CHAPTER ONE

Solutions for Section 1.1

1. **(a)** The story in (a) matches Graph (IV), in which the person forgot her books and had to return home.
(b) The story in (b) matches Graph (II), the flat tire story. Note the long period of time during which the distance from home did not change (the horizontal part).
(c) The story in (c) matches Graph (III), in which the person started calmly but sped up later.

The first graph (I) does not match any of the given stories. In this picture, the person keeps going away from home, but his speed decreases as time passes. So a story for this might be: *I started walking to school at a good pace, but since I stayed up all night studying calculus, I got more and more tired the farther I walked.*

2. The height is going down as time goes on. A possible graph is shown in Figure 1.1. The graph is decreasing.

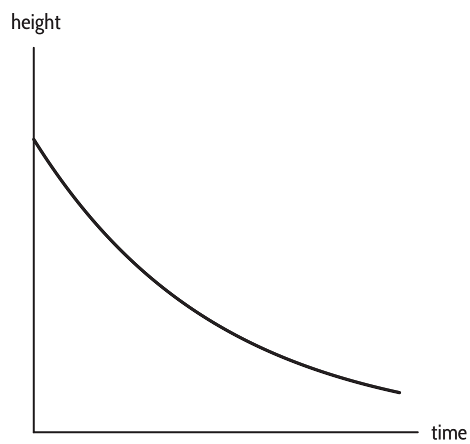


Figure 1.1

3. The amount of carbon dioxide is going up as time goes on. A possible graph is shown in Figure 1.2. The graph is increasing.

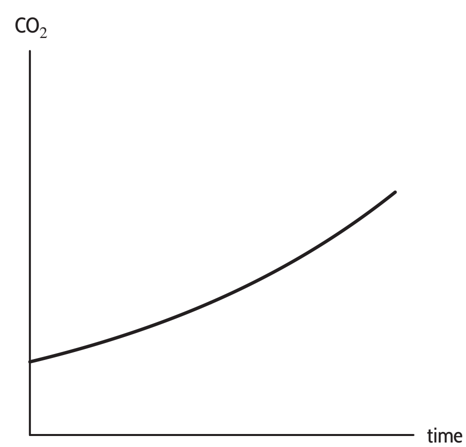


Figure 1.2

4. The number of air conditioning units sold is going up as temperature goes up. A possible graph is shown in Figure 1.3. The graph is increasing.

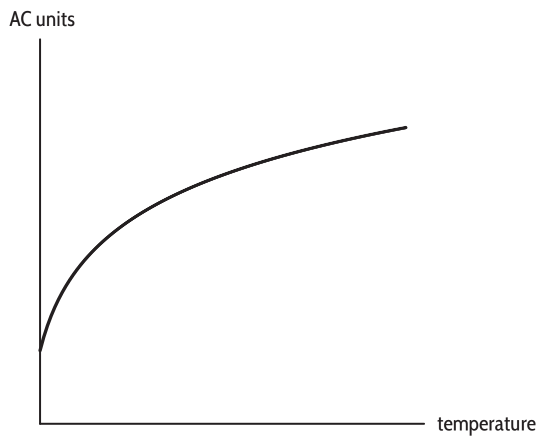


Figure 1.3

5. The noise level is going down as distance goes up. A possible graph is shown in Figure 1.4. The graph is decreasing.

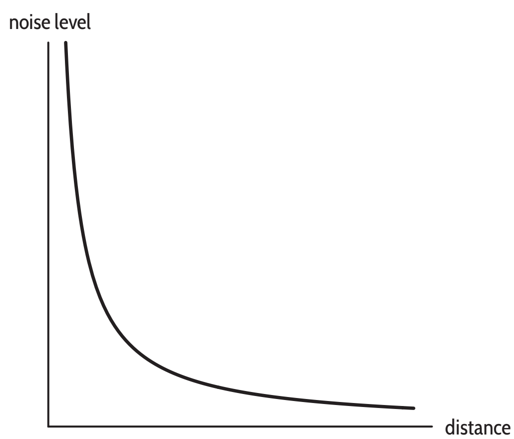


Figure 1.4

6. If we let t represent the number of years since 1900, then the population increased between $t = 0$ and $t = 40$, stayed approximately constant between $t = 40$ and $t = 50$, and decreased for $t \geq 50$. Figure 1.5 shows one possible graph. Many other answers are also possible.

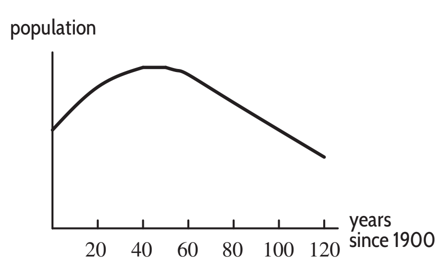


Figure 1.5

7. Amount of grass $G = f(r)$ increases as the amount of rainfall r increases, so $f(r)$ is an increasing function.

8. We are given information about how atmospheric pressure $P = f(h)$ behaves when the altitude h decreases: as altitude h decreases the atmospheric pressure P increases. This means that as altitude h increases the atmospheric pressure P decreases. Therefore, $P = f(h)$ is a decreasing function.

9. We are given information about how battery capacity $C = f(T)$ behaves when air temperature T decreases: as T decreases, battery capacity C also decreases. This means that increasing temperature T increases battery capacity C. Therefore, $C = f(T)$ is an increasing function.

10. Time $T = f(m)$ increases as m increases, so $f(m)$ is increasing.

11. The attendance $A = f(P)$ decreases as the price P increases, so $f(P)$ is a decreasing function.

12. The cost of manufacturing $C = f(v)$ increases as the number of vehicles manufactured v increases, so $f(v)$ is an increasing function.

13. We are given information about how commuting time, $T = f(c)$ behaves as the number of cars on the road c decreases: as c decreases, commuting time T also decreases. This means that increasing the number of cars on the road c increases commuting time T. Therefore, $T = f(c)$ is an increasing function.

14. The statement $f(4) = 20$ tells us that $W = 20$ when $t = 4$. In other words, in 2019, Argentina produced 20 million metric tons of wheat.

15. **(a)** If we consider the equation

$$C = 4T - 160$$

simply as a mathematical relationship between two variables C and T, any T value is possible. However, if we think of it as a relationship between cricket chirps and temperature, then C cannot be less than 0. Since $C = 0$ leads to $0 = 4T - 160$, and so $T = 40°\text{F}$, we see that T cannot be less than 40°F. In addition, we are told that the function is not defined for temperatures above 134°. Thus, for the function $C = f(T)$ we have

$$\begin{aligned}\text{Domain} &= \text{All } T \text{ values between } 40°\text{F and } 134°\text{F}\\ &= \text{All } T \text{ values with } 40 \le T \le 134\\ &= [40, 134].\end{aligned}$$

(b) Again, if we consider $C = 4T - 160$ simply as a mathematical relationship, its range is all real C values. However, when thinking of the meaning of $C = f(T)$ for crickets, we see that the function predicts cricket chirps per minute between 0 (at $T = 40°\text{F}$) and 376 (at $T = 134°\text{F}$). Hence,

$$\begin{aligned}\text{Range} &= \text{All } C \text{ values from 0 to 376}\\ &= \text{All } C \text{ values with } 0 \le C \le 376\\ &= [0, 376].\end{aligned}$$

16. **(a)** The statement $f(19) = 415$ means that $C = 415$ when $t = 19$. In other words, in the year 2019, the concentration of carbon dioxide in the atmosphere was 415 ppm.

(b) The expression $f(22)$ represents the concentration of carbon dioxide in the year 2022.

17. **(a)** At $p = 0$, we see $r = 8$. At $p = 3$, we see $r = 7$.

(b) When $p = 2$, we see $r = 10$. Thus, $f(2) = 10$.

18. Substituting $x = 5$ into $f(x) = 2x + 3$ gives

$$f(5) = 2(5) + 3 = 10 + 3 = 13.$$

19. Substituting $x = 5$ into $f(x) = 10x - x^2$ gives

$$f(5) = 10(5) - (5)^2 = 50 - 25 = 25.$$

20. We want the y-coordinate of the graph at the point where its x-coordinate is 5. Looking at the graph, we see that the y-coordinate of this point is 3. Thus

$$f(5) = 3.$$

21. Looking at the graph, we see that the point on the graph with an x-coordinate of 5 has a y-coordinate of 2. Thus

$$f(5) = 2.$$

22. In the table, we must find the value of $f(x)$ when $x = 5$. Looking at the table, we see that when $x = 5$ we have

$$f(5) = 4.1$$

23. **(a)** We are asked for the value of y when x is zero. That is, we are asked for $f(0)$. Plugging in we get

$$f(0) = (0)^2 + 2 = 0 + 2 = 2.$$

(b) Substituting we get

$$f(3) = (3)^2 + 2 = 9 + 2 = 11.$$

(c) Asking what values of x give a y-value of 11 is the same as solving

$$y = 11 = x^2 + 2$$
$$x^2 = 9$$
$$x = \pm\sqrt{9} = \pm 3.$$

We can also solve this problem graphically. Looking at Figure 1.6, we see that the graph of $f(x)$ intersects the line $y = 11$ at $x = 3$ and $x = -3$. Thus, when x equals 3 or x equals -3 we have $f(x) = 11$.

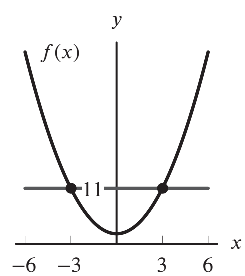


Figure 1.6

(d) No. No matter what, x^2 is greater than or equal to 0, so $y = x^2 + 2$ is greater than or equal to 2.

24. The year 2018 was 2 years before 2020 so 2018 corresponds to $t = 2$. Thus, an expression that represents the statement is:

$$f(2) = 7.088.$$

25. The year 2020 was 0 years before 2020 so 2020 corresponds to $t = 0$. Thus, an expression that represents the statement is:

$$f(0) \text{ meters.}$$

26. The year 1949 was $2020 - 1949 = 71$ years before 2020 so 1949 corresponds to $t = 71$. Similarly, we see that the year 2000 corresponds to $t = 20$. Thus, an expression that represents the statement is:

$$f(71) = f(20).$$

27. The year 2018 was 2 years before 2020 so 2018 corresponds to $t = 2$. Similarly, $t = 3$ corresponds to the year 2017. Thus, $f(3)$ and $f(2)$ are the average annual sea level values, in meters, in 2017 and 2018, respectively. Because 11 millimeters is the same as 0.011 meters, the average sea level in 2018, $f(2)$, is 0.011 less than the sea level in 2017 which is $f(3)$. An expression that represents the statement is:

$$f(2) = f(3) - 0.011.$$

Note that there are other possible equivalent expressions, such as: $f(2) - f(3) = -0.011$.

28. **(a)** Since the potato is getting hotter, the temperature is increasing. The temperature of the potato eventually levels off at the temperature of the oven, so the best answer is graph (III).

(b) The vertical intercept represents the temperature of the potato at time $t = 0$ or just before it is put in the oven.

29. **(a)** The graph shows that the yield increases as more kg of seeds are planted. Moreover the yield initially increases very rapidly but then levels off at about 2.5 tons, meaning there is little benefit from continuing to increase the quantity of seed.

(b) We see that at 80 kg the graph is almost a horizontal line, meaning that any additional seed planted will not impact the crop yield, so it is not worth the cost of planting additional seed.

30. **(a)** $f(30) = 10$ means that the value of f at $t = 30$ was 10. In other words, the temperature at time $t = 30$ minutes was 10°C. So, 30 minutes after the object was placed outside, it had cooled to 10 °C.

(b) The intercept a measures the value of $f(t)$ when $t = 0$. In other words, when the object was initially put outside, it had a temperature of a°C. The intercept b measures the value of t when $f(t) = 0$. In other words, at time b the object's temperature is 0 °C.

31. **(a)** Since $N = f(h)$, the statement $f(500) = 100$, means that if $h = 500$, then $N = 100$. This tells us that at an elevation of 500 feet above sea level, there are 100 species of bats.

(b) The vertical intercept k is on the N-axis, so it represents the value of N when $h = 0$. The intercept, k, is the number of bat species at sea level. The horizontal intercept, c, represents the value of h when $N = 0$. The intercept, c, is the lowest elevation above which no bats are found.

32. The number of species of algae is low when there are few snails or lots of snails. The greatest number of species of algae (about 10) occurs when the number of snails is at a medium level (around 125 snails per square meter.) The graph supports the statement that diversity peaks at intermediate predation levels, when there are an intermediate number of snails.

