

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.Determine the third Taylor polynomial of the function at $x = 0$.

1) $f(x) = \frac{1}{x+3}$

1) _____

A) $\frac{1}{3} - \frac{1}{9}x + \frac{1}{27}x^2 - \frac{1}{81}x^3$

B) $\frac{1}{3} + \frac{1}{9}x + \frac{1}{27}x^2 + \frac{1}{81}x^3$

C) $\frac{1}{3}x + \frac{1}{9}x^2 + \frac{1}{27}x^3 + \frac{1}{81}x^4$

D) $\frac{1}{3}x - \frac{1}{9}x^2 + \frac{1}{27}x^3 - \frac{1}{81}x^4$

2) $f(x) = \frac{1}{7-x}$

2) _____

A) $\frac{1}{7} - \frac{1}{49}x + \frac{1}{343}x^2 - \frac{1}{2401}x^3$

B) $\frac{1}{7}x - \frac{1}{49}x^2 + \frac{1}{343}x^3 - \frac{1}{2401}x^4$

C) $\frac{1}{7}x + \frac{1}{49}x^2 + \frac{1}{343}x^3 + \frac{1}{2401}x^4$

D) $\frac{1}{7} + \frac{1}{49}x + \frac{1}{343}x^2 + \frac{1}{2401}x^3$

3) $f(x) = e^{-4x}$

3) _____

A) $1 + 4x + 8x^2 + \frac{32}{3}x^3$

B) $1 - 4x + 8x^2 - \frac{32}{3}x^3$

C) $1 - 4x + 8x^2 - \frac{64}{3}x^3$

D) $1 - 16x + 128x^2 - \frac{1024}{3}x^3$

4) $f(x) = e^{5x}$

4) _____

A) $1 + 5x + \frac{25}{2}x^2 + \frac{125}{3}x^3$

B) $1 + 25x + \frac{625}{2}x^2 + \frac{15625}{12}x^3$

C) $1 + 5x + \frac{25}{2}x^2 + \frac{125}{18}x^3$

D) $1 + 5x + \frac{25}{2}x^2 + \frac{125}{6}x^3$

5) $f(x) = \sqrt{x+36}$

5) _____

A) $6 - \frac{1}{12}x + \frac{1}{1728}x^2 - \frac{1}{62,208}x^3$

B) $6 - \frac{1}{12}x + \frac{1}{1728}x^2 - \frac{1}{124,416}x^3$

C) $6 + \frac{1}{12}x - \frac{1}{1728}x^2 + \frac{1}{124,416}x^3$

D) $6 + \frac{1}{12}x - \frac{1}{1728}x^2 + \frac{1}{62,208}x^3$

Find the n th Taylor polynomial for the function at $x = 0$, and use it to estimate the value of the function at the given value of x . Round to seven decimal places.

6) $f(x) = \cos x$, $n = 3$, estimate $\cos(0.1)$

6) _____

A) 0.9950250

B) 0.9998334

C) 1.0050042

D) 0.9950042

- 7) $f(x) = e^x$, $n = 4$, estimate $e^{0.345}$ 7) _____
 A) 0.9410770 B) 0.9931967 C) 1.4113564 D) 0.7213564
- 8) $f(x) = \ln x$, $n = 4$, estimate $\ln(0.865)$ 8) _____
 A) -0.1450157 B) -0.1445364 C) -0.1266246 D) 0.8737024

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

- 9) Determine the second Taylor polynomial of $\sin x^2$ at $x = 0$. 9) _____
 Enter an unlabeled polynomial in x in standard form (i.e., highest powers first).
- 10) Determine the third Taylor polynomial of $f(x) = x^3 - 3x$ at $x = 0$. 10) _____
 Enter an unlabeled polynomial in x in standard form (i.e., highest powers first).
- 11) Determine the third Taylor polynomial of $f(x) = \frac{1}{\sqrt{1-x}}$ at $x = 0$. 11) _____
 Enter your answer as an unlabeled polynomial in x in standard form (i.e., highest powers first).

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

- 12) Let $f(x) = \ln(1+x)$. Determine the third Taylor polynomial of $f(x)$ at $x = 0$. 12) _____
 A) $x + \frac{1}{2}x^2 + \frac{1}{3}x^3$ B) $x + \frac{1}{2}x^2 + x^3$ C) $x + x^2 + x^3$ D) $x - \frac{1}{2}x^2 + \frac{1}{3}x^3$
- 13) Let $f(x) = \frac{1}{1-x}$. Determine the fourth Taylor polynomial at $x = 0$. 13) _____
 A) $1 + x + x^2 + x^3 + x^4$ B) $1 - x + 2x^2 - \frac{1}{2}x^3 + \frac{1}{6}x^4$
 C) $1 - x + x^2 - x^3 + x^4$ D) $1 + x + 2x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

- 14) Find the second Taylor polynomial for $f(x) = \frac{1}{\sqrt{x+4}}$ at $x = 0$ and use it to approximate 14) _____
 $\frac{1}{\sqrt{4.1}}$.
 Enter just a real number rounded off to two decimal places.

- 15) Find the second Taylor polynomial of $f(x) = \sin x^2$ at $x = 0$ and use it to approximate the area under the curve $f(x)$ between 0 and $\frac{\pi}{2}$. 15) _____
 Enter an unlabeled polynomial in x in standard form followed by a comma and then just a quotient representing the area (π in the numerator).
- 16) Find the third Taylor polynomial of $f(x) = x^2 + \sin x$ at $x = 0$. 16) _____
 Enter an unlabeled polynomial in x in standard form (i.e., highest powers first).
- 17) Find the third Taylor polynomial of $f(x) = e^x$ at $x = 0$ and use it to approximate e . 17) _____
 Enter just a reduced fraction of form $\frac{a}{b}$.
- 18) Find the third Taylor polynomial of $f(x) = \sin x$ at $x = 0$ and use it to approximate $\sin \frac{1}{2}$. 18) _____
 Enter just a real number rounded off to two decimal places.
- 19) Write down the fourth Taylor polynomial of $f(x) = e^{-x^2}$ at $x = 0$. 19) _____
 Enter your answer as an unlabeled polynomial in x in standard Taylor polynomial form (i.e., constant first, highest power last).

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

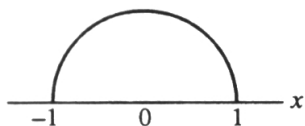
- 20) Suppose $f(x) = x^4 - 7x^3 + 2$. The fifth Taylor polynomial of $f(x)$ at $x = 0$ is $p_5(x) = x^4 - 7x^3 + 2$. 20) _____
 A) True B) False
- 21) Suppose $f(x) = x^4 - 7x^3 + 2$. The third Taylor polynomial of $f(x)$ at $x = 0$ is $p_3(x) = 2 - 7x^3$. 21) _____
 A) True B) False

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

- 22) The area of a circle with radius 1 is π . If $f(x) = \sqrt{1 - x^2}$ gives the top half of this circle, as illustrated below, use the second Taylor polynomial of $f(x)$ at $x = 0$ to find an approximate value for π . Is the following correct? 22) _____

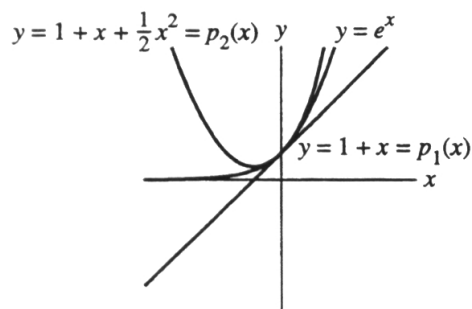
$$p_2(x) = 1 - \frac{1}{2}x^2;$$

$$\frac{\pi}{2} \approx \int_{-1}^1 \left(1 - \frac{1}{2}x^2\right) dx = \frac{5}{3} \text{ so } \pi \approx \frac{10}{3}$$



Enter "yes" or "no".

