brackets.

χ^2_{tab} at $\alpha = 0.01$; df = 1 = 6.635

$$\chi^2_{cal} = \frac{(18 - 16.9)^2}{16.9} + ... + \frac{(13 - 11\ 9)^2}{11.9}$$

$= 0.0715 + 0.109 + 0.067 + 0.1016$

$= 0.349$

$\chi^2_{cal} < \chi^2_{tab}$

Do not reject H_o, and conclude lack of association between child's sex and hours of play.

6. (a)The null hypothesis would be that the population proportions would be the same -that trial would be the same in each city.

   (b) That trial would be higher in Tulsa than Fresno.

   (c) It is significant at 0.06 level. The p-level (0.06) would be significant at the .10 level but not the .05 level. The null hypothesis would be rejected at the .10 level but not the 0.05 level.

   (d) The hypothesis test only provides the p-level, a measure of the strength of the evidence against the null hypothesis—it does not show it true or false. To determine whether to use a $.50 coupon we would need much more information, such as costs of various types.

   (e) The differences between the audiences can be accounted for by using matched groups of people in both the cities or by using a random group of people in both cities

7. A random sample of 100 automobiles.

   H_o: $\mu \geq 5$ miles/gallon
   H_a: $\mu < 5$ miles/gallon

   <u>n</u> = 100
   x = 4.4
   S = 1.8
   $\alpha = 0.05$

   <u>One tailed test</u> (left tailed)

Therefore, $Z = \frac{\bar{x} - \mu}{S_{x}}$

$= \frac{4.4 - 5}{S_{x}}$

Standard error of the mean six $S_{x} = \frac{S}{\sqrt{n}} = \frac{1.8}{\sqrt{100}} = \underline{0.18}$

$Z_{calc} = \frac{4.4 - 5}{0.18}$

$= -3.33$

Z_{tab} at $\alpha = 0.05 = -1.45$

$Z_{calc} > Z_{tab}$

Therefore, reject H_o and conclude that the population mean is less than 5 miles/gallon.

P value < 0.001

A small p-value is observed and hence greater the researcher's confidence in the sample findings.

The p-value is the largest significance level at which we fail to reject H_o.

8.

H_o: $\mu \geq 40$
H_a: $\mu < 40$ cases
$\alpha =$ 0.05
s = 12.2

$$S_{\bar{x}} = \frac{12.2}{\sqrt{n}} = \frac{12.2}{\sqrt{25}} = 2.44$$

Z_{value} from the table = -1.645 (left tailed)

$Z_{cal} = \frac{31.3 - 40}{2.44} = -3.56$

$Z_{cal} > Z_{tab}$

Therefore, reject H_o and conclude that the population mean sales increase is less than 40 cases.

9. The data for this can be statistically described as:

$p_n = 0.75$

Hypothesized value of people with business experience prior to opening the business.

$q_n = 0.25$

Hypothesized value of people with no business experience prior to opening the business.

p = 0.70 (sample proportion of people with prior business experience)
q = 0.30 (sample proportion of people with no prior business experience)

H_o: $p \geq 0.75$
H_1: $p < 0.75$

Standard error of the proportion $\sigma_p = \frac{p_o q_o}{\sqrt{n}}$

$= 0.0228$

Therefore, one tailed test of proportion

$Z_{cal} = \frac{0.7-0.75}{0.0228} = -2.19$

$Z_{tab} = -1.645$

$Z_{cal} > Z_{tab}$

Therefore, reject H_o and conclude that more than 25% of members had no prior business experience.

10. H_o: $p = 0.5$
H_a: $p \neq 0.5$
$\alpha = 0.05$

Two tailed test of proportions

Therefore, $Z_{cal} = \frac{(172/400) - 0.5}{(0.5)(0.5)} = \frac{-0.07}{0.025}$

$= -2.8$

Z_{tab} at $\alpha = 0.05$ (two tailed) $= \pm 1.96$

Since $Z_{cal} > Z_{tab}$; reject H_o and conclude that half of all purchases are not women.

11. H_o: $p \geq 0.45$
    H_a: $p < 0.45$
    $\alpha =$ 0.05

    One tailed (left tailed) test of proportions

    $$Z_{cal} = \frac{(70/200) - 0.45}{\sqrt{(0.55*0.45)/200}} = -2.84$$

    $Z_{tab} = -1.645$

    Since $Z_{cal} > Z_{tab}$: reject H_o and conclude that members opting for international marketing research is lower.

12. $n_1 = 120$ $\quad$ $n_2 = 100$
    $\bar{x}_1 = 3.355$ $\quad$ $\bar{x}_2 = 9.5$
    $s_{\bar{x}_1} = 2.03$ $\quad$ $s_{\bar{x}_2} = 2.1$

    H_o: $\mu_1 - \mu_2 = 0$ (Population means are equal)

    H_a: $\mu_1 - \mu_2 \neq 0$ (Population means are not equal.)

    $$S_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

    $= \sqrt{0.0343} + \sqrt{0.0441}$

    $= 0.28$

    $$Z_{cal} = \frac{(3.355 - 9\ 5) - (\mu_1 - \mu_2)}{0.28}$$

    $$= \frac{(3.355 - 9.5) - 0}{0.28} = -21.9$$

    Z_{tab} at $\alpha = 0.05$; $Z_{\alpha/2} = \pm 1.96$

    Since $|Z_{cal}| > Z_{tab}$; reject H_o and conclude that population means are not equal.

13. $\alpha = 0.10$

$\bar{x} = 5.1$

$\sigma = 0.1$

$\mu = 5.0$

$n = 5$

$H_o: \mu = 5.0$

$H_a: \mu \neq 5.0$

Two tailed test at $\alpha = 0.1$

$Z_{tab} = \pm 1.645$

$Z_{cal} = \frac{5.1 - 5.0}{0.1/\sqrt{25}} = \frac{0.1}{0.02} = 5$

$|Z_{cal}| > Z_{tab}$

Therefore, reject Ho and conclude that the mean preference is not 5.0.

14. $n = 9$

$\mu = 2.0$

$\sigma = 0.06$

$\bar{x} = 1.95$

$H_o: \mu \geq 2$

$H_a: \mu < 2$

Left tailed test at $\alpha = 0.05$

$Z_{tab} = -1.645$

$Z_{cal} = \frac{1.95 - 2}{0.06/\sqrt{9}} = -\frac{0.05}{0.02} = -2.5$

$|Z_{cal}| > Z_{tab}$

Therefore, reject H_o and conclude that the mean is less than 2 units.

15. (a) Null hypothesis is that the "population" means are equal—there is, no difference between the three advertisements. The alternate hypothesis is that there is some difference—they are not all equally effective.

(b) The F-ratio is: $\frac{6.0}{2.0} = 3.0$

The p-value is about .055.

(c) The p-value is significant at the .10 level but not at the .05 level.

(d) There may be. The evidence against the null hypothesis is fairly strong but we don't know for sure.