33. **(a)** From the graph, we estimate $f(3) = 0.14$. This means that after 3 hours, the level of nicotine is about 0.14 mg.

(b) About 4 hours.

(c) The vertical intercept is 0.4. It represents the level of nicotine in the blood right after the cigarette is smoked.

(d) A horizontal intercept would represent the value of t when $N = 0$, or the number of hours until all nicotine is gone from the body.

34. **(a)** The original deposit is the balance, B, when $t = 0$, which is the vertical intercept. The original deposit was $1000.

(b) It appears that $f(10) \approx 2200$. The balance in the account after 10 years is about $2200.

(c) When $B = 5000$, it appears that $t \approx 20$. It takes about 20 years for the balance in the account to reach $5000.

35. **(a)** The statement $f(10) = 0.2$ means that $C = 0.2$ when $t = 10$. In other words, CFC consumption was 0.2 million tons in 1997.

(b) The vertical intercept is the value of C when $t = 0$. It represents CFC consumption in 1987.

(c) The horizontal intercept is the value of t when $C = 0$. It represents the year when CFC consumption is expected to be zero.

36. **(a)** The graph shows maximum range is about 78 miles and it occurs at approximately 65°F.

(b) The graph is decreasing for temperatures above 65°F, so above 65°F, every increase in the temperature will reduce the range.

37. **(a)** We see that the deficit is decreasing during this time period.

(b) The statement $f(4) = 485$ means that the deficit was 485 billion dollars in 2014.

(c) We can estimate the location of 4 on the horizontal axis, since the graph covers 2011–2015, corresponding to $1 \leq t \leq 5$. The value 485 on the vertical axis is then the corresponding y-coordinate. These values, and a dot at the corresponding point, are shown in Figure 1.7.

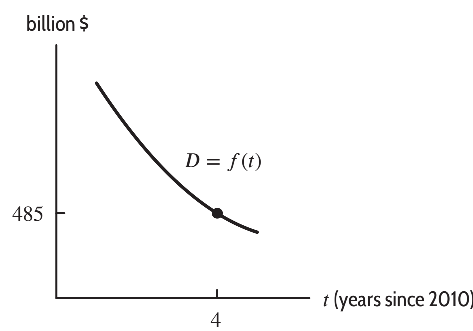


Figure 1.7

(d) A vertical intercept represents the value of D when $t = 0$, which is the US deficit (in billions of dollars) in the year 2010.

(e) A horizontal intercept represents the value of t when $D = 0$, or the number of years after 2010 when the deficit is zero: the horizontal intercept occurs at a moment when the deficit is zero.

38. See Figure 1.8.

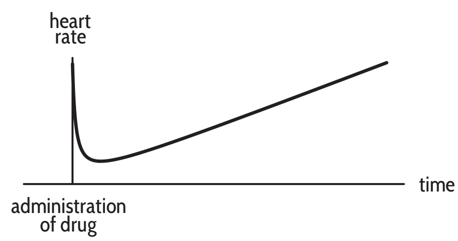


Figure 1.8

39. Figure 1.9 shows one possible graph of gas mileage increasing to a high at a speed of 45 mph and then decreasing again. Other graphs are also possible.

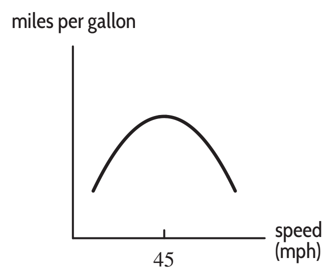


Figure 1.9

40. One possible graph is shown in Figure 1.10. Since the patient has none of the drug in the body before the injection, the vertical intercept is 0. The peak concentration is labeled on the concentration (vertical) axis and the time until peak concentration is labeled on the time (horizontal) axis. See Figure 1.10.

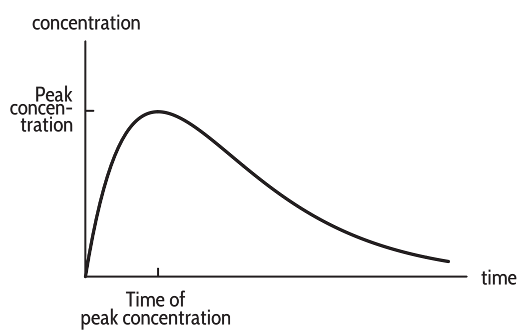


Figure 1.10

41. **(a)** We want a graph of expected return as a function of risk, so expected return is on the vertical axis and risk is on the horizontal axis. The return increases as the risk increases, so the graph might look like Figure 1.11. However, the graph may not be a straight line; many other answers are also possible.

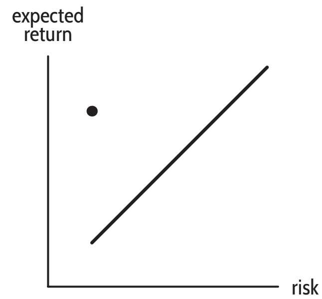


Figure 1.11

(b) High expected return and low risk is in the top left corner. See Figure 1.11.

42. **(a)** The statement $f(1000) = 3500$ means that when $a = 1000$, we have $S = 3500$. In other words, when \$1000 is spent on advertising, the number of sales per month is 3500.
(b) Graph I, because we expect that as advertising expenditures go up, sales will go up (not down).
(c) The vertical intercept represents the value of S when $a = 0$, or the sales per month if no money is spent on advertising.

43. **(a)** In 1970, fertilizer use in the US was about 13 million tons, in India about 2 million tons, and in the former Soviet Union about 10 million tons.
(b) Fertilizer use in the US rose steadily between 1950 and 1980 and has stayed relatively constant at about 18 million tons since then. Fertilizer use in India has risen steadily throughout this 50-year period. Fertilizer use in the former Soviet Union rose rapidly between 1950 and 1985 and then declined very rapidly between 1985 and 2000.

44. Since $P(t)$ is the total cases, it can't decrease. So (I) is the graph of the function $N(t)$, and (II) is the graph of $P(t)$.

45. **(a)** $P(100)$ is the total number of confirmed cases of the disease up to day 100, which is the last day of the outbreak, so it is the total number of people who contract the disease during the outbreak. Therefore $P(100) = 20{,}000$.
$N(100)$ is the number of new cases on day 100. Since the outbreak is over on day 100 there are no new cases, so $N(100) = 0$.
(b) Since the outbreak is over on day 100, there are no new cases after day 100, so $P(t) = P(100)$ for $t > 100$.

46. **(a)** On January 8, 2021, there were 1419 new Covid-19 cases reported in Iowa.
(b) On January 9, 2021, there were 898 fewer Covid-19 cases reported than on January 8, 2021.
(c) On January 10 there were 581 more cases reported than on January 9.

47. **(a)** On February 3, 2021, there were 2296 new Covid-19 cases reported in Arizona.
(b) On February 4, 2021, there were 2121 more Covid-19 cases reported than on February 3, 2021.
(c) On February 5 there were 591 fewer cases reported than on February 4.

48. **(a)** (I) The incidence of cancer increase with age, but the rate of increase slows down slightly. The graph is nearly linear. This type of cancer is closely related to the aging process.
(II) In this case a peak is reached at about age 55, after which the incidence decreases.
(III) This type of cancer has an increased incidence until the age of about 48, then a slight decrease, followed by a gradual increase.
(IV) In this case the incidence rises steeply until the age of 30, after which it levels out completely.
(V) This type of cancer is relatively frequent in young children, and its incidence increases gradually from about the age of 20.
(VI) This type of cancer is not age-related – all age-groups are equally vulnerable, although the overall incidence is low (assuming each graph has the same vertical scale).
(b) Graph (V) shows a relatively high incidence rate for children. Leukemia behaves in this way.
(c) Graph (III) could represent cancer in women with menopause as a significant factor. Breast cancer is a possibility here.
(d) Graph (I) shows a cancer which might be caused by toxins building up in the body. Lung cancer is a good example of this.

49. **(a)** Since 2014 corresponds to $t = 0$, the average annual sea level in Aberdeen in 2014 was 7.071 meters.
(b) Looking at the table, we see that the average annual sea level was 7.083 twenty five years before 2014, or in the year 1989. Similar reasoning shows that the average sea level was 6.985 meters 100 years before 2014, or in 1914.
(c) Because 1889 was 125 years before 2014, we see that the sea level value corresponding to the year 1889 is 6.900 (this is the sea level value corresponding to $t = 125$). Similar reasoning yields Table 1.1.

Table 1.1

Year	1889	1914	1939	1964	1989	2014
S	6.900	6.985	6.964	6.990	7.083	7.071

50. **(a)** When the temperature is 14°F the graph is at 60%, so on average an electrical vehicle will achieve 60% of its advertised range.

(b) The temperatures that give 80% of advertised range are approximately 32°F and 104°F.

(c) If the vehicle has a shorter range than the advertised range, then the percentage of the advertised range achieved will be less than 100%. All temperatures where the percentage is less than 100% correspond to temperatures where the vehicle will not achieve its advertised range. From the graph we see that these correspond to temperatures below 50°F and above approximately 90°F.

51. See Figure 1.12.

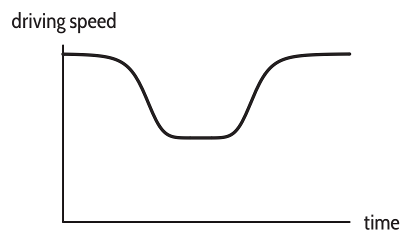


Figure 1.12

52. See Figure 1.13.

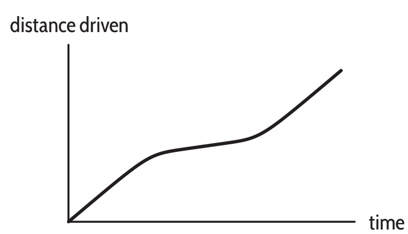


Figure 1.13

53. See Figure 1.14.

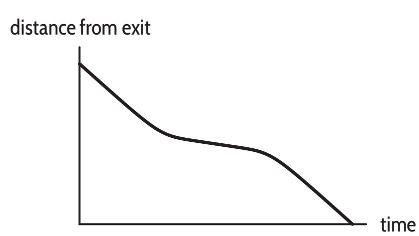


Figure 1.14

54. See Figure 1.15.

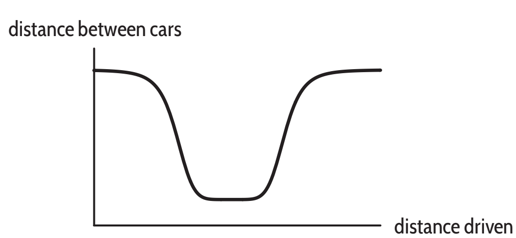


Figure 1.15

Solutions for Section 1.2

1. The slope is $(3-2)/(2-0) = 1/2$. So the equation of the line is $y = (1/2)x + 2$.

2. The slope is $(1-0)/(1-0) = 1$, so the equation of the line is $y = x$.

3. The slope is

$$\text{Slope} = \frac{3-1}{2-(-2)} = \frac{2}{4} = \frac{1}{2}.$$

Now we know that $y = (1/2)x + b$. Using the point $(-2, 1)$, we have $1 = -2/2 + b$, which yields $b = 2$. Thus, the equation of the line is $y = (1/2)x + 2$.

4. The slope is

$$\text{Slope} = \frac{-1-5}{2-4} = \frac{-6}{-2} = 3.$$

Substituting $m = 3$ and the point $(4, 5)$ into the equation of the line, $y = b + mx$, we have

$$\begin{aligned} 5 &= b + 3 \cdot 4 \\ 5 &= b + 12 \\ b &= -7. \end{aligned}$$

The equation of the line is $y = -7 + 3x$.

5. Rewriting the equation as

$$y = -\frac{12}{7}x + \frac{2}{7}$$

shows that the line has slope $-12/7$ and vertical intercept $2/7$.

6. Rewriting the equation as

$$y = 4 - \frac{3}{2}x$$

shows that the line has slope $-3/2$ and vertical intercept 4.

7. Rewriting the equation of the line as

$$\begin{aligned} y &= \frac{12}{6}x - \frac{4}{6} \\ y &= 2x - \frac{2}{3}, \end{aligned}$$

we see that the line has slope 2 and vertical intercept $-2/3$.

s

8. Rewriting the equation of the line as

$$\begin{aligned} -y &= \frac{-2}{4}x - 2 \\ y &= \frac{1}{2}x + 2, \end{aligned}$$

we see the line has slope $1/2$ and vertical intercept 2.