- 23) Is this the graph of $y = e^x$ and are its first two Taylor polynomials at $x = 0$ on the same axis? 23) _____



Enter "yes" or "no".

- 24) Estimate $\int_0^1 e^{x^2} dx$ by using the second Taylor polynomial for $f(x) = e^{x^2}$. Is 24) _____

$$\int_0^1 e^{x^2} dx \approx \frac{4}{3} \text{ the solution?}$$

Enter "yes" or "no".

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

- 25) Let $f(x) = \frac{1}{x+1}$. Determine the second Taylor polynomial $p_2(x)$ of $f(x)$ at $x = 0$. 25) _____

- A) $1 - x - x^2$
- B) $1 - x$
- C) $1 - 2x + 2x^2$
- D) $1 - x + x^2$
- E) none of these

- 26) Let $f(x) = x^3 - 4x - 1$. Which of the following statements is true? (All Taylor polynomials are at $x = 0$.) 26) _____
- A) $p_1(3) = -11$
 - B) $p_2(-1) = 0$
 - C) $p_n = f(x)$ for all $n \geq 3$
 - D) $p_3(1) = 7$
 - E) none of these

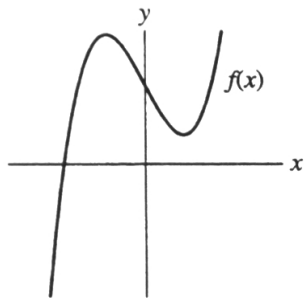
- 27) The function $f(x) = \sin x^2$ is approximated by its second Taylor polynomial $p_2(x)$ at $x = 0$. Which 27) _____ of the following statements is NOT true?
- A) $p_2(x) = x^2$
 - B) $p_2(x) = \frac{1}{3} + x^2$
 - C) $f'(0) = 0$
 - D) $f''(0) = 2$
 - E) none of these

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

- 28) If $f(x) = 2 + 3x - 2x^2 + 2x^3$, then what is $f'''(0)$? 28) _____
Enter just an integer.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

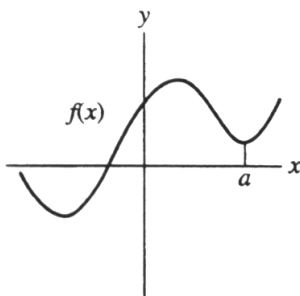
- 29) Below is a graph of the function $f(x)$. Which of the following could be the first Taylor polynomial of $f(x)$ at $x = 0$? 29) _____



- A) $p_1(x) = -2 + 2x$
- B) $p_1(x) = 3 + 4x$
- C) $p_1(x) = -2 - 3x$
- D) $p_1(x) = 3 - 2x$
- E) none of these

30) Below is a graph of function $f(x)$. Which of the following could be the second Taylor polynomial of $f(x)$ at $x = a$?

30) _____



A) $p_2(x) = \frac{13}{3} - \frac{5}{3}(x - a) + \frac{2}{3}(x - a)^2$

B) $p_2(x) = \frac{13}{3} - \frac{2}{3}(x - a)^2$

C) $p_2(x) = \frac{13}{3} + \frac{5}{3}(x - a)$

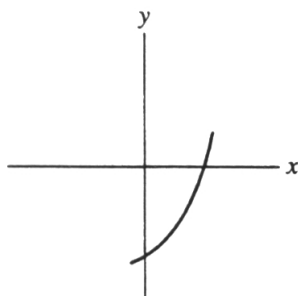
D) $p_2(x) = \frac{13}{3} + \frac{2}{3}(x - a)^2$

E) $p_2(x) = \frac{5}{3}(x - a) - \frac{2}{3}(x - a)^2$

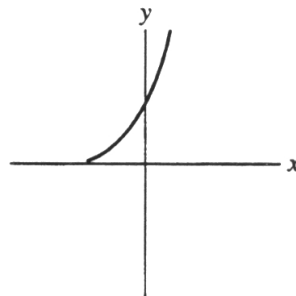
31) Suppose that the first Taylor polynomial of a function $f(x)$ at $x = 0$ is $p_1(x) = 2 - 3x$. Which of the following could be a graph of $f(x)$?

31) _____

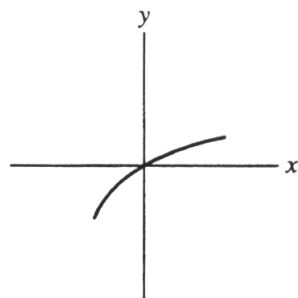
A)



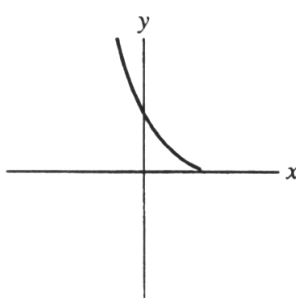
B)



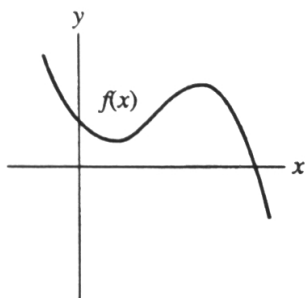
C)



D)



- 32) If the following is a graph of $f(x)$, which of the following could be the first Taylor polynomial of f at $x = 0$? 32) _____



- A) $p_1(x) = 2x - 2$ B) $p_1(x) = 2 + 2x$ C) $p_1(x) = 2x + 5$ D) $p_1(x) = 2 - 2x$

Determine the third Taylor polynomial of the function at $x = a$.

- 33) $f(x) = x^2$, $a = 3$ 33) _____

- A) $1 + 6(x - 9) + 9(x - 9)^2 + 12(x - 9)^3$ B) $1 + 18(x - 9) + 81(x - 9)^2 + 324(x - 9)^3$
 C) $9 + 6(x - 9) + 9(x - 9)^2 + 12(x - 9)^3$ D) $9 + 6(x - 9) + (x - 9)^2$

- 34) $f(x) = x^3$, $a = 9$ 34) _____

- A) $729 + 81(x - 81) + 81(x - 81)^2 + (x - 81)^3$
 B) $729 + 243(x - 81) + 27(x - 81)^2 + (x - 81)^3$
 C) $2916 + 243(x - 81) + 18(x - 81)^2 + (x - 81)^3$
 D) $6 + 3(x - 81) + (x - 81)^2 + (x - 81)^3$

- 35) $f(x) = x^2 + x + 1$, $a = 4$ 35) _____

- A) $1 + 3(x - 4) + 3(x - 4)^2 + (x - 4)^3$ B) $5 + 9(x - 4) + 13(x - 4)^2$
 C) $21 + 9(x - 4) + 9(x - 4)^2 + 21(x - 4)^3$ D) $21 + 9(x - 4) + (x - 4)^2$

- 36) Let $f(x) = e^{x/2}$. Determine the second Taylor polynomial of $f(x)$ at $x = 2$. 36) _____