9. The slope is positive for lines l_1 and l_2 and negative for lines l_3 and l_4. The y-intercept is positive for lines l_1 and l_3 and negative for lines l_2 and l_4. Thus, the lines match up as follows:

(a) l_1
(b) l_3
(c) l_2
(d) l_4

10. **(a)** Lines l_2 and l_3 are parallel and thus have the same slope. Of these two, line l_2 has a larger y-intercept.
(b) Lines l_1 and l_3 have the same y-intercept. Of these two, line l_1 has a larger slope.

11. **(a)** The slope is constant at 750 people per year so this is a linear function. The vertical intercept (at $t = 0$) is 28,600. We have

$$P = 28{,}600 + 750t.$$

(b) The population in 2020, when $t = 5$, is predicted to be

$$P = 28{,}600 + 750 \cdot 5 = 32{,}350.$$

(c) We use $P = 35{,}000$ and solve for t:

$$\begin{aligned} P &= 28{,}600 + 750t \\ 35{,}000 &= 28{,}600 + 750t \\ 6{,}400 &= 750t \\ t &= 8.53. \end{aligned}$$

The population will reach 35,000 people in about 8.53 years, or during the year 2023.

12. This is a linear function with vertical intercept 35 and slope 0.20. The formula for the monthly charge is

$$C = 35 + 0.20m.$$

13. **(a)** The first company's price for a day's rental with m miles on it is $C_1(m) = 40 + 0.15m$. Its competitor's price for a day's rental with m miles on it is $C_2(m) = 50 + 0.10m$.

(b) See Figure 1.16.

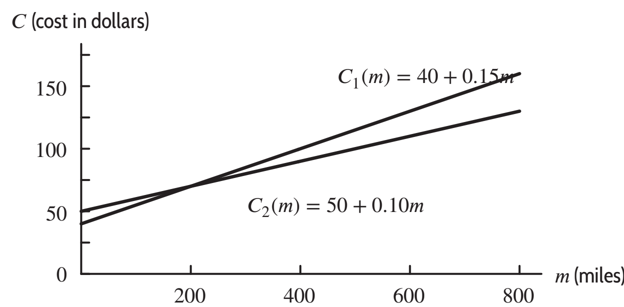


Figure 1.16

(c) To find which company is cheaper, we need to determine where the two lines intersect. We let $C_1 = C_2$, and thus

$$\begin{aligned} 40 + 0.15m &= 50 + 0.10m \\ 0.05m &= 10 \\ m &= 200. \end{aligned}$$

If you are going more than 200 miles a day, the competitor is cheaper. If you are going less than 200 miles a day, the first company is cheaper.

14. This is a linear function with two parts to the formula:
For $0 \le d \le 2$, we pay only the \$25, so

$$P = 25.$$

For $2 < d$, the \$5 charge is only on the data above 2 gigabytes, that is $d - 2$ gigabytes:

$$P = 25 + 5(d - 2).$$

15. **(a)** The vertical intercept appears to be about 300 miles. When this trip started (at $t = 0$) the person was 300 miles from home.

(b) The slope appears to be about $(550 - 300)/5 = 50$ miles per hour. This person is moving away from home at a rate of about 50 miles per hour.

(c) We have $D = 300 + 50t$.

16. (a) On the interval from 0 to 1 the value of y decreases by 2. On the interval from 1 to 2 the value of y decreases by 2. And on the interval from 2 to 3 the value of y decreases by 2. Thus, the function has a constant rate of change and it could therefore be linear.

(b) On the interval from 15 to 20 the value of s increases by 10. On the interval from 20 to 25 the value of s increases by 10. And on the interval from 25 to 30 the value of s increases by 10. Thus, the function has a constant rate of change and could be linear.

(c) On the interval from 1 to 2 the value of w increases by 5. On the interval from 2 to 3 the value of w increases by 8. Thus, we see that the slope of the function is not constant and so the function is not linear.

17. For the function given by table (a), we know that the slope is

$$\text{slope} = \frac{27-25}{0-1} = -2.$$

We also know that at $x = 0$ we have $y = 27$. Thus we know that the vertical intercept is 27. The formula for the function is

$$y = -2x + 27.$$

For the function in table (b), we know that the slope is

$$\text{slope} = \frac{72-62}{20-15} = \frac{10}{5} = 2.$$

Thus, we know that the function will take on the form

$$s = 2t + b.$$

Substituting in the coordinates (15, 62) we get

$$\begin{aligned} s &= 2t + b \\ 62 &= 2(15) + b \\ &= 30 + b \\ 32 &= b. \end{aligned}$$

Thus, a formula for the function would be

$$s = 2t + 32.$$

18. No. Linear growth corresponds to the same increase in revenue every year. Facebook's ad revenue growth was faster than linear.

19. (a) The vertical intercept is the value of S when $t = 0$, so it is $S = 124$ mph. This tells us that according to the linear model, the top speed of the fastest car in 1949 is $S = 124$ mph.

(b) The slope is 2.25 mph/year. This tells us that according to the linear model, the top speed of the fastest car is increasing by 2.25 mph each year.

20. (a) For the first function, from $t = 0$ to $t = 5$, the slope is \$15.5 million dollars per quarter. This tells us that for this period, Zoom's revenue increased, on average, by \$15.5 million per quarter.

For the second function, from $t = 6$ onward in 2020, the slope is \$22.1 million dollars per quarter. This tells us that for this period, Zoom's revenue increased, on average, by \$22.1 million per quarter.

(b) The sharp change in the rate at which Zoom earned revenue corresponds to the start of the Covid-19 pandemic in March 2020. With schools and businesses moving online, many people were spending much more time on Zoom.

21. (a) The slope is -1.22 billion dollars per year. McDonald's revenue is decreasing at a rate of 1.22 billion dollars per year.

(b) The vertical intercept is 25.44 billion dollars. In 2015, McDonald's revenue was 25.44 billion dollars.

(c) Substituting $t = 6$, we have $R = 25.44 - 1.22 \cdot 6 = 18.12$ billion dollars.

(d) We substitute $R = 17$ and solve for t:

$$\begin{aligned} 25.44 - 1.22t &= 17 \\ -1.22t &= -8.44 \\ t &= \frac{8.44}{1.22} \approx 6.92 \text{ years.} \end{aligned}$$

The function predicts that annual revenue will fall to 17 billion dollars in 2022.

22. **(a)** Reading coordinates from the graph, we see that rainfall $r = 100$ mm corresponds to about $Q = 600$ kg/hectare, and $r = 600$ mm corresponds to about $Q = 5800$ kg/hectare. Using a difference quotient we have

$$\text{Slope} = \frac{\Delta Q}{\Delta r} = \frac{5800 - 600}{600 - 100} = 10.4 \text{ kg/hectare per mm.}$$

(b) Every additional 1 mm of annual rainfall corresponds to an additional 10.4 kg of grass per hectare.

(c) Using the slope, we see that the equation has the form

$$Q = b + 10.4r.$$

Substituting $r = 100$ and $Q = 600$ we can solve for b.

$$\begin{aligned} b + 10.4(100) &= 600 \\ b &= -440. \end{aligned}$$

The equation of the line is

$$Q = -440 + 10.4r.$$

23. **(a)** Reading coordinates from the graph, we see that rainfall $r = 100$ mm corresponds to about $Q = 300$ kg/hectare, and $r = 600$ mm corresponds to about $Q = 3200$ kg/hectare. Using a difference quotient we have

$$\text{Slope} = \frac{\Delta Q}{\Delta r} = \frac{3200 - 300}{600 - 100} = 5.8 \text{ kg/hectare per mm.}$$

(b) Every additional 1 mm of annual rainfall corresponds to an additional 5.8 kg of grass per hectare.

(c) Using the slope, we see that the equation has the form

$$Q = b + 5.8r.$$

Substituting $r = 100$ and $Q = 300$ we can solve for b.

$$\begin{aligned} b + 5.8(100) &= 300 \\ b &= -280. \end{aligned}$$

The equation of the line is

$$Q = -280 + 5.8r.$$

24. The difference quotient $\Delta Q/\Delta r$ equals the slope of the line and represents the increase in the quantity of grass per millimeter of rainfall. We see from the graph that the slope of the line for 1939 is larger than the slope of the line for 1997. Thus, each additional 1 mm of rainfall in 1939 led to a larger increase in the quantity of grass than in 1997.

25. **(a)** (i) The slope $-0.0566 = \Delta A/\Delta t$ has units seconds/year. This tells us that the time it takes the fastest car to accelerate from 0–60 mph drops by 0.0566 seconds or about 1/20 of a second every year.

(ii) The vertical intercept has the same units as A, and is therefore seconds. It corresponds to the value of A when $t = 0$, which is the prediction of the model for the fastest car in 1955. It tells us that the model predicts the fastest car in 1955 took 6 seconds to accelerate from 0 to 60 miles per hour.

(b) We solve for t in $A(t) = 1$:

$$\begin{aligned} -0.0566t + 6 &= 1 \\ t &= \frac{-5}{-0.0566} \\ &= 88.339 \text{ years.} \end{aligned}$$

Thus, we expect the 0–60 mph time to reach 1 second a little more than 88 years after 1955, so in 2044.

26. We know $W = 22.42$ when $t = 2001$ and $W = 2.17$ when $t = 2019$. We use these two points to find the slope:

$$m = \frac{\Delta W}{\Delta t} = \frac{2.17 - 22.42}{2019 - 2001} = \frac{-20.25}{18} = -1.125 \text{ million metric tons/year.}$$

We substitute the point $W = 22.42$ and $t = 2001$ and the slope $m = -1.125$ into the point-slope form to find the equation:

$$\begin{aligned} W - W_0 &= m(t - t_0) \\ W - 22.42 &= -1.125(t - 2001) \\ W - 22.42 &= -1.125t + 2251.125 \\ W &= -1.125t + 2273.545. \end{aligned}$$

The equation of the line is $W = -1.125t + 2273.545$.

27. (a) We select two points on the line. Using the values (0, 12.5) and (10, 4) we have

$$\text{Slope} = \frac{4 - 12.5}{10 - 0} = -0.85\ \frac{\text{million barrels per day}}{\text{year}}.$$

(b) With t in years since 2006, and $f(t)$ as the quantity of net imports in millions of barrels per day, we have

$$f(t) = 12.5 - 0.85t.$$

(c) We have

$$\begin{aligned} 12.5 - 0.85t &= 0 \\ t &= 14.71. \end{aligned}$$

The model predicts US will stop being a net importer of oil 14.71 years after 2006, so in the year 2006 + 14.71 = 2020.71. This agrees with the actual date net US imports of oil dropped below 0 million barrels per day.

28. (a) We know that the function for q in terms of p will take on the form

$$q = mp + b.$$

We know that the slope will represent the change in q over the corresponding change in p. Thus

$$m = \text{slope} = \frac{4-3}{12-15} = \frac{1}{-3} = -\frac{1}{3}.$$

Thus, the function will take on the form

$$q = -\frac{1}{3}p + b.$$

Substituting the values $q = 3$, $p = 15$, we get

$$\begin{aligned} 3 &= -\frac{1}{3}(15) + b \\ 3 &= -5 + b \\ b &= 8. \end{aligned}$$

Thus, the formula for q in terms of p is

$$q = -\frac{1}{3}p + 8.$$

(b) We know that the function for p in terms of q will take on the form

$$p = mq + b.$$

We know that the slope will represent the change in p over the corresponding change in q. Thus

$$m = \text{slope} = \frac{12-15}{4-3} = -3.$$

Thus, the function will take on the form

$$p = -3q + b.$$

Substituting the values $q = 3$, $p = 15$ again, we get

$$\begin{aligned} 15 &= (-3)(3) + b \\ 15 &= -9 + b \\ b &= 24. \end{aligned}$$

Thus, a formula for p in terms of q is

$$p = -3q + 24.$$

29. (a) The vertical intercept appears to be about 495. In 2015, world milk production was about 495 million tons of milk.

(b) World milk production appears to be about 495 at $t = 0$ and about 525 at $t = 4$. We estimate

$$\text{Slope} = \frac{525 - 495}{4 - 0} = 7.5 \text{ million tons/year}.$$

World milk production increased at a rate of about 7.5 million tons per year during this 5-year period.

(c) We have $M = 495 + 7.5t$.

30. **(a)** Let x be the average minimum daily temperature (°C), and let y be the date the marmot is first sighted. Reading coordinates from the graph, we see that temperature $x = 12$°C corresponds to date $y = 137$ days after Jan 1, and $x = 22$°C corresponds to $y = 109$ days. Using a difference quotient, we have

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{109 - 137}{22 - 12} = -2.8\,\frac{\text{days}}{°\text{C}}.$$

(b) The slope is negative. An increase in the temperatures corresponds to an earlier sighting of a marmot. Marmots come out of hibernation earlier in years with warmer average daily minimum temperature.

(c) We have

$$\Delta y = \text{Slope} \times \Delta x = (-2.8)(6) = 16.8 \text{ days}.$$

If temperatures are 6°C higher, then marmots come out of hibernations about 17 days earlier.

(d) Using the slope, we see that the equation has the form

$$y = b - 2.8x.$$

Substituting $x = 12$ and $y = 137$ we solve for b.

$$\begin{aligned} b - 2.8(12) &= 137 \\ b &= 171. \end{aligned}$$

The equation of the line is

$$y = 171 - 2.8x.$$

31. **(a)** If the first date of bare ground is 140, then, according to the figure, the first bluebell flower is sighted about day 150, that is $150 - 140 = 10$ days later.

(b) Let x be the first date of bare ground, and let y be the date the first bluebell flower is sighted. Reading coordinates from the graph, we see that date $x = 130$ days after Jan 1 corresponds to date $y = 142$ days after Jan 1, and $x = 170$ days corresponds to $y = 173$ days. Using a difference quotient we have

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{173 - 142}{170 - 130} = 0.775 \text{ days per day}.$$

(c) The slope is positive, so an increase in the x-variable corresponds to an increase in the y-variable. These variables represents days in the year, and larger values indicate days that are later in the year. Thus if bare ground first occurs later in the year, then bluebells first flower later in the year. Positive slope means that bluebells flower later when the snow cover lasts longer.

(d) Using the slope, we see that the equation has the form

$$y = b + 0.775x.$$

Substituting $x = 130$ and $y = 142$ we can solve for b.

$$\begin{aligned} b + 0.775(130) &= 142 \\ b &= 41.25. \end{aligned}$$

The equation of the line is

$$y = 41.25 + 0.775x.$$

32. **(a)** We have $m = \dfrac{256}{18} = 14.222$ cm per hour. When the snow started, there were 100 cm on the ground, so

$$f(t) = 100 + 14.222t.$$

(b) The domain of f is $0 \le t \le 18$ hours. The range is $100 \le f(t) \le 356$ cm.

33. **(a)** Substituting into the function $g(t)$:

$$\begin{aligned} g(10) &= 300 + 1.4 \cdot (10) = 314 \text{ ppm} \\ g(20) &= 300 + 1.4 \cdot (20) = 328 \text{ ppm} \\ g(100) &= 300 + 1.4 \cdot (100) = 440 \text{ ppm} \end{aligned}$$

(b) Substituting into $h(t)$:

$$h(10) = 300 + 1.5 \cdot (10) = 315 \text{ ppm}$$
$$h(20) = 300 + 1.5 \cdot (20) = 330 \text{ ppm}$$
$$h(100) = 300 + 1.5 \cdot (100) = 450 \text{ ppm}$$

(c) They differ in slope, 1.4 vs. 1.5 ppm per year. This creates a larger difference between $g(t)$ and $h(t)$ for larger values of t:
For $t = 10$, the difference is $h(10) - g(10) = 315 - 314 = 1$ ppm.
For $t = 100$, the difference is $h(100) - g(100) = 450 - 440 = 10$ ppm.

34. (a) A linear function has the form

$$P = b + mt$$

where t is in years since 2015. The slope is

$$\text{Slope } = \frac{\Delta P}{\Delta t} = -0.6.$$

The function takes the value 13.5 in the year 2015 (when $t = 0$). Thus, the formula is

$$P = 13.5 - 0.6t.$$

(b) In 2019, we have $t = 4$. We predict $P = 13.5 - 0.6 \cdot 4 = 11.1$, that is, there are 11.1% below the poverty level in 2019.
(c) The difference is $11.1 - 10.5 = 0.6\%$; the prediction is too high.

35. (a) We want $P = b + mt$, where t is in years since 2005. The slope is

$$\text{Slope } = m = \frac{\Delta P}{\Delta t} = \frac{2170 - 2048}{14 - 0} = 8.71429$$

Since $P = 2048$ when $t = 0$, the vertical intercept is 2048 so

$$P = 2048 + 8.71429t.$$

(b) The slope tells us that world grain production has been increasing at a rate of about 8.7 million tons per year.
(c) The vertical intercept tells us that world grain production was 2048 million tons in 2005 (when $t = 0$).
(d) We substitute $t = 15$ to find $P = 2048 + 8.71429t \cdot 15 = 2178.714$ million tons of grain.
(e) Substituting $P = 2250$ and solving for t, we have

$$2250 = 2048 + 8.71429t$$
$$202 = 8.71429t$$
$$t = 23.180.$$

Grain production is predicted to reach 2250 million tons during the year 2028.

36. (a) Let G be the increase in average global temperature above the benchmark average between 1951 and 1980 as a function of year, t. The slope of the line joining the two points (2010, 0.72) and (2020, 1.02) is

$$m = \frac{\Delta G}{\Delta t} = \frac{1.02 - 0.72}{2020 - 2010} = \frac{0.3}{10} = 0.03^\circ \text{ C/year.}$$

We use the point-slope form to find the equation of the line. We substitute the point (2010, 0.72) and the slope $m = 0.03$ into the equation:

$$G - G_0 = m(t - t_0)$$
$$G - 0.72 = 0.03(t - 2010)$$
$$G - 0.72 = 0.03t - 60.3$$
$$G = 0.03t - 59.58.$$

The equation of the line is $G = 0.03t - 59.58$.
(b) To calculate the global average temperature increase over the benchmark predicted for the year 2025, we substitute $t = 2025$ into the equation of the line and calculate:

$$G = 0.03(2025) - 59.58 = 1.17.$$

The formula predicts that in the year 2025, the average global temperature will have increased by 1.17°C over the benchmark.

37. **(a)** We are given two points: $S = 0.9$ when $t = 0$ and $S = 18.84$ when $t = 13$. We use these two points to find the slope. Since $S = f(t)$, we know the slope is $\Delta S/\Delta t$:

$$m = \frac{\Delta S}{\Delta t} = \frac{18.84 - 0.9}{13 - 0} = 1.38.$$

The vertical intercept (when $t = 0$) is 0.9, so the linear equation is

$$S = 0.9 + 1.38t.$$

(b) The slope is 1.38 million vinyl records per year. Sales of vinyl records were increasing at a rate of 1.38 million vinyl records a year. The vertical intercept is 0.9 million vinyl records and represents the sales in the year 2006.

(c) In the year 2025, we have $t = 19$ and we predict

$$\text{Sales } = 0.9 + 1.38 \cdot 19 = 27.12 \text{ million vinyl records.}$$

38. **(a)** Since the initial value is \$25,000 and the slope is -2000, the value of the vehicle at time t is

$$V(t) = 25000 - 2000t.$$

The cost of repairs has initial value 0 and slope 1500, so

$$C(t) = 1500t.$$

Figure 1.17 shows the graphs of these two functions.

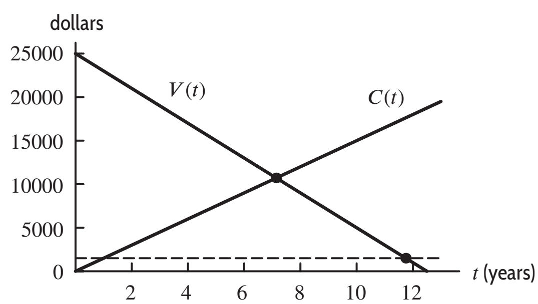


Figure 1.17

(b) The vehicle value equals the repair cost, when $V(t) = C(t)$.

$$\begin{aligned} 25000 - 2000t &= 1500t \\ 25000 &= 3500t \\ 7.143 &= t. \end{aligned}$$

Thus, replace during the eighth year (because the first year ends at $t = 1$). Since 0.143 years is $0.143(12) = 1.7$ months, replacement should take place in the second month of the eighth year.

(c) Since 6% of the orginal value is 1500, the vehicle should be replaced when $V(t) = 1500$.

$$\begin{aligned} 25000 - 2000t &= 1500 \\ 25000 &= 1500 + 2000t \\ 23500 &= 2000t \\ 11.750 &= t. \end{aligned}$$

Thus, in the twelfth year (because the first year ends at $t = 1$). Since 0.75 years is 9 months, replacement should take place at the end of the ninth month of the twelfth year.

39. **(a)** It appears that P is a linear function of d since P decreases by 10 on each interval as d increases by 20.

(b) The slope of the line is

$$m = \frac{\Delta P}{\Delta d} = \frac{-10}{20} = -0.5.$$

We use this slope and the first point in the table to find the equation of the line. We have

$$P = 100 - 0.5d.$$

(c) The slope is -0.5 %/ft. The percent found goes down by 0.5% for every additional foot separating the rescuers.

(d) The vertical intercept is 100%. If the distance separating the rescuers is zero, the percent found is 100%. This makes sense: if the rescuers walk with no distance between them (touching shoulders), they will find everyone.

The horizontal intercept is the value of d when $P = 0$. Substituting $P = 0$, we have

$$\begin{aligned} P &= 100 - 0.5d \\ 0 &= 100 - 0.5d \\ d &= 200. \end{aligned}$$

The horizontal intercept is 200 feet. This is the separation distance between rescuers at which, according to the model, no one is found. This is unreasonable, because even when the searchers are far apart, the search will sometimes be successful. This suggests that at some point, the linear relationship ceases to hold. We have extrapolated too far beyond the given data.

40. **(a)** We find the slope m and intercept b in the linear equation $C = b + mw$. To find the slope m, we use

$$m = \frac{\Delta C}{\Delta w} = \frac{1098.32 - 694.55}{8 - 5} = 134.59 \text{ dollars per ton.}$$

We substitute to find b:

$$\begin{aligned} C &= b + mw \\ 694.55 &= b + (134.59)(5) \\ b &= 21.6 \text{ dollars.} \end{aligned}$$

The linear formula is $C = 21.6 + 134.59w$.

(b) The slope is 134.59 dollars per ton. Each additional ton of waste collected costs \$134.59.

(c) The intercept is \$21.60. The flat fee for waste collection is \$21.60. This is the amount charged even if there is no waste.

41. **(a)** Since the differences in P-values are constant ($\Delta P = 147$) for equally spaced h-values ($\Delta h = 2$), P could be a linear function of h.

(b) To find a formula for $P = f(h)$, we first find the slope:

$$\frac{\Delta P}{\Delta t} = \frac{147}{2} = 73.5 \text{ torr per meter.}$$

Thus, the equation of the line has the form $P = 73.5h + b$, where b is the vertical intercept. To find b, we choose a point from the table: for example, when $h = 6$, we have $P = 1201$. Substituting these values, we have

$$\begin{aligned} P &= 73.5h + b \\ 1201 &= 73.5(6) + b \\ b &= 1201 - 73.5(6) = 760. \end{aligned}$$

Therefore, $P = f(h) = 73.5h + 760$.

The slope is 73.5 torr per meter which means that the pressure increases by 73.5 torr for each additional meter of depth below the surface of the lake. The vertical intercept is 760 torr, which means that the pressure at the surface of the lake is 760 torr.

(c) The pressure at the surface of the lake is $f(0) = 760$ torr. Since we want the value of h when the pressure is twice that at the surface, we solve $f(h) = 2 \cdot 760$:

$$\begin{aligned} 2 \cdot 760 &= f(h) \\ 1520 &= 73.5h + 760 \\ 73.5h &= 760 \\ h &= \frac{760}{73.5} = 10.34. \end{aligned}$$

42. **(a)** We are given two points: $N = 34$ when $l = 11$ and $N = 26$ when $l = 44$. We use these two points to find the slope. Since $N = f(l)$, we know the slope is $\Delta N/\Delta l$:

$$m = \frac{\Delta N}{\Delta l} = \frac{26 - 34}{44 - 11} = -0.2424.$$

We use the point $l = 11$, $N = 34$ and the slope $m = -0.2424$ to find the vertical intercept:

$$\begin{aligned} N &= b + ml \\ 34 &= b - 0.2424 \cdot 11 \\ 34 &= b - 2.67 \\ b &= 36.67. \end{aligned}$$

The equation of the line is

$$N = 36.67 - 0.2424l.$$

(b) The slope is -0.2424 species per degree latitude. In other words, the number of coastal dune plant species decreases by about 0.2424 for every additional degree South in latitude. The vertical intercept is 36.67 species, which represents the number of coastal dune plant species at the equator, if the model (and Australia) extended that far.