- A) $e + \frac{e}{2}(x - 2) + \frac{e}{8}(x - 2)^2$ B) $1 + \frac{1}{2}(x - 2) + \frac{1}{6}(x - 2)^2$
 C) $1 - \frac{1}{2}(x - 2) + \frac{1}{8}(x - 2)^2$ D) $1 + \frac{1}{2}(x - 2) + \frac{1}{8}(x - 2)^2$

- 37) Let $f(x) = \ln x$. Find the second Taylor polynomial of $f(x)$ at $x = 2$. 37) _____

- A) $1 + \frac{1}{2}(x - 2) - \frac{1}{8}(x - 2)^2$ B) $\ln 2 - \frac{1}{2}(x - 2) - \frac{1}{8}(x - 2)^2$
 C) $\ln 2 + \frac{1}{2}(x - 2) - \frac{1}{8}(x - 2)^2$ D) $1 + \frac{1}{2}(x - 2) + \frac{1}{8}(x - 2)^2$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

38) Find the third Taylor polynomial of $f(x) = \cos x$ at $x = \frac{\pi}{2}$. 38) _____

Enter your answer as an unlabeled polynomial in $x - \frac{\pi}{2}$ in standard form (i.e., highest powers first).

39) Find the second Taylor polynomial of $f(x) = \sqrt{x}$ at $x = 9$ and use it to approximate $\sqrt{9.1}$. 39) _____
Enter just a real number rounded off to two decimal places.

40) Determine the third Taylor polynomial of $f(x) = \ln(2 - x)$ at $x = 1$ and use it to estimate $\ln(1.3)$. 40) _____
[Hint: $\ln(1.3) = f(0.7)$.]
Enter your answer as an unlabeled polynomial in $x - 1$ in standard form (i.e., highest powers first) followed by a comma and "yes" or "no" depending on whether or not the following is the correct estimate of $\ln(1.3)$: $\ln(1.3) \approx p_3(0.7) = 0.264$

41) Determine the third Taylor polynomial of $f(x) = x^3 - 2x + 4$ at $x = 1$. 41) _____
Enter an unlabeled polynomial in $x - 1$ in standard form (i.e., highest powers first).

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

42) Suppose $f(x) = x^4 - 7x^3 + 2$. The fifth Taylor polynomial of $f(x)$ at $x = 1$ is $p_5(x) = x^5 + x^4 - 7x^3 + 2$. 42) _____
A) True B) False

43) Suppose $f(x) = x^4 - 7x^3 + 2$. The third Taylor polynomial of $f(x)$ at $x = 1$ is $p_3(x) = 2 - 7x^3$. 43) _____
A) True B) False

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

44) Suppose $f(0) = 1$, $f'(0) = 1$, and $f''(0) = -1$. Use a Taylor polynomial of degree two to 44) _____
approximate $f\left(\frac{1}{2}\right)$. Is $f\left(\frac{1}{2}\right) \approx \frac{11}{8}$ the solution?
Enter "yes" or "no".

45) Use the second Taylor polynomial at $x = 1$ to estimate $\int_1^2 \ln x^2 dx$. 45) _____
Enter just a reduced fraction.

46) Suppose the second Taylor polynomial for $f(x)$ at $x = 3$ is $p_2(x) = 2(x - 3) - \frac{1}{3}(x - 3)^2$. Find $f''(3)$. Enter just a reduced fraction. 46) _____

47) If $f(x) = 1 - 3(x - 2) + 4(x - 2)^2 + 6(x - 2)^3$, then what is $f''(2)$? Enter just an integer. 47) _____

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

48) A polynomial $f(x)$ of degree 3 for which $f(1) = -1$, $f'(1) = 2$, $f''(1) = -1$, and $f'''(1) = -2$ is given by 48) _____
 A) $f(x) = x^3 - 2x^2 - 3x + 2$
 B) $f(x) = -1 + 2x - x^2 - 2x^3$
 C) $f(x) = -1 + 2(x - 1) - \frac{1}{2}(x - 1)^2 - \frac{1}{3}(x - 1)^3$
 D) $f(x) = -1 + 2(x - 1) - 1(x - 1)^2 - 2(x - 1)^2$
 E) none of these

Use the Newton-Raphson algorithm to approximate the given root to the nearest thousandth.

49) $\sqrt{2}$ 49) _____
 A) 2.000 B) 1.419 C) 1.411 D) 1.414

50) $\sqrt[3]{4}$ 50) _____
 A) 1.593 B) 1.583 C) 1.578 D) 1.587

Use the Newton-Raphson algorithm to find a zero of the function on the given interval. Round your answer to the nearest hundredth.

51) $f(x) = 4x^2 + 14x - 13$; between 0 and 1 51) _____
 A) 0.77 B) 0.76 C) 0.78 D) 0.75

52) $f(x) = e^x + 6x - 6$; between 0 and 1 52) _____
 A) 0.65 B) 0.66 C) 0.64 D) 0.67

53) The Newton-Raphson algorithm is used to approximate the zero of $f(x) = x^3 + x - 5$ between $x = 1$ and $x = 2$. If $x_0 = 1$, find x_1 . 53) _____
 A) $\frac{7}{4}$
 B) $\frac{1}{4}$
 C) $\frac{3}{4}$
 D) $\frac{7}{3}$
 E) none of these

54) The Newton-Raphson algorithm is applied to estimate $\sqrt{10}$. If $x_0 = 3$, find x_2 .

54) _____

- A) $\frac{5}{3}$
- B) $\frac{758}{521}$
- C) $\frac{721}{228}$
- D) $\frac{700}{237}$
- E) none of these

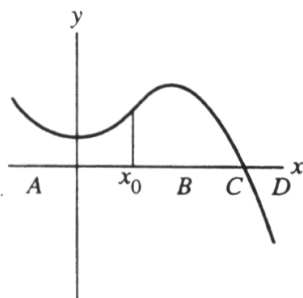
55) The Newton-Raphson algorithm is applied to estimate a zero of $f(x)$ with $x_0 = 3$. Which of the following statements is true?

55) _____

- A) $x_1 = 3 - \frac{f'(3)}{f(3)}$
- B) $x_1 = \frac{f'(3)}{f(3)}$
- C) $x_1 = 3 - \frac{f(3)}{f'(3)}$
- D) $x_1 = 3 + \frac{f'(3)}{f(3)}$
- E) none of these

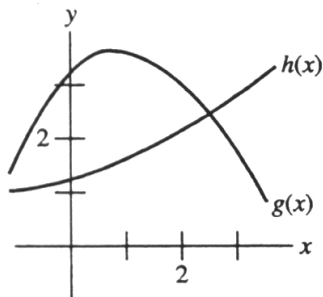
56) Below is a graph of the function $f(x)$. If x_0 is taken as the initial approximation of the zero of $f(x)$, then which of the following points, A, B, C, or D could be given by the Newton-Raphson algorithm as the next approximation?

56) _____



- A) A
- B) B
- C) C
- D) D

- 57) Below is a graph of the functions $h(x)$ and $g(x)$. In using the Newton-Raphson algorithm to find where $h(x) = g(x)$, which of the following statements is false? 57) _____



- A) Use the Newton-Raphson algorithm to find the zeroes of $f(x) = h(x) + g(x)$.
 B) $x_0 = 3$ could be used as the initial approximation.
 C) Use the Newton-Raphson algorithm to find the zeroes of $f(x) = h(x) - g(x)$.
 D) $x_0 = 4$ could be used as the initial approximation.
 E) Use the Newton-Raphson algorithm to find the zeroes of $f(x) = g(x) - h(x)$.
- 58) Suppose x_0 is an initial approximation of a zero of the function $f(x)$. Using the Newton-Raphson algorithm, a second approximation, x_1 is obtained. Which of the following must be true? 58) _____
- A) $x_1 = x_0 - \frac{f'(x_0)}{f(x_0)}$
 B) x_1 is the x -coordinate of the x -intercept of the tangent line to $f(x)$ at $(x_0, f(x_0))$
 C) $f(x_1) = 0$
 D) x_1 is closer to the zero of $f(x)$ than x_0 .
 E) all of these

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

- 59) Let $x_0 = 2$. Use three repetitions of the Newton-Raphson algorithm to approximate $\sqrt[3]{5}$. 59) _____
 Enter just a real number rounded off to two decimal places (no label).
- 60) Use three repetitions of the Newton-Raphson algorithm to approximate $\sqrt{3}$. Let $x_0 = 4$. 60) _____
 Enter just a real number rounded off to two decimal places.
- 61) Use two repetitions of the Newton-Raphson algorithm to approximate $\sqrt{15}$. 61) _____
 Enter just a real number rounded off to two decimal places.
- 62) $f(x) = x^5 + x - 3$ has a zero between 1 and 2. 62) _____
 Use two repetitions of the Newton-Raphson algorithm to approximate this zero with $x_0 = 1$.
 Enter just a real number rounded off to two decimal places.

- 63) Use the Newton-Raphson algorithm with two repetitions to estimate the positive solution of $\sin x = \frac{1}{2}x$. Use $x_0 = 2$. 63) _____
Enter just a real number rounded off to two decimal places.
- 64) Use the Newton-Raphson algorithm with three repetitions to approximate the zero of $f(x) = e^x - 2$ near $x = 1$. 64) _____
Enter just a real number rounded off to two decimal places.
- 65) Use the Newton-Raphson algorithm with three repetitions to approximate the zero of $f(x) = \cos x + x - 2$ near $x = 3$. 65) _____
Enter just a real number rounded off to two decimal places.
- 66) Use the Newton-Raphson algorithm with three repetitions to approximate the solution to $e^{-x} = 2 - x$ near $x = 2$. 66) _____
Enter just a real number rounded off to two decimal places.
- 67) Use two repetitions of the Newton-Raphson algorithm to approximate the zero of $f(x) = \sin x - \cos x$ near $x = 0$. 67) _____
Enter just a real number rounded off to two decimal places.
- 68) Use two repetitions of the Newton-Raphson algorithm to approximate the value of x for which $e^x = 3x$. Use $x = 0$ as the first approximation. 68) _____
Enter just a real number rounded off to two decimal places.
- 69) Use two repetitions of the Newton-Raphson algorithm to find the value of x near zero for which $\cos x = x$. 69) _____
Enter just a real number rounded off to two decimal places.
- 70) Use two repetitions of the Newton-Raphson algorithm to find the value of x near zero for which $e^x = 2 \cos x$. 70) _____
Enter just a real number rounded off to two decimal places.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

- 71) Suppose that an investment of \$1000 yields returns of \$267.38, \$343.28, and \$500.00 at the end of the first, second, and third months, respectively. Determine the internal rate of return on this investment. 71) _____
A) 5.9% B) 5.13% C) 4.9% D) 4.59%
- 72) A mortgage of \$123,981 is repaid in 300 monthly payments of \$1000. Determine the monthly rate of interest. 72) _____
A) 0.5% B) 0.81% C) 0.71% D) 7.1%

Determine whether the given series converges or diverges, and find the sum if it converges.

73) $30 + 5 + \frac{5}{6} + \frac{5}{36} + \dots$ 73) _____

A) Converges; sum = 36

B) Converges; sum = 40

C) Converges; sum = 37

D) Converges; sum = 35.5

74) $\frac{1}{23} + \frac{4}{23} + \frac{16}{23} + \frac{64}{23} + \dots$ 74) _____

A) Converges; sum = 3.98

B) Converges; sum = 2.78

C) Converges; sum = 3.70

D) Diverges

75) $1 + \frac{1}{7} + \frac{1}{49} + \frac{1}{343} + \dots$ 75) _____

A) Converges; sum = 1.3125

B) Converges; sum = 1.1667

C) Converges; sum = 1.75

D) Diverges

76) $13 + 7.8 + 4.68 + 2.808 + \dots$ 76) _____

A) Converges; sum = 19.5

B) Converges; sum = 48.75

C) Converges; sum = 32.5

D) Diverges

77) $0.98 + 0.0098 + 0.000098 + \dots$ 77) _____

A) Converges; sum = $\frac{49}{50}$

B) Converges; sum = $\frac{23}{25}$

C) Converges; sum = $\frac{98}{99}$

D) Diverges

Solve the problem.

78) The repeating decimal 0.44444... can be expressed as infinite geometric series 78) _____

$$0.4 + 0.4\left(\frac{1}{10}\right) + 0.4\left(\frac{1}{10}\right)^2 + 0.4\left(\frac{1}{10}\right)^3 + \dots$$

By finding the sum of the series, determine the rational number whose decimal expansion is 0.44444... .

A) $\frac{4}{17}$

B) $\frac{4}{11}$

C) $\frac{4}{13}$

D) $\frac{4}{9}$

79) The repeating decimal 0.90909090... can be expressed as infinite geometric series 79) _____

$$0.90 + 0.90\left(\frac{1}{100}\right) + 0.90\left(\frac{1}{100}\right)^2 + 0.90\left(\frac{1}{100}\right)^3 + \dots$$

By finding the sum of the series, determine the rational number whose decimal expansion is 0.90909090... .

A) $\frac{10}{13}$

B) $\frac{10}{9}$

C) $\frac{10}{17}$

D) $\frac{10}{11}$

- 80) Determine the sum of the series $e^{-1} + e^{-2} + e^{-3} + \dots$ if it converges. 80) _____
 A) e B) diverges C) $\frac{1}{e-1}$ D) $\frac{e}{e-1}$
- 81) Determine the sum of the series $\sum_{n=0}^{\infty} (-1)^n \left(\frac{2}{e}\right)^n$. 81) _____
 A) $\frac{2}{e}$ B) $\frac{e}{e+2}$ C) $1 - \frac{2}{e}$ D) none of these
- 82) Determine the sum of the series $\sum_{n=0}^{\infty} \frac{1-2^n}{3^n}$. 82) _____
 A) $\frac{2}{3}$ B) 3 C) $-\frac{3}{2}$ D) none of these
- 83) Determine the sum of the series $\sum_{n=1}^{\infty} \left(\frac{\pi}{e}\right)^n$. 83) _____
 A) $\frac{\pi}{e-\pi}$ B) $\frac{e}{\pi}$ C) $\frac{e}{e-\pi}$ D) none of these
- 84) Determine the sum of the series $1 + \frac{1}{1.01} + \frac{1}{(1.01)^2} + \frac{1}{(1.01)^3} + \dots$. 84) _____
 A) $\frac{1}{1.01}$ B) 10.1 C) 101 D) none of these
- 85) Determine the sum of the series $\sum_{n=1}^{\infty} \frac{3^n}{4^n - 1}$. 85) _____
 A) $\frac{1}{4}$ B) 3 C) $\frac{3}{4}$ D) 12
- 86) Determine the sum of the series $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots$. 86) _____
 A) 2 B) 1 C) $\frac{1}{2}$ D) none of these
- 87) Determine the sum of the series $\sum_{n=2}^{\infty} \left(-\frac{1}{2}\right)^n$. 87) _____
 A) $\frac{1}{3}$ B) $\frac{1}{4}$ C) $\frac{1}{6}$ D) none of these

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

88) Determine the sum of the following geometric series: $1 - \frac{1}{2^3} + \frac{1}{2^6} - \frac{1}{2^9} + \frac{1}{2^{12}} - \dots$ 88) _____

Enter just a reduced fraction of form $\frac{a}{b}$.