(c) See Figure 1.18.

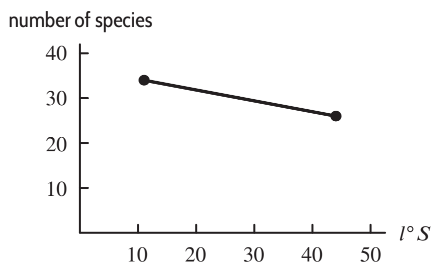


Figure 1.18

43. **(a)** This could be a linear function because w increases by 3.768 as h increases by 4.

(b) We find the slope m and the intercept b in the linear equation $w = b + mh$. We first find the slope m using the first two points in the table. Since we want w to be a function of h, we take

$$m = \frac{\Delta w}{\Delta h} = \frac{80.032 - 76.264}{176 - 172} = 0.942.$$

Substituting the first point and the slope $m = 0.942$ into the linear equation $w = b + mh$, we have $76.264 = b + (0.942)(172)$, so $b = -85.76$. The linear function is

$$w = 0.942h - 85.76.$$

The slope, $m = 0.942$, is in units of kg per cm.

(c) We find the slope and intercept in the linear function $h = b + mw$ using $m = \Delta h/\Delta w$ to obtain the linear function

$$h = 1.0616w + 91.04.$$

Alternatively, we could solve the linear equation found in part (b) for h. The slope, $m = 1.0616$, has units cm per kg.

44. Both formulas are linear, with slope -1, which means that in one year there is a decrease of one beat per minute, so (c) is correct.

45. If M is the male's maximum heart rate and F is the female's, then

$$\begin{aligned} M &= 220 - a \\ F &= 226 - a, \end{aligned}$$

so F is 6 larger than M. Thus, female's maximum heart rate exceeds the male's if they are the same age.

46. If f is the age of the female, and m the age of the male, then if they have the same MHR,

$$226 - f = 220 - m,$$

so

$$f - m = 6.$$

Thus the female is 6 years older than the male.

47. **(a)** If old and new formulas give the same MHR for females, we have

$$226 - a = 206.9 - 0.67a,$$

so

$$a = 57.88 \text{ years}.$$

Thus, a woman's MHR is the same under both methods when she is approximately 58 years old.

For males we need to solve the equation

$$220 - a = 206.9 - 0.67a,$$

so

$$a = 39.7 \text{ years}.$$

Thus, a man's MHR is the same under both methods when he is approximately 40 years old.

(b) The old formula starts at either 226 or 220 and decreases. The new formula starts at 206.9 and decreases slower than the old formula. Thus, the new formula predicts a lower MHR for young people and a higher MHR for older people, so the answer is (ii).

(c) Under the old formula,

$$\text{Heart rate reached } = 0.85(220 - 65) = 131.75 \text{ beats/minute},$$

whereas under the new formula,

$$\text{Heart rate reached } = 0.85(206.9 - 0.67 \cdot 65) = 138.85 \text{ beats/minute}.$$

The difference is $138.85 - 131.75 = 7.1$ beats/minute more under the new formula.

48. If the MHR is linear with age then the rate of change, in beats per minute per year, is approximately the same at any age. Between 0 and 21 years,

$$\text{Rate of change } = -\frac{2}{21} = -0.095 \text{ beats per minute per year},$$

whereas between 0 and 33 years,

$$\text{Rate of change } = -\frac{5}{33} = -0.152 \text{ beats per minute per year}.$$

These are not approximately the same, so are not consistent with MHR being approximately linear with age.

49. If the MHR is linear with age then the slope, in beats per minute per year, would be approximately the same at any age. Between 0 and 21 years,

$$\text{Rate of change } = -\frac{6}{21} = -0.286 \text{ beats per minute per year},$$

whereas between 0 and 33 years,

$$\text{Rate of change } = -\frac{11}{33} = -0.333 \text{ beats per minute per year}.$$

These are not approximately the same, so they are not consistent with MHR being approximately linear with age.

50. **(a)** We have $y = 3.5$ when $u = 0$, which occurs when the unemployment rate does not change. When the unemployment rate is constant, Okun'a law states that US production increases by 3.5% annually.

(b) When unemployment rises from 5% to 8% we have $u = 3$ and therefore

$$y = 3.5 - 2u = 3.5 - 2 \cdot 3 = -2.5.$$

National production for the year decreases by 2.5%.

(c) If annual production does not change, then $y = 0$. Hence $0 = 3.5 - 2u$ and so $u = 1.75$. There is no change in annual production if the unemployment rate goes up 1.75%.

(d) The coefficient -2 is the slope $\Delta y/\Delta u$. Every 1% increase in the unemployment rate during the course of a year results in an additional 2% decrease in annual production for the year.

51. **(a)** The average hourly wage for men is AU\$29.40 and the height premium is 3%, so

$$\text{Slope of men's line} = \frac{(3\%)\ \text{AU\$29.40}}{10\ \text{cm}} = 0.0882\ \text{AU\$/cm}.$$

Since the wage is AU\$29.40 for height $x = 178$, we have

$$\text{Men's wage} = 0.0882(x - 178) + 29.40$$

so

$$\begin{aligned}\text{Men's wage} &= 29.40 + 0.0882x - 0.0882 \cdot 178\\ \text{Men's wage} &= 13.70 + 0.0882x.\end{aligned}$$

(b) The average hourly wage for women is AU\$24.78 and the height premium is 2%, so

$$\text{Slope of women's line} = \frac{(2\%)\ \text{AU\$24.78}}{10\ \text{cm}} = 0.0496\ \text{AU\$/cm}.$$

Since the wage is AU\$24.78 for height $y = 164$, we have

$$\text{Women's wage} = 0.0496(y - 164) + 24.78$$

so

$$\begin{aligned}\text{Women's wage} &= 24.78 + 0.0496y - 0.0496 \cdot 164\\ \text{Women's wage} &= 16.65 + 0.0496y.\end{aligned}$$

(c) For $x = 178$, we are given that

$$\text{Men's average wage} = \text{AU\$29.40}.$$

For $y = 178$, we find

$$\text{Women's average wage} = 16.65 + 0.0496 \cdot 178 = \text{AU\$25.48}.$$

Thus the wage difference is $29.40 - 25.48 =$AU\$3.92.

(d) The line representing men's wages has vertical intercept 13.70 and slope 0.0082; the line representing women's wages has vertical intercept 16.65 and slope 0.0496. Since the vertical intercept of men's wages is lower but the slope is larger, the two lines intersect. Thus, the model predicts there is a height making the two average hourly wages equal. For men and women of the same height, $x = y$. If the average hourly wages are the same

$$\begin{aligned}\text{Men's wage} &= \text{Women's wage}\\ 13.70 + 0.0882x &= 16.65 + 0.0496x\\ x &= \frac{16.65 - 13.70}{0.0882 - 0.0496} = 76.425\ \text{cm}.\end{aligned}$$

However, this height 76.425 cm, or about 30 inches, or 2 foot 6 inches, is an unrealistic height. Thus for all realistic heights, men's average hourly wage is predicted to be larger than women's average hourly wage.

Solutions for Section 1.3

1. The graph shows a concave down function.

2. The graph shows a concave up function.

3. The graph is concave up.

4. This graph is neither concave up or down.

5. As t increases w decreases, so the function is decreasing. The rate at which w is decreasing is itself decreasing: as t goes from 0 to 4, w decreases by 42, but as t goes from 4 to 8, w decreases by 36. Thus, the function is concave up.

6. One possible answer is shown in Figure 1.19.

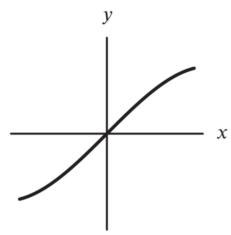


Figure 1.19

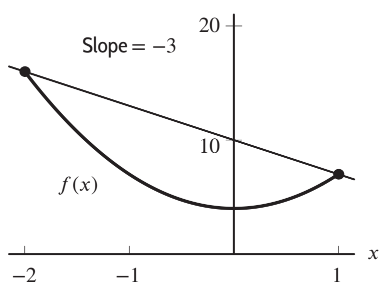
7. The function is increasing and concave up between D and E, and between H and I. It is increasing and concave down between A and B, and between E and F. It is decreasing and concave up between C and D, and between G and H. Finally, it is decreasing and concave down between B and C, and between F and G

8. The average rate of change R between $x = 1$ and $x = 3$ is

$$\begin{aligned} R &= \frac{f(3) - f(1)}{3 - 1} \\ &= \frac{18 - 2}{2} \\ &= \frac{16}{2} \\ &= 8. \end{aligned}$$

9. The average rate of change R between $x = -2$ and $x = 1$ is:

$$R = \frac{f(1) - f(-2)}{1 - (-2)} = \frac{3(1)^2 + 4 - (3(-2)^2 + 4)}{1 + 2} = \frac{7 - 16}{3} = -3.$$



10. When $t = 0$, we have $B = 1000(1.02)^0 = 1000$. When $t = 5$, we have $B = 1000(1.02)^5 = 1104.08$. We have

$$\text{Average rate of change} = \frac{\Delta B}{\Delta t} = \frac{1104.08 - 1000}{5 - 0} = 20.82 \text{ dollars/year.}$$

During this five year period, the amount of money in the account increased at an average rate of \$20.82 per year.

11. In February of 2019, there were 14.4 million square kilometers of Arctic Sea covered by ice.

12. In February of 2009, there were 14.9 million square kilometers of Arctic Sea covered by ice.

13. Over the 10 year period from February 2009 to February 2019, the area of Arctic Sea covered by ice, shrunk, on average, by 40,000 square kilometers per year.

14. On February 1st, 2019, there were about 14.11 million square kilometers of Arctic Sea covered by ice.

15. Over the period between February 1 and March 1, the area of Arctic Sea covered by ice, grew, on average, by about 15,000 square kilometers per day.

16. **(a)** Between 2017 and 2019,

$$\begin{aligned}\text{Change in net sales} &= \text{Net sales in 2019} - \text{Net sales in 2017}\\ &= 16.38 - 15.86\\ &= 0.52 \text{ billion dollars.}\end{aligned}$$

(b) Between 2015 and 2019,

$$\begin{aligned}\begin{array}{c}\text{Average rate of change}\\ \text{in net sales}\end{array} &= \frac{\text{Change in net sales}}{\text{Change in time}}\\ &= \frac{\text{Net sales in 2019} - \text{Net sales in 2015}}{2019-2015}\\ &= \frac{16.38-15.80}{2019-2015}\\ &= 0.145 \text{ billion dollars per year.}\end{aligned}$$

This means that the Gap's net sales increased on average by 0.145 billion (145 million) dollars per year between 2015 and 2019.

(c) The average rate of change was negative from 2015 to 2016, and from 2018 to 2019, when sales decreased.

17. **(a)** Between 2006 and 2018

$$\begin{aligned}\text{Change in number of bicyclists} &= \text{Bicyclists in 2018} - \text{Bicyclists in 2006}\\ &= 47.88 - 39.69 \quad \text{(in millions)}\\ &= 8.19 \text{ million bicyclists.}\end{aligned}$$

(b) The average rate of change R is the change in the number (from part (a)) divided by the change in time.

$$\begin{aligned}R &= \frac{47.88-39.69}{2018-2006}\\ &= \frac{8.19}{12}\\ &= 0.6825 \text{ million bicyclists per year.}\end{aligned}$$

This means that number of bicyclists has increased on average by 682,500 per year between 2006 and 2018.