89) Determine the sum of the following geometric series: $\frac{2^2}{5^3} + \frac{2^4}{5^5} + \frac{2^6}{5^7} + \frac{2^8}{5^9} + \frac{2^{10}}{5^{11}} + \dots$ 89) _____

Enter just a reduced fraction of form $\frac{a}{b}$.

90) Determine the sum of the following geometric series: $1 + \frac{1}{\sqrt{2}} + \frac{1}{2} + \dots$ 90) _____

Enter your answer exactly in the form $\frac{\sqrt{a}}{\sqrt{a} - b}$.

91) Determine the sum of the following geometric series: $2 + \frac{4}{3} + \frac{8}{9} + \dots$ 91) _____

Enter just an integer.

92) Determine the sum of the following geometric series: $3 - 1.8 + 1.08 + .648 - \dots$ 92) _____
Enter just a real number rounded off to three decimal places.

93) Determine the sum of the following geometric series: $\frac{27}{5} + \frac{18}{5} + \frac{12}{5} + \frac{8}{5} + \dots$ 93) _____

Enter a reduced fraction of form $\frac{a}{b}$.

94) Determine the sum of the following geometric series: $1 + (0.25)^2 + (0.25)^4 + (0.25)^8 + \dots$ 94) _____

Enter a reduced fraction of form $\frac{a}{b}$.

95) Sum an appropriate infinite series to find the rational number whose decimal expansion is: $0.\overline{37}$. 95) _____

Enter just a reduced fraction of form $\frac{a}{b}$.

96) Sum an appropriate infinite series to find the rational number whose decimal expansion is: $0.4\overline{98}$. 96) _____

Enter just a reduced fraction of form $\frac{a}{b}$.

97) Sum an appropriate infinite series to find the rational number whose decimal expansion is: $0.185\overline{185}$. Enter just a reduced fraction of form $\frac{a}{b}$. 97) _____

98) Sum an appropriate infinite series to find the rational number whose decimal expansion is: $0.196\overline{96}$. Enter just a reduced fraction of form $\frac{a}{b}$. 98) _____

99) Determine the sum of the following infinite series: $\sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k (2)^{k+1}$. Enter just an integer. 99) _____

100) Determine the sum of the following infinite series: $\sum_{k=0}^{\infty} (-1)^k \frac{2}{7^k}$. Enter just a reduced fraction of form $\frac{a}{b}$. 100) _____

101) Determine the sum of the following infinite series: $\sum_{k=0}^{\infty} (1 - \sqrt{2})^k$. Enter your answer exactly in the reduced form $\frac{a}{\sqrt{b}}$. 101) _____

102) Determine the sum of the following infinite series: $\sum_{n=1}^{\infty} 2^n \left(\frac{1}{3}\right)^{n-1}$. Enter just an integer. 102) _____

103) Determine the sum of the following infinite series: $\sum_{n=1}^{\infty} \frac{2^n + (-1)^n}{3^n}$. Enter just a reduced fraction of form $\frac{a}{b}$. 103) _____

104) Determine the sum of the infinite geometric series $\sum_{k=0}^{\infty} 3^{-k} \ln 2$. Enter your answer exactly in the reduced form $\frac{a \ln b}{c}$. 104) _____

105) Determine the sum of the following series: $10 + 4 + \frac{8}{5} + \frac{16}{25} + \dots$.

105) _____

Enter just a reduced fraction of form $\frac{a}{b}$.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

106) Determine the sum of the geometric series $\frac{2^2}{5^2} - \frac{2^3}{5^3} + \frac{2^4}{5^4} - \dots$, if it is convergent.

106) _____

A) $\frac{5}{7}$

B) $\frac{3}{5}$

C) $\frac{4}{35}$

D) $\frac{7}{5}$

E) none of these

107) Determine the sum of the infinite series: $\sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^{2k}$.

107) _____

A) $\frac{4}{3}$

B) $\frac{2}{3}$

C) $\frac{4}{5}$

D) 2

E) none of these

108) The geometric series $1 + (0.2)^3 + (0.2)^6 + (0.2)^9 + \dots$

108) _____

(I) converges

(II) is equal to $\sum_{k=0}^{\infty} \left(\frac{1}{125}\right)^k$

(III) is equal to $\sum_{k=0}^{\infty} (0.2)^{3k}$

A) II and III

B) I and III

C) I and II

D) I, II, and III

E) III only

Solve the problem.

109) The infinite series $a_1 + a_2 + a_3 + \dots$ has partial sums given by $S_n = 1 - \frac{3}{n}$. Find $\sum_{k=1}^{10} a_k$. 109) _____

A) 0.97 B) 0.7 C) 7 D) 1.3

110) The infinite series $a_1 + a_2 + a_3 + \dots$ has partial sums given by $S_n = 8 - \frac{3}{n}$. Does the infinite series converge? If so, to what value does it converge? 110) _____

A) No, $\lim_{n \rightarrow \infty} \left(8 - \frac{3}{n}\right)$ does not exist B) Yes, 11

C) Yes, 5 D) Yes, 8

111) England decides to decrease its taxes by £7 billion. It is estimated that of each pound received, a typical citizen will spend 90%. The level of economic activity generated by the tax cut is therefore estimated to be: $7 \cdot (0.90) + 7 \cdot (0.90)^2 + 7 \cdot (0.90)^3 + \dots$ billion pounds. This amount is equal to what? 111) _____

A) £3.6 billion

B) £45 billion

C) £63 billion

D) £70 billion

E) none of these

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

112) A student receives \$1000 at the start of each month from his parents. Every month the student spends 70% of all the money he has. If the only money the student receives is the money from his parents, estimate how much money the student will have at the beginning of each month after an extended period of time. 112) _____

Enter just a reduced fraction of form $\frac{a}{b}$.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

- 113) A patient receives M milligrams of a certain drug every hour. Each hour the body eliminates a fraction p of the amount of drug in the body. After an extended period of time, which of the following series approximates the amount of drugs present in the patient's body immediately before receiving an hourly dose? 113) _____

A) $\sum_{k=1}^{\infty} M(1-p)^k$

B) $\sum_{k=0}^{\infty} Mp^k$

C) $\sum_{k=1}^{\infty} Mp^k$

D) $\sum_{k=0}^{\infty} M(1-p)^k$

E) $\sum_{k=1}^{\infty} Mp^{-k}$

- 114) Consider the following geometric series: $\frac{25}{2} - \frac{15}{2} + \frac{9}{2} - \frac{27}{10} + \dots$. Which of the following statements is true? 114) _____

A) This series diverges.

B) The sum of this series is $\frac{125}{4}$.

C) The ratio r of this series is $\frac{3}{5}$.

D) Another way writing this series is $\sum_{k=1}^{\infty} \left(-\frac{3}{5}\right)^k$

E) none of these

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

- 115) Suppose $\sum_{k=0}^{\infty} \left(\frac{2}{m}\right)^k$ converges. What can you say about the value of m ? 115) _____

Enter your answer in standard interval notation (no variables).

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

- 116) A ball is dropped from a height of 14 meters and returns to about $\frac{6}{7}$ of its previous height on each bounce. About how far will the ball travel before it comes to rest? 116) _____
 A) 98 m B) 266 m C) 350 m D) 182 m

- 117) A pendulum bob swings through an arc 40 centimeters long on its first swing. For each swing thereafter, it swings only 95% as far as on the previous swing. How far will it swing altogether before coming to a complete stop? 117) _____
 A) $\frac{800}{39}$ cm B) $\frac{475}{7}$ cm C) $\frac{475}{3}$ cm D) 800 cm

- 118) After being struck with a hammer, a gong vibrates 30 vibrations in the first second and in each second thereafter makes $\frac{3}{4}$ as many vibrations as in the previous second. Find how many vibrations the gong makes before it stops vibrating. 118) _____
 A) 35 vibrations B) 120 vibrations C) 130 vibrations D) 40 vibrations

Use the Integral Test to determine whether the series converges.

- 119) $\sum_{n=1}^{\infty} \frac{15}{\sqrt{n}}$ 119) _____
 A) Diverges B) Converges

- 120) $\sum_{n=1}^{\infty} 7n^{-3/2}$ 120) _____
 A) Converges B) Diverges

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

- 121) Use the integral test to determine whether the infinite series $\sum_{k=1}^{\infty} \frac{1}{k\sqrt{k}}$ is convergent or divergent. 121) _____
 Enter just the word "divergent" or "convergent".

- 122) Use the integral test to determine whether the infinite series $\sum_{k=1}^{\infty} \frac{1}{(k+2)^2}$ is convergent or divergent? 122) _____
 Enter just the word "convergent" or "divergent".

- 123) Use the integral test to determine whether the infinite series $\sum_{k=2}^{\infty} \frac{2}{2k+1}$ is convergent or divergent. Then use the comparison test to determine whether the infinite series $\sum_{k=1}^{\infty} \frac{4}{k+1}$ is convergent or divergent.
Enter just two words which answer the two questions above in order (separated by a comma) where each word is either "convergent" or "divergent". 123) _____
- 124) Use the integral test to determine whether the infinite series $\sum_{k=1}^{\infty} \frac{k}{(k^2+2)^2}$ is convergent or divergent.
Enter just "convergent" or "divergent". 124) _____
- 125) Use the integral test to determine whether the infinite series $\sum_{k=1}^{\infty} \frac{5k^2+1}{2k^3-1}$ is convergent or divergent.
Enter just "convergent" or "divergent". 125) _____
- 126) Use the integral test to determine whether the infinite series $\sum_{k=2}^{\infty} \frac{\ln k}{k}$ is convergent or divergent.
Enter just "convergent" or "divergent". 126) _____
- 127) It can be shown that $\int_0^{\infty} xe^{-x} dx = 1$. Use this fact and the integral test to construct an appropriate convergent infinite series. Is $\sum_{k=0}^{\infty} e^{-k}$ the correct series?
Enter just "yes" or "no". 127) _____
- 128) Use the comparison test to determine whether the infinite series $\sum_{k=1}^{\infty} \frac{1}{k^3+k-1}$ is convergent or divergent.
Enter just "convergent" or "divergent". 128) _____
- 129) Use the comparison test to determine whether the infinite series $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k+1}}$ is convergent or divergent.
Enter just "convergent" or "divergent". 129) _____

130) Use the comparison test to determine whether the infinite series $\sum_{k=1}^{\infty} \frac{1}{2 + e^k}$ is 130) _____
 convergent or divergent.
 Enter just "convergent" or "divergent".

131) Use the comparison test to determine whether the infinite series $\sum_{k=1}^{\infty} \frac{k^2}{k + k^2 + k^{5/2}}$ is 131) _____
 convergent or divergent.
 Enter just "convergent" or "divergent".

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

132) Which of the following series converge? 132) _____

A) $\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$
 B) $\sum_{n=1}^{\infty} \frac{n}{n + 1}$
 C) $\sum_{n=1}^{\infty} \frac{n^{1/2}}{n^2 + 1}$
 D) all of these
 E) none of these

133) Which of the following series converge? 133) _____

A) $\sum_{n=1}^{\infty} \frac{n}{(1 + n)^3}$
 B) $\sum_{n=1}^{\infty} \frac{1}{n^2 + \sin n}$
 C) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$
 D) all of these
 E) none of these

134) Which of the following series converge?

134) _____

A) $\sum_{n=1}^{\infty} \frac{1}{(\ln n)^3}$

B) $\sum_{n=1}^{\infty} \frac{e^n}{ne^n + 10}$

C) $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$

D) all of these

E) none of these

135) Which of the following series converge?

135) _____

A) $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

B) $\sum_{n=1}^{\infty} \frac{n^2}{10n^3 + 100}$

C) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^{3/2} + n}$

D) none of these

E) all of these

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

136) Find the Taylor series of $f(x) = \frac{1}{1+x}$ for $|x| < 1$ then multiply the series by x to obtain a

136) _____

series expansion of $\frac{x}{x+1}$, and then use these two series to obtain a series expansion of

$\frac{1-x}{1+x}$ for $|x| < 1$.

Is $\frac{1-x}{1+x} = 1 + 2x + 2x^2 + 2x^3 + 2x^4 + \dots$ the correct expansion?

Enter "yes" or "no".

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the Taylor series for the given function.

137) $f(x) = \frac{5}{1+9x}$

137) _____

A) $5 + 45x + \dots + (-1)^n 5 \cdot 9^n x^n + \dots$

B) $5 - 45x + \dots + (-1)^n 5 \cdot 9^n x^n + \dots$

C) $5x + 45x^2 + \dots + (-1)^n 5 \cdot 9^n x^{n+1} + \dots$

D) $5x - 45x^2 + \dots + (-1)^n 5 \cdot 9^n x^{n+1} + \dots$

138) $f(x) = \frac{5}{4-x}$

138) _____

A) $\frac{5}{4}x + \frac{5}{16}x^2 + \frac{5}{64}x^3 + \dots + \frac{5}{4^n}x^n + \dots$

B) $5 + \frac{5}{16}x + \frac{5}{64}x^2 + \dots + \frac{5}{4^n}x^n + \dots$

C) $\frac{5}{4} + \frac{5}{16}x^2 + \frac{5}{64}x^4 + \dots + \frac{5}{4^{n+1}}x^{2n} + \dots$

D) $\frac{5}{4} + \frac{5}{16}x + \frac{5}{64}x^2 + \dots + \frac{5}{4^{n+1}}x^n + \dots$

139) $f(x) = \ln(1+2x)$

139) _____

A) $2x - \frac{2^2}{2!}x^2 + \frac{2^3}{3!}x^3 - \frac{2^4}{4!}x^4 + \dots + \frac{(-1)^n 2^{n+1}}{(n+1)!}x^{n+1} + \dots$

B) $-2x - \frac{2^2}{2!}x^2 + \frac{2^3}{3!}x^3 - \frac{2^4}{4!}x^4 + \dots + \frac{(-1)^n 2^n}{(n+1)!}x^n + \dots$