18. **(a)** The average rate of change is the change in attendance divided by the change in time. Between 2015 and 2019,

$$\begin{aligned}\text{Average rate of change} = \frac{16.67-17.26}{2019-2015} &= -0.1475 \text{ million people per year}\\ &= -147{,}500 \text{ people per year.}\end{aligned}$$

(b) For each of the years from 2015–2019, the annual change in the average attendance, in million people per year, was:

$$\begin{aligned}\text{2015 to 2016} &: 17.79 - 17.26 = 0.53\\ \text{2016 to 2017} &: -0.53\\ \text{2017 to 2018} &: -0.16\\ \text{2018 to 2019} &: -0.43.\end{aligned}$$

(c) We average the four yearly rates of change:

$$\begin{aligned}\text{Average of the four figures in part (b)} &= \frac{-0.53+0.53-0.16-0.43}{4}\\ &= -\frac{0.59}{4} = -0.1475, \text{ which is the same as part (a).}\end{aligned}$$

19. The average velocity over a time period is the change in position divided by the change in time. Since the function $s(t)$ gives the distance of the particle from a point, we find the values of $s(3) = 72$ and $s(10) = 144$. Using these values, we find

$$\text{Average velocity} = \frac{\Delta s(t)}{\Delta t} = \frac{s(10)-s(3)}{10-3} = \frac{144-72}{7} = \frac{72}{7} = 10.286 \text{ cm/sec.}$$

20. The average velocity over a time period is the change in position divided by the change in time. Since the function $s(t)$ gives the distance of the particle from a point, we read off the graph that $s(0) = 1$ and $s(3) = 4$. Thus,

$$\text{Average velocity } = \frac{\Delta s(t)}{\Delta t} = \frac{s(3) - s(0)}{3 - 0} = \frac{4 - 1}{3} = 1 \text{ meter/sec.}$$

21. The average velocity over a time period is the change in position divided by the change in time. Since the function $s(t)$ gives the distance of the particle from a point, we read off the graph that $s(1) = 2$ and $s(3) = 6$. Thus,

$$\text{Average velocity } = \frac{\Delta s(t)}{\Delta t} = \frac{s(3) - s(1)}{3 - 1} = \frac{6 - 2}{2} = 2 \text{ meters/sec.}$$

22. **(a)** Average velocity $= (10 - 5)/1 = 5$ ft/sec.
(b) 0 ft/sec, because the distance is not changing.

23. **(a)** Average velocity $= (-5 - 5)/1 = -10$ ft/sec.
(b) Average velocity $= (5 - (-5))/1 = 10$ ft/sec.

24. **(a)** Since

$$\text{Average velocity } = \frac{f(20) - f(0)}{20 - 0} = \frac{3.6 - 0}{20} = 0.18,$$

the average velocity of the plane is about 0.18 thousand feet per second, or 180 feet per second.

(b) Since

$$\text{Average velocity } = \frac{f(40) - f(20)}{40 - 20} = \frac{5.6 - 3.6}{40 - 20} = 0.1,$$

the average velocity of the plane is about 0.1 thousand feet per second, or 100 feet per second.

25. **(a)** The value of imports was higher in 2020 than in 2005. The 2005 figure is about 1.7 and the 2020 figure is about 2.3, making the 2020 figure 0.6 trillion dollars higher than the 2005 figure.
(b) The average rate of change R is the change in values divided by the change in time.

$$\begin{aligned} R &= \frac{2.3 - 1.7}{2020 - 2005} \\ &= \frac{0.6}{15} \\ &= 0.04 \text{ trillion dollars per year.} \end{aligned}$$

Between 2005 and 2020, the value of US imports has increased by an average of about 40 billion dollars per year.

26. **(a)** During the years 1997 through 2016 the use of the internet increased dramatically, replacing earlier newspaper readership which declined. Hence graph A records newspaper ad revenue and graph B gives internet ad revenue.
(b) We compute average rates of change using difference quotients. Newspaper ad revenues were about 50 billion dollars in 1997, corresponding to $t = 0$, and about 15 billion dollars in 2016, corresponding to $t = 19$. So

$$\text{Average rate newspapers} = \frac{15 - 50}{2016 - 1997} = -1.842.$$

The average rate of change is negative because newspaper revenue decreased. Newspaper ad revenue decreased at an average rate of about 1.8 billion dollars per year.

Internet ad revenues were about 1 billion dollars in 1997 and 60 billion dollars in 2016. So

$$\text{Average rate internet} = \frac{60 - 1}{2016 - 1997} = 3.105.$$

The average rate of change is positive because internet ad revenue increased. Internet ad revenue increased at an average rate of about 3.1 billion dollars per year.

27. **(a)** (i) The average velocity of the car over the interval $0 \le t \le 0.5$ is

$$\frac{10 - 0}{0.5 - 0} = 20 \text{ miles per hour.}$$

(ii) The average velocity over the interval $0.5 \le t \le 1$ is

$$\frac{40 - 10}{1 - 0.5} = 60 \text{ miles per hour.}$$

(iii) The average velocity over the interval $3 \leq t \leq 3.5$ is

$$\frac{25-65}{3.5-3} = -80 \text{ miles per hour.}$$

(iv) The average velocity over the interval $3.5 \leq t \leq 4$ is

$$\frac{0-25}{4-3.5} = -50 \text{ miles per hour.}$$

(b) The average velocity over the time interval $0 \leq t \leq 5$ is given by

$$\frac{0-0}{5-0} = 0 \text{ miles per hour.}$$

This means that, after 5 hours, the car has returned home, the same position it started at when $t = 0$.

28. **(a)** We have

$$\begin{aligned}
\text{Average rate of change of } P &= \frac{7.34-6.51}{2015-2005} = 0.083 \text{ billion people per year.}\\
\text{Average rate of change of } A &= \frac{90.8-59.7}{2015-2005} = 3.1 \text{ million cars per year.}\\
\text{Average rate of change of } C &= \frac{2168-91}{2015-2005} = 0.444 \text{ billion subscribers per year.}
\end{aligned}$$

(b) (i) The number of people is increasing faster since the population is increasing at 83 million per year and car production is increasing at about 3 million per year.

(ii) The number of cell phone subscribers is increasing faster since the population is increasing at 83 million per year and the number of cell phone subscribers is increasing at about 444 million per year.

29. **(a)** Between 2007 and 2018, we have

$$\begin{aligned}
\text{Change in sales} &= \text{Sales in 2018} - \text{Sales in 2007}\\
&= 64.66 - 39.47\\
&= 25.19 \text{ billion dollars.}
\end{aligned}$$

(b) Over the same period, we have

$$\begin{aligned}
\begin{array}{c}\text{Average rate of change in sales}\\ \text{between 2007 and 2018}\end{array} &= \frac{\text{Change in sales}}{\text{Change in time}}\\
&= \frac{\text{Sales in 2018} - \text{Sales in 2007}}{2018-2007}\\
&= \frac{64.66-39.47}{2018-2007}\\
&= 2.29 \text{ billion dollars per year.}
\end{aligned}$$

This means that Pepsico's sales increased on average by 2.29 billion dollars per year between 2007 and 2018.

30. **(a)** Negative. Rain forests are being continually destroyed to make way for housing, industry and other uses.

(b) Positive. Virtually every country has a population which is increasing, so the world's population must also be increasing overall.

(c) Negative. Since cell phones became widely available in the 1990s, the number of pay phones has dropped every year to almost zero today.

(d) Negative. As time passes and more sand is eroded, the height of the sand dune decreases.

(e) Positive. As time passes the price of just about everything tends to increase.

31. **(a)** The average rate of change R is the difference in amounts divided by the change in time.

$$\begin{aligned}
R &= \frac{468-728}{2019-2006}\\
&= \frac{-260}{13}\\
&= -20 \quad \text{million pounds/yr.}
\end{aligned}$$

This means that in the years between 2006 and 2019, the production of tobacco decreased at a rate of approximately 20 million pounds per year.

(b) To have a positive rate of change, the production has to increase during one of these 1 year intervals. Looking at the data, we can see that between 2006 and 2007, production of tobacco increased by 60 million pounds. Thus, the average rate of change is positive between 2003 and 2004.

Likewise, the annual average rate of change is positive between 2007 and 2008, between 2008 and 2009, between 2011 and 2012, between 2013 and 2014, and between 2016 and 2017. Nevertheless, the average rate of change over the period 2006 to 2019 is negative.

32. **(a)** This function is increasing and the graph is concave down.

(b) From the graph, we estimate that when $t = 5$, we have $L = 70$ and when $t = 15$, we have $L = 130$. Thus,

$$\text{Average rate of change of length } = \frac{\Delta L}{\Delta t} = \frac{130 - 70}{15 - 5} = 6 \text{ cm/year.}$$

During this ten year period, the sturgeon's length increased at an average rate of 6 centimeters per year.

33. Between 2000 and 2020:

$$\begin{aligned}\text{Average rate of change} &= \frac{160.74 - 142.58}{20}\\ &= \frac{18.16}{20}\\ &= 0.908 \quad \text{million people/year.}\end{aligned}$$

This means that, between 2000 and 2020, the labor force increased by an average of 908,000 workers per year. Between 2000 and 2010:

$$\begin{aligned}\text{Average rate of change} &= \frac{153.89 - 142.58}{10}\\ &= \frac{11.31}{10}\\ &= 1.131 \quad \text{million people/year.}\end{aligned}$$

This means that, between 2000 and 2010, the labor force increased by an average of 1,131,000 workers per year. Between 2010 and 2020:

$$\begin{aligned}\text{Average rate of change} &= \frac{160.74 - 153.89}{10}\\ &= \frac{6.85}{10}\\ &= 0.685 \quad \text{million people/year.}\end{aligned}$$

This means that, between 2010 and 2020, the labor force increased by an average of about 685,000 workers per year.

34. Between 1950 and 2016, we have

$$\text{Average rate of change} = \frac{\text{Change in marine catch}}{\text{Change in years}} = \frac{90.91 - 18.71}{2016 - 1950} = 1.09 \text{ million tons/year.}$$

Between 1950 and 2016, marine catch increased at an average rate of 1.09 million tons each year.

35. **(a)** The change is given by

$$\begin{aligned}\text{Change between 2013 and 2019} &= \text{Revenues in 2019} - \text{Revenues in 2013}\\ &= 137.2 - 138.79\\ &= -1.59 \text{ billion dollars.}\end{aligned}$$

(b) The rate of change is given by

$$\begin{aligned}\begin{matrix}\text{Average rate of change}\\ \text{between 2013 and 2019}\end{matrix} &= \frac{\text{Change in revenues}}{\text{Change in time}}\\ &= \frac{\text{Revenues in 2019} - \text{Revenues in 2013}}{2019 - 2013}\\ &= \frac{137.2 - 138.79}{2019 - 2013}\\ &= -0.265 \text{ billion dollars per year.}\end{aligned}$$

This means that General Motors' revenues decreased on average by 0.265 billion dollars per year, or 265 million dollars per year, between 2013 and 2019.

(c) To have a positive rate of change, the revenue has to increase during one of these intervals. Looking at the data, we can see that between 2015 and 2016, revenues increased by 13.45 billion dollars. Thus the average rate of change is positive between 2015 and 2016.

Likewise, the annual average rate of change is positive between 2017 and 2018.

36. We have

$$\text{Average rate of change} = \frac{88.4 - 104.1}{2019 - 2010} = \frac{-15.7}{9} = -1.744 \text{ million households/year.}$$

During this 9-year period, the number of US households with cable television decreased, on average, by 1.74 million households per year.

37. **(a)** As time passes after a cigarette is smoked, the nicotine level in the body decreases, so the average rate of change is negative. The slope of a secant line between $t = 0$ and $t = 3$ is negative.

(b) The nicotine level at $t = 0$ is $N = 0.4$ and the nicotine level at $t = 3$ is about $N = 0.14$. We have

$$\text{Average rate of change} = \frac{f(3) - f(0)}{3 - 0} = \frac{0.14 - 0.4}{3 - 0} = -0.087 \text{ mg/hour.}$$

During this three hour period, nicotine decreased at an average rate of 0.087 mg of nicotine per hour.

38. Between 1804 and 1927, the world's population increased 1 billion people in 123 years, for an average rate of change of $1/123$ billion people per year. We convert this to people per minute:

$$\frac{1{,}000{,}000{,}000}{123} \text{ people/year} \cdot \frac{1}{60 \cdot 24 \cdot 365} \text{ years/minute} = 15.468 \text{ people/minute.}$$

Between 1804 and 1927, the population of the world increased at an average rate of 15 people per minute. Similarly, we find the following:
Between 1927 and 1960, the increase was 58 people per minute.
Between 1960 and 1974, the increase was 136 people per minute.
Between 1974 and 1987, the increase was 146 people per minute.
Between 1987 and 1999, the increase was 159 people per minute.
Between 1999 and 2012, the increase was 146 people per minute.

39. **(a)** (i) Between the 6^{th} and 8^{th} minutes the concentration changes from 0.336 to 0.298 so we have

$$\text{Average rate of change} = \frac{0.298 - 0.336}{8 - 6} = -0.019 \text{ (mg/ml)/min.}$$

(ii) Between the 8^{th} and 10^{th} minutes we have

$$\text{Average rate of change} = \frac{0.266 - 0.298}{10 - 8} = -0.016 \text{ (mg/ml)/min.}$$

(b) The signs are negative because the concentration is decreasing. The magnitude of the decrease is decreasing because, as the concentration falls, the rate at which it falls decreases.