C) $2x + \frac{2^2}{2}x^2 - \frac{2^3}{3}x^3 + \frac{2^4}{4}x^4 + \dots + \frac{(-1)^n 2^{n+1}}{n+1}x^{n+1} + \dots$

D) $2x - \frac{2^2}{2}x^2 + \frac{2^3}{3}x^3 - \frac{2^4}{4}x^4 + \dots + \frac{(-1)^n 2^{n+1}}{n+1}x^{n+1} + \dots$

140) $f(x) = e^{2x^2}$

140) _____

A) $1 + 2x + \frac{2^2}{2!}x^2 + \frac{2^3}{3!}x^3 + \dots + \frac{2^n}{n!}x^n + \dots$

B) $1 + 2x^2 + \frac{2^2}{2!}x^4 + \frac{2^3}{3!}x^6 + \dots + \frac{2^n}{n!}x^{2n} + \dots$

C) $1 + 2x^2 + \frac{2^2}{2}x^4 + \frac{2^3}{3}x^6 + \dots + \frac{2^n}{n}x^{2n} + \dots$

D) $1 - 2x^2 + \frac{2^2}{2!}x^4 - \frac{2^3}{3!}x^6 + \dots + \frac{(-1)^n 2^n}{n!}x^{2n} + \dots$

141) $f(x) = x^3 e^{-x}$

141) _____

A) $x^3 - x^4 + \frac{1}{2}x^5 - \frac{1}{6}x^6 + \dots + \frac{(-1)^n}{n!}x^{3+n} + \dots$

B) $1 - x^3 + x^4 - \frac{1}{2}x^5 + \frac{1}{6}x^6 - \dots + \frac{(-1)^n}{n!}x^{3+n} + \dots$

C) $1 + x^3 - x^4 + \frac{1}{2}x^5 - \frac{1}{6}x^6 + \dots + \frac{(-1)^n}{n!}x^{3+n} + \dots$

D) $x^3 + x^4 - \frac{1}{2}x^5 + \frac{1}{6}x^6 + \dots + \frac{(-1)^{2n}}{n!}x^{3+n} + \dots$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

142) Find the Taylor series expansion at $x = 0$ of $\int 2xe^{-x} dx$. Is 142) _____

$$\left[x^2 + \frac{2}{3}x^3 + \frac{1}{4}x^4 + \frac{1}{15}x^5 + \dots \right] + C \text{ correct?}$$

Enter "yes" or "no".

143) Find the Taylor series expansion at $x = 0$ of $\int \frac{x}{1+x^3} dx$. Is 143) _____

$$\left[\frac{1}{2}x^2 + \frac{1}{5}x^5 + \frac{1}{8}x^8 + \frac{1}{11}x^{11} \dots \right] + C \text{ correct?}$$

Enter "yes" or "no".

144) Find an infinite series that converges to the value of $\int_0^1 x^2e^{-x^3} dx$. Is 144) _____

$$\frac{1}{3} - \frac{1}{6} + \frac{1}{18} - \frac{1}{72} \dots \text{ correct?}$$

Enter "yes" or "no".

145) Find an infinite series that converges to the value of $\int_0^1 2xe^{-x} dx$. Is $1 - \frac{2}{3} + \frac{1}{4} - \frac{1}{15} \dots$ 145) _____

correct?

Enter "yes" or "no".

146) Find the Taylor series at $x = 0$ of $f(x) = e^{3x}$ by computing four derivatives and using the 146) _____

definition of the Taylor series. Is $1 - 3x + \frac{3^2}{2!}x^2 - \frac{3^3}{3!}x^3 + \frac{3^4}{4!}x^4 - \dots$ the correct answer?

Enter "yes" or "no".

147) Determine the first four non-zero terms of the Taylor series at $x = 0$ for $f(x) = \sin x^3$. 147) _____

$$\text{Is } f(x) = x^3 + \frac{x^9}{3!} + \frac{x^{15}}{5!} + \frac{x^{21}}{7!} \text{ the correct answer?}$$

Enter "yes" or "no".

148) Determine the first three non-zero terms of the Taylor series at $x = 0$ for 148) _____

$f(x) = x \cos x - \sin x$.

$$\text{Is } \frac{(3! - 2!)x^3}{2! 3!} + \frac{(5! - 4!)x^5}{4! 5!} + \frac{(7! - 6!)x^7}{6! 7!} \text{ the correct answer?}$$

Enter "yes" or "no".

- 149) Determine the first four non-zero terms of the Taylor series at $x = 0$ for $f(x) = xe^{(1/2)x}$. 149) _____
 Is $x - \frac{x^2}{2} + \frac{x^3}{2^2 \cdot 2!} - \frac{x^4}{2^3 \cdot 3!}$ the correct answer?
 Enter "yes" or "no".
- 150) Find the first four non-zero terms of the Taylor series at $x = 0$ for $f(x) = 1 + xe^x$. 150) _____
 Is $1 + x + x^2 + \frac{x^3}{2!}$ the correct answer?
 Enter "yes" or "no".
- 151) Find the Taylor series at $x = 0$ of $f(x) = x^2e^{2x}$. Use enough terms to calculate $0.25e$ to two decimal places of accuracy. 151) _____
 Enter just a real number to 2 decimal places.
- 152) Find the first four non-zero terms of the Taylor series at $x = 0$ of $f(x) = \cos 3x + \sin 2x$. 152) _____
 Is $1 + 2x - \frac{9x^2}{2!} - \frac{8x^3}{3!} + \dots$ the correct answer?
 Enter "yes" or "no".
- 153) Find the first four non-zero terms of the Taylor series at $x = 0$ of $f(x) = \sin 2x$. 153) _____
 Is $f(x) = 2x - \frac{8x^3}{3!} + \frac{32x^5}{5!} - \frac{128x^7}{7!}$ the correct answer?
 Enter "yes" or "no".
- 154) Find the first four non-zero terms of the Taylor series at $x = 0$ of $f(x) = \sqrt{4 - x}$. 154) _____
 Is $f(x) = 2 - \frac{x}{4} - \frac{x^2}{32 \cdot 2!} - \frac{3x^3}{256 \cdot 3!}$ correct?
 Enter "yes" or "no".
- 155) Find the first four non-zero terms of the Taylor series at $x = 0$ of $f(x) = e^x$. 155) _____
 Is $f(x) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!}$ correct?
 Enter "yes" or "no".
- 156) Find the first four non-zero terms of the Taylor series at $x = 0$ of $f(x) = \ln(x + 1)$. 156) _____
 Is $f(x) = x + \frac{x^2}{2!} + \frac{2x^3}{3!} + \frac{6x^4}{4!}$ correct?
 Enter "yes" or "no".
- 157) Find the first four non-zero terms of the Taylor series at $x = 0$ of $f(x) = e^{-2x}$. 157) _____
 Is $f(x) = 1 + 2x + \frac{4x^2}{2!} + \frac{8x^3}{3!}$ correct?
 Enter "yes" or "no".

158) Find the first three non-zero terms of the Taylor series at $x = 0$ of $f(x) = e^x \sin x$. 158) _____

Is $f(x) = x + x^2 + \frac{x^3}{3}$ correct?

Enter "yes" or "no".