40. **(a)** The muscle contracts at 30 cm/sec when it is not pulling any load at all.

(b) If the load is 4 kg, the contraction velocity is zero. Thus, the maximum load the muscle can move is 4 kg.

41. **(a)** The contraction velocity for 1 kg is about 13 cm/sec and for 3 kg it is about 2 cm/sec. The change is $2 - 13 = -11$ cm/sec.

(b) Using the answer to part (a), we have

$$\text{Average rate of change} = \frac{2 - 13}{3 - 1} = -5.5 \text{ (cm/sec)/kg.}$$

42. **(a)** Between 2005 and 2020

$$\begin{aligned}\text{Change in sales} &= \text{Sales in 2020} - \text{Sales in 2005}\\ &= 77.87 - 38.8\\ &= 39.07 \text{ billion dollars.}\end{aligned}$$

(b) Between 2005 and 2020

$$\begin{matrix}\text{Average rate of change}\\ \text{in sales}\end{matrix} = \frac{\text{Change in sales}}{\text{Change in time}}$$

$$= \frac{\text{Sales in 2020} - \text{Sales in 2005}}{2020 - 2005}$$
$$= \frac{77.87 - 38.3}{15}$$
$$= 2.60 \text{ billion dollars per year.}$$

This means that Intel's sales increased on average by 2.60 billion dollars per year between 2005 and 2020.

43. **(a)** (i) $f(2001) = 272$

(ii) $f(2019) = 607$

(b) The average yearly increase is the rate of change.

$$\text{Yearly increase} = \frac{f(2019) - f(2001)}{2019 - 2001} = \frac{607 - 272}{18} = \frac{335}{18} = 18.611 \text{ billionaires per year.}$$

So on average, there were between 18 and 19 more billionaires in the US each year.

(c) Since we assume the rate of increase remains constant, we use a linear function with slope $335/18 = 18.611$ billionaires per year. The equation is

$$f(t) = b + \frac{335}{18}t$$

where $f(2001) = 272$, so

$$272 = b + \frac{335}{18}(2001)$$
$$b = -36{,}968.833.$$

Thus, $f(t) = 18.611t - 36{,}968.833$.

44. **(a)** See Figure 1.20.

(b) The average velocity between two times can be represented by the slope of the line joining these points on the position curve. In Figure 1.21, the average velocity between $t = 0$ and $t = 3$ is equal to the slope of l_1, while the average velocity between $t = 3$ and $t = 6$ is equal to the slope of l_2. We can see that the slope of l_2 is greater than the slope of l_1 and so the average velocity between $t = 3$ and $t = 6$ is greater.

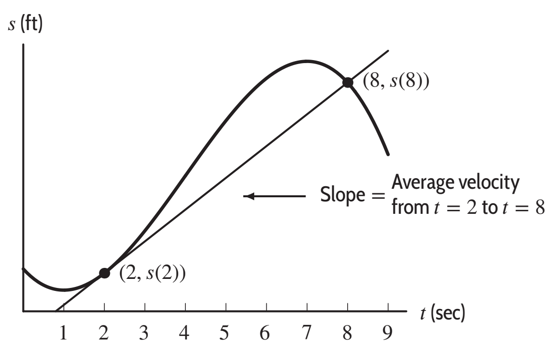


Figure 1.20

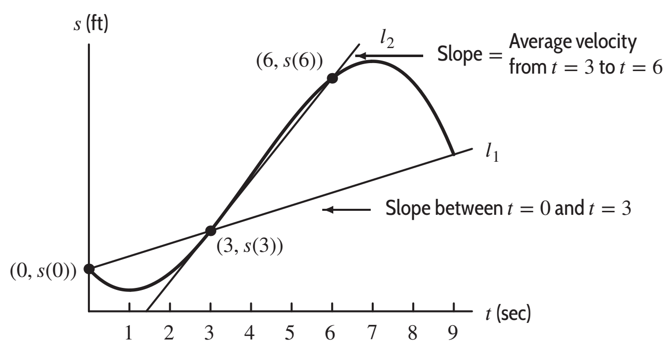


Figure 1.21

(c) Since the average velocity can be represented as the slope of the line between the two values given, we just have to look at the slope of the line passing through $(6, s(6))$ and $(9, s(9))$. We notice that the distance traveled at $t = 6$ is greater than the distance traveled at $t = 9$. Thus, the line between those two points will have negative slope, and thus, the average velocity will be negative.

45. **(a)** If the lizard were running faster and faster, the graph would be concave up. In fact, the graph looks linear; it is not concave up or concave down. The lizard is running at a relatively constant velocity during this short time interval.

(b) During this 0.8 seconds, the lizard goes about 2.1 meters, so the average velocity is about $2.1/0.8 = 2.6$ meters per second.

46. We know that a function is linear if its slope is constant. A function is concave up if the slope increases as t gets larger. And a function is concave down if the slope decreases as t gets larger. Looking at the points $t = 10$, $t = 20$ and $t = 30$ we see that the slope of $F(t)$ decreases since

$$\frac{F(20) - F(10)}{20 - 10} = \frac{22 - 15}{10} = 0.7$$

while

$$\frac{F(30)-F(20)}{30-20}=\frac{28-22}{10}=0.6.$$

Looking at the points $t=10$, $t=20$ and $t=30$ we see that the slope of $G(t)$ is constant

$$\frac{G(20)-G(10)}{20-10}=\frac{18-15}{10}=0.3$$

and

$$\frac{G(30)-G(20)}{30-20}=\frac{21-18}{10}=0.3.$$

Also note that the slope of $G(t)$ is constant everywhere.

Looking at the points $t=10$, $t=20$ and $t=30$ we see that the slope of $H(t)$ increases since

$$\frac{H(20)-H(10)}{20-10}=\frac{17-15}{10}=0.2$$

while

$$\frac{H(30)-H(20)}{30-20}=\frac{20-17}{10}=0.3.$$

Thus $F(t)$ is concave down, $G(t)$ is linear, and $H(t)$ is concave up.

47. Between 0 and 21 years,

$$\text{Average rate of change} = -\frac{9}{21} = -0.429 \text{ beats per minute per year,}$$

whereas between 0 and 33 years,

$$\text{Average rate of change} = -\frac{26}{33} = -0.788 \text{ beats per minute per year.}$$

Because the average rate of change is negative, a decreasing function is suggested. Also, as age increases, the average rate of change decreases, suggesting the graph of the function is concave down. (Since the average rate of change is negative and increasing in absolute value, this rate is decreasing.)

48. Velocity is the slope of the graph of distance against time, so we draw a graph with a slope that starts out small and increases (so the graph gets steeper). Then the slope decreases until the car stops. When the car is stopped, the distance stays constant and the slope is zero. See Figure 1.22.

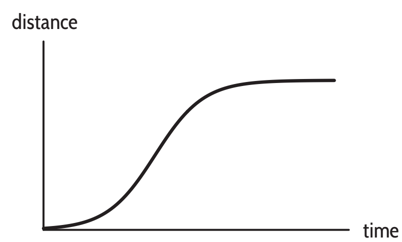


Figure 1.22

49. **(a)** Using a rate of change of 2.25 miles per hour per year, we calculate that the top speed in 2016 is 2.25 mph less than that in 2017, so

$$\text{2016 Top speed} = 277.9 - 2.25 = 275.65 \text{ mph.}$$

In addition, we calculate

$$\text{2020 Top speed} = 3\cdot 2.25 + 277.9 = 284.65 \text{ mph.}$$

(b) The top speed increased $316.1 - 277.9 = 38.2$ mph over three years so

$$\text{Average rate of change} = \frac{38.2}{3} = 12.7 \text{ mph per year.}$$

(c) The difference may have been due to disruptive technological advances, which caused the rate to increase. It may also be due to the fact that the 2.25 mph per year rate is an average. Some years may be higher than 2.25 mph and some may be lower, even though the average is 2.25 mph per year.

(d) The top speed in 2020 is already at 316.1 mph, so half the speed of sound requires a gain of 67.9 mph. If the rate of change is 2.25 mph per year

$$\text{Time for 67.9 mph gain } = \frac{67.9}{2.25} = 30.178 \text{ years.}$$

Thus, top speed is expected to reach 384 mph by 2051 if the rate of change is 2.25 mph per year.

If the rate is 12.7 mph per year as in part (b), then

$$\text{Time for 67.9 mph gain } = \frac{67.9}{12.7} = 5.35 \text{ years}$$

which tells us top speed is expected to reach 384 mph by 2026.

50. We have

$$\text{Relative change } = \frac{15{,}000 - 12{,}000}{12{,}000} = 0.25.$$

The quantity B increases by 25%.

51. We have

$$\text{Relative change } = \frac{450 - 400}{400} = 0.125.$$

The quantity S increases by 12.5%.

52. We have

$$\text{Relative change } = \frac{0.05 - 0.3}{0.3} = -0.833.$$

The quantity W decreases by 83.3%.

53. We have

$$\text{Relative change } = \frac{47 - 50}{50} = -0.06.$$

The quantity R decreases by 6%.

54. The changes are −86.7 in 1931 and −4485.4 in 2008 (they are negative because the Dow Jones Industrial average fell in both years), but the relative changes are

$$\text{Relative change, 1931, } = -\frac{86.7}{164.6} = -52.7\%$$

$$\text{Relative change, 2008, } = -\frac{4485.4}{13{,}261.8} = -33.8\%.$$

The 1931 relative change is bigger in magnitude.

55. The changes are 2.0 million for 1800-1810 and 28.0 million for 1950-1960. The relative changes are

$$\frac{2.0}{5.2} = 38.5\% \quad \text{and} \quad \frac{28.0}{151.3} = 18.5\%.$$

The relative change is bigger for 1800-1810. On the other hand, the change in 1950-1960 represents the baby boom that has had various important consequences for the US in the last 50 years so many would consider the implications of that change greater.

56. The changes are 5 students for the small class and 20 students for the larger class. The relative changes are

$$\frac{5}{5} = 100\% \quad \text{and} \quad \frac{20}{30} = 66.667\%.$$

The relative change for the small class is bigger than the relative change for the large class, but the extra 20 students added to a class of 30 has a potentially larger effect on the class.

57. The changes are 400,000 and 500,000. The relative changes are

$$\frac{400{,}000}{100{,}000} = 400\% \quad \text{and} \quad \frac{500{,}000}{20{,}000{,}000} = 2.5\%.$$

The relative change for the first sales is much larger and would have a much larger effect on the company.

58. **(a)** We have

$$\text{Relative change} = \frac{2000-1000}{1000} = 1 = 100\%.$$

(b) We have

$$\text{Relative change} = \frac{1000-2000}{2000} = -0.5 = -50\%.$$

(c) We have

$$\text{Relative change} = \frac{1{,}001{,}000-1{,}000{,}000}{1{,}000{,}000} = 0.001 = 0.1\%.$$

59. We have

$$\text{Relative change} = \frac{260.64-298.97}{298.97} = -0.128 = -12.8\%.$$

The Dow Jones average dropped by 12.8%.

60. We have

$$\text{Relative change} = \frac{\text{Change in cost}}{\text{Initial cost}} = \frac{55-50}{50} = 0.1.$$

The cost to mail a letter increased by 10%.