159) The Taylor series at $x = 0$ for $f(x) = \ln\left(\frac{1+x}{1-x}\right)$ is $2x + \frac{2}{3}x^3 + \frac{2}{5}x^5 + \frac{2}{7}x^7 + \dots, |x| < 1$. Find 159) _____

$f^{(6)}(0)$.

Enter just an integer.

160) The Taylor series at $x = 0$ for $f(x) = \tan x$ is $x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \dots$. Find $f^{(5)}(0)$. 160) _____

Enter just an integer.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

161) Find the Taylor series at $x = 0$ of the function $f(x) = \frac{1}{1-3x}$ by computing three or four derivatives 161) _____

and using the definition of the Taylor series.

A) $1 + \frac{3x}{1}x + \frac{9x}{2}x^2 + \dots$

B) $1 - 3x + 9x^2 - 27x^3 + \dots$

C) $1 + (3x)^2 + (3x)^4 + (3x)^6 + \dots$

D) $1 + 3x + 9x^2 + 27x^3 + \dots$

E) none of these

162) Find the Taylor series at $x = 0$ of the function $f(x) = x \ln(1 + 2x)$ by computing three or four derivatives and using the definition of the Taylor series. 162) _____

A) $2x^2 - \frac{4x^3}{3}$

B) $2x^2 + \frac{4x^3}{2!}$

C) $2x^2 - \frac{4x^3}{2}$

D) $2x - \frac{4x^2}{2!} + \frac{8x^3}{3!}$

E) none of these

163) Find the Taylor series expansion for $f(x) = \frac{x}{1-x}$ and use it to determine which of the following is false? 163) _____

false?

A) $\frac{x}{1-x} = x + x^2 + x^3 + x^4 + \dots$

B) $\frac{1}{2} = \frac{1}{3} + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \dots$

C) $-\frac{3}{5} = \left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^3 + \dots$

D) $-\frac{1}{3} = \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^3 + \dots$

E) All the statements are true.

164) The Bessel function $f(x)$ of order zero has the Taylor series at $x = 0$ given by 164) _____

$f(x) = 1 - \frac{x^2}{4} + \frac{x^4}{64} - \frac{x^6}{2304} + \dots$. What is $f^{(4)}(0)$?

A) $\frac{1}{64}$

B) $\frac{1}{4}$

C) $\frac{3}{8}$

D) none of these

165) The Taylor Series for $\frac{1}{(1-x)^2}$ at $x = 0$ is given by $f(x) = 1 + 2x + 3x^2 + 4x^3 + \dots$. Find $f^{(3)}(0)$. 165) _____

A) 3

B) 4

C) $\frac{2}{3}$

D) none of these

166) Find the Taylor Series at $x = 0$ for $f(x) = \frac{\sin x}{x}$. 166) _____

A) $1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \dots$

B) $x^2 - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

C) $x - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \dots$

D) none of these

167) Find the Taylor Series at $x = 0$ for $f(x) = e^x + 1$. 167) _____

A) $\frac{1}{e} + \frac{x}{e} + \frac{x^2}{e2!} + \frac{x^3}{e3!} + \dots$

B) $e + ex + \frac{ex^2}{2!} + \frac{ex^3}{3!} + \dots$

C) $e - ex + \frac{ex^2}{2!} - \frac{ex^3}{3!} + \dots$

D) none of these

168) The Bessel function of order 1 has the Taylor Series at $x = 0$ given by $f(x) = \frac{x}{2} - \frac{x^3}{16} + \frac{x^5}{384} - \dots$, find 168) _____

$f^{(5)}(0)$.

A) $\frac{1}{384}$

B) $\frac{5}{16}$

C) $\frac{5}{384}$

D) none of these

Answer Key

Testname: UNTITLED11

- 1) A
- 2) D
- 3) B
- 4) D
- 5) C
- 6) D
- 7) C
- 8) A
- 9) x^2
- 10) $x^3 - 3x$
- 11) $\frac{5}{16}x^3 + \frac{3}{8}x^2 + \frac{1}{2}x + 1$
- 12) D
- 13) A
- 14) 0.49
- 15) $x^2, \frac{\pi^3}{24}$
- 16) $-\frac{x^3}{3} + x^2 + x$
- 17) $\frac{8}{3}$
- 18) 0.48
- 19) $1 - x^2 + \frac{1}{2}x^4$
- 20) A
- 21) A
- 22) yes
- 23) yes
- 24) yes
- 25) D
- 26) C
- 27) B
- 28) 12
- 29) D
- 30) D
- 31) D
- 32) D
- 33) D
- 34) B
- 35) D
- 36) A
- 37) C

Answer Key

Testname: UNTITLED11

38) $\frac{1}{6}\left(x - \frac{\pi}{2}\right)^3 - \left(x - \frac{\pi}{2}\right)$

39) 3.02

40) $-\frac{1}{3}(x - 1)^3 - \frac{1}{2}(x - 1)^2 - (x - 1)$, yes

41) $(x - 1)^3 + 3(x - 1)^2 + (x - 1) + 3$

42) B

43) B

44) yes

45) $\frac{2}{3}$

46) $-\frac{2}{3}$

47) 8

48) C

49) D

50) D

51) B

52) D

53) A

54) C

55) C

56) A

57) A

58) B

59) 1.71

60) 1.73

61) 3.87

62) 1.14

63) 1.90

64) 0.69

65) 2.99

66) 1.84

67) 0.78

68) 0.61

69) 0.75

70) 0.63

71) C

72) C

73) A

74) D

75) B

76) C

Answer Key

Testname: UNTITLED11

77) C

78) D

79) D

80) C

81) B

82) C

83) A

84) C

85) D

86) B

87) C

88) $\frac{8}{9}$

89) $\frac{4}{105}$

90) $\frac{\sqrt{2}}{\sqrt{2} - 1}$

91) 6

92) 1.875

93) $\frac{81}{5}$

94) $\frac{16}{15}$

95) $\frac{37}{99}$

96) $\frac{247}{495}$

97) $\frac{5}{27}$

98) $\frac{13}{66}$

99) 6

100) $\frac{7}{4}$

101) $\frac{1}{\sqrt{2}}$

102) 6

103) $\frac{7}{4}$

104) $\frac{3 \ln 2}{2}$

105) $\frac{50}{3}$

106) C

Answer Key

Testname: UNTITLED11

- 107) A
- 108) D
- 109) B
- 110) D
- 111) C
- 112) $\frac{1000}{3}$
- 113) A
- 114) E
- 115) $(-\infty, -2) \cup (2, \infty)$
- 116) D
- 117) D
- 118) B
- 119) A
- 120) A
- 121) convergent
- 122) convergent
- 123) divergent, divergent
- 124) convergent
- 125) divergent
- 126) divergent
- 127) no
- 128) convergent
- 129) divergent
- 130) convergent
- 131) divergent
- 132) C
- 133) D
- 134) E
- 135) A
- 136) no
- 137) B
- 138) D
- 139) D
- 140) B
- 141) A
- 142) no
- 143) no
- 144) yes
- 145) yes
- 146) no
- 147) no
- 148) no

Answer Key

Testname: UNTITLED11

- 149) no
- 150) yes
- 151) 0.68
- 152) yes
- 153) yes
- 154) yes
- 155) no
- 156) no
- 157) no
- 158) yes
- 159) 0
- 160) 16
- 161) D
- 162) C
- 163) C
- 164) C
- 165) C
- 166) A
- 167) B
- 168) B