61. Let $f(t)$ be the CPI in January of year t. Since the inflation rate is the relative rate of change with $\Delta t = 1$, we have

$$\text{Inflation rate} = \frac{1}{f}\frac{\Delta f}{\Delta t} = \frac{1}{f(t)}\frac{f(t+1)-f(t)}{1} = \frac{f(t+1)-f(t)}{f(t)}$$

we have

$$\begin{aligned}
\text{2014 inflation} &= \frac{237.017-236.736}{236.736} = 0.0012 = 0.12\% \text{ per year}\\
\text{2015 inflation} &= \frac{240.007-237.017}{237.017} = 0.0126 = 1.26\% \text{ per year}\\
\text{2016 inflation} &= \frac{245.120-240.007}{240.007} = 0.0213 = 2.13\% \text{ per year}\\
\text{2017 inflation} &= \frac{251.107-245.120}{245.120} = 0.0244 = 2.44\% \text{ per year}\\
\text{2018 inflation} &= \frac{255.657-251.107}{251.107} = 0.0181 = 1.81\% \text{ per year}
\end{aligned}$$

Rounding to two decimal places, we have

Table 1.2

Year	2014	2015	2016	2017	2018
Inflation (% per year)	0.12%	1.26%	2.13%	2.44%	1.81%

62. **(a)** Let $f(t)$ be the GDP at time t. We are using $\Delta t = 0.25$, so

$$\text{Relative growth rate} = \frac{(\Delta f(t))/\Delta t}{f(t)} \approx \frac{(f(t+0.25)-f(t))/0.25}{f(t)}$$

we have

$$\begin{aligned}
\text{Relative growth rate at 0} &\approx \frac{(14.81-14.67)/0.25}{14.67} = 0.038 = 3.8\% \text{ per year}\\
\text{Relative growth rate at 0.25} &\approx \frac{(14.84-14.81)/0.25}{14.81} = 0.008 = 0.8\% \text{ per year}\\
\text{Relative growth rate at 0.5} &\approx \frac{(14.55-14.84)/0.25}{14.84} = -0.078 = -7.8\% \text{ per year}\\
\text{Relative growth rate at 0.75} &\approx \frac{(14.38-14.55)/0.25}{14.55} = -0.047 = -4.7\% \text{ per year.}
\end{aligned}$$

We have

Table 1.3

t (years since 2008)	0	0.25	0.5	0.75
GDP growth rate (% per year)	3.8	0.8	−7.8	−4.7

(b) Yes, since the relative growth rate was negative in the last two quarters of 2008, the GDP decreased then, and the economy was in recession.

63. (a) We know $P(37)$ is the total number of Covid-19 cases confirmed up to and including day 37 in Minnesota. Similarly, $P(36)$ is the total number of cases confirmed up to and including day 36. Thus the difference $P(37) - P(36)$ is the number of new cases confirmed on day 37 (April 11). This means that $P(37) - P(36) = N(37)$. There were 80 new cases on day $t = 37$ (April 11).

(b) We have

$$\begin{aligned}\frac{P(37) - P(36)}{P(36)} &= 0.074\\ P(37) - P(36) &= 0.074P(36)\\ P(37) &= 1.074P(36).\end{aligned}$$

There are 7.4% more total cases by day 37 than by day 36. In other words, the total number of cases increased by 7.4% from day 36 to day 37 (so, from April 10 to April 11).

(c) The number $P(37) - P(36)$ is the number of new cases confirmed on day 37. Thus, $(P(37) - P(36))/P(37)$ gives the fraction of confirmed cases by day 37 that were first confirmed on day 37. Hence 6.9% of the total cases confirmed by day 37 are new cases on that day.

(d) The sum $P(35) + N(36) + N(37)$ gives the total number of cases up to day 35, plus the number of new cases on day 36, plus the number of new cases on day 37. The total, 1164, is thus the total number of Covid-19 cases up to and including day 37 (April 11). Note that $P(35) + N(36) + N(37) = P(37)$.

64. (a) The quantity $P(102)$ is the total number of Covid-19 cases confirmed up to and including day 102 (June 15) in Maryland, and $P(95)$ is the total number of cases confirmed up to and including day 95 (June 8). The difference $P(102) - P(95)$ is thus the number of cases that were confirmed during the 7 days from $t = 96$ to $t = 102$ (from June 9 to June 15). In other words, there were 4081 cases confirmed in the week ending on day 102 (June 15).

(b) This expression is the average rate of change of $P(t)$ between $t = 95$ and $t = 102$. Thus, the total number of cases was growing on average by 583 cases each day between $t = 95$ and $t = 102$. In other words, there were on average 583 new reported cases each day between $t = 95$ and $t = 102$.

(c) This is the relative rate of change of $P(t)$ between $t = 95$ and $t = 102$. Thus, it says that there were 7.2% more total cases by day 102 than by day 95.

65. (a) The largest time interval was 2015–2017 since the percentage growth rate increased each year from 2015 to 2016 to 2017. This means that from 2015 to 2017 the US consumption of hydroelectric power grew relatively more with each successive year.

(b) The largest time interval was 2012–2015 since the percentage growth rates were negative for each of these four consecutive years. This means that the amount of hydroelectric power consumed by the US industrial sector steadily decreased during this period.

66. (a) We have

$$\text{Relative change in price of candy} = \frac{\text{Change in price}}{\text{Initial price}} = \frac{1.25 - 1}{1} = 0.25.$$

The price of the candy has gone up by 25%.

(b) We have

$$\text{Relative change in quantity of sold candy} = \frac{\text{Change in quantity}}{\text{Initial quantity}} = \frac{2440 - 2765}{2765} = -0.12.$$

The number of candy sold dropped down by 12%.

(c) We have

$$\left|\frac{\text{Relative change in quantity}}{\text{Relative change in price}}\right| = \left|\frac{-0.12}{0.25}\right| = 0.48.$$

So for a 1% increase in a \$1 candy the quantity sold drops by 0.48%.

67. **(a)** The largest time interval was 2016–2018 since the percentage growth rate increased each year from 0.33 to 6.20 to 10.10 from 2016 to 2018. This means US exports of biofuels were growing faster and faster between 2016 and 2018. (Note that the percentage growth rate was a decreasing function of time over 2012–2014 and in 2015–2016.)

(b) The largest time interval was 2015–2018 since the percentage growth rates were positive for each of these consecutive years. This means that the amount of biofuels exported from the US steadily increased during the four years from 2015 to 2018, after a large fall in 2014.

68. **(a)** The largest time interval was 2008–2010 since the percentage growth rate decreased each year from 2008 to 2009 to 2010. This means that from 2008 to 2010 the US price per watt of a solar panel fell relatively more with each successive year.

(b) The largest time interval was 2009–2010 since the percentage growth rates were negative for each of these two consecutive years. This means that the US price per watt of a solar panel decreased during these two years, after an increase in the previous year.

Solutions for Section 1.4

1. **(a)** The company makes a profit when the revenue is greater than the cost of production. Looking at the figure in the problem, we see that this occurs whenever more than roughly 335 items are produced and sold.

(b) If $q = 600$, revenue ≈ 2400 and cost ≈ 1750 so profit is about \$650.

2. **(a)** The fixed costs are the price of producing zero units, or $C(0)$, which is the vertical intercept. Thus, the fixed costs are roughly \$75. The marginal cost is the slope of the line. We know that

$$C(0) = 75$$

and looking at the graph we can also tell that

$$C(30) = 300.$$

Thus, the slope or the marginal cost is

$$\text{Marginal cost} = \frac{300 - 75}{30 - 0} = \frac{225}{30} = 7.50 \text{ dollars per unit.}$$

(b) Looking at the graph it seems that

$$C(10) \approx 150.$$

Alternatively, using what we know from parts (a) and (b) we know that the cost function is

$$C(q) = 7.5q + 75.$$

Thus,

$$C(10) = 7.5(10) + 75 = 75 + 75 = \$150.$$

The total cost of producing 10 items is \$150.

3. See Figure 1.23

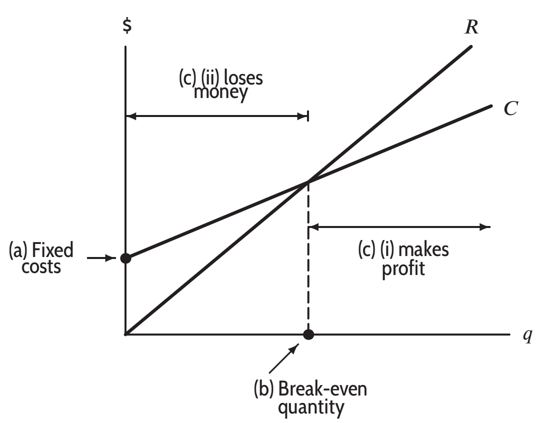


Figure 1.23

4. We know that the fixed cost is the cost that the company would have to pay if no items were produced. Thus

$$\text{Fixed cost} = C(0) = \$5000.$$

We know that the cost function is linear and we know that the slope of the function is exactly the marginal cost. Thus

$$\text{Slope} = \text{Marginal cost} = \frac{5020 - 5000}{5 - 0} = \frac{20}{5} = 4 \text{ dollars per unit produced.}$$

Thus, the marginal cost is 4 dollars per unit produced. We know that since $C(q)$ is linear

$$C(q) = m \cdot q + b,$$

where m is the slope and b is the value of $C(0)$, the vertical intercept. Or in other words, m is equal to the marginal cost and b is equal to the fixed cost. Thus

$$C(q) = 4q + 5000.$$

5. The 5.7 represents the fixed cost of \$5.7 million.

The 0.002 represents the variable cost per unit in millions of dollars. Thus, each additional unit costs an additional \$0.002 million = \$2000 to produce.

6. **(a)** A company with little or no fixed costs would be one that does not need much start-up capital and whose costs are mainly on a per unit basis. An example of such a company is a consulting company, whose major expense is the time of its consultants. Such a company would have little fixed costs to worry about.

(b) A company with marginal cost of zero (or almost) would be one that can produce a product with little or no additional costs per unit. An example is a computer software company. The major expense of such a company is software development, a fixed cost. Additional copies of its software can be very easily made. Thus, its marginal costs are rather small.

7. **(a)** The statement $f(12) = 60$ says that when $p = 12$, we have $q = 60$. When the price is \$12, we expect to sell 60 units.

(b) Decreasing, because as price increases, we expect less to be sold.

8. **(a)** The cost of producing 500 units is

$$C(500) = 6000 + 10(500) = 6000 + 5000 = \$11{,}000.$$

The revenue the company makes by selling 500 units is

$$R(500) = 12(500) = \$6000.$$

Thus, the cost of making 500 units is greater than the money the company will make by selling the 500 units, so the company does not make a profit.

The cost of producing 5000 units is

$$C(5000) = 6000 + 10(5000) = 6000 + 50000 = \$56{,}000.$$

The revenue the company makes by selling 5000 units is

$$R(5000) = 12(5000) = \$60{,}000.$$

Thus, the cost of making 5000 units is less than the money the company will make by selling the 5000 units, so the company does make a profit.

(b) The break-even point is the number of units that the company has to produce so that in selling those units, it makes as much as it spent on producing them. That is, we are looking for q such that

$$C(q) = R(q).$$

Solving we get

$$\begin{aligned} C(q) &= R(q) \\ 6000 + 10q &= 12q \\ 2q &= 6000 \\ q &= 3000. \end{aligned}$$

Thus, if the company produces and sells 3000 units, it will break even.

Graphically, the break-even point, which occurs at (3000, \$36,000), is the point at which the graphs of the cost and the revenue functions intersect. (See Figure 1.24.)

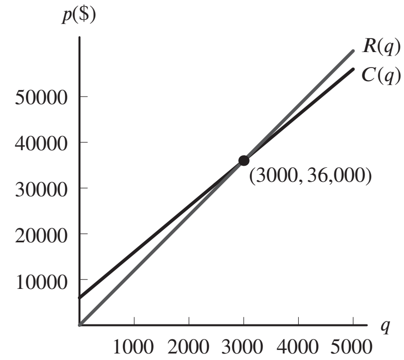


Figure 1.24

9. The 5500 tells us the quantity that would be demanded if the price were \$0. The 100 tells us the change in the quantity demanded if the price increases by \$1. Thus, a price increase of \$1 causes demand to drop by 100 units.

10. Since the price p is on the vertical axis and the quantity q is on the horizontal axis we would like a formula which expresses p in terms of q. Thus

$$\begin{aligned} 75p + 50q &= 300 \\ 75p &= 300 - 50q \\ p &= 4 - \frac{2}{3}q. \end{aligned}$$

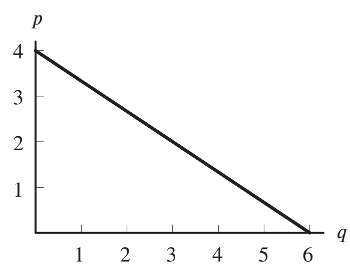


Figure 1.25

From the graph in Figure 1.25 we see that the vertical intercept occurs at $p = 4$ dollars and the horizontal intercept occurs at $q = 6$. This tells us that at a price of \$4 or more nobody would buy the product. On the other hand even if the product were given out for free, no more than 6 units would be demanded.

11. The cost function $C(q) = b + mq$ satisfies $C(0) = 500$, so $b = 500$, and $MC = m = 6$. So

$$C(q) = 500 + 6q.$$

The revenue function is $R(q) = 12q$, so the profit function is

$$\pi(q) = R(q) - C(q) = 12q - 500 - 6q = 6q - 500.$$

12. The cost function $C(q) = b + mq$ satisfies $C(0) = 35{,}000$, so $b = 35{,}000$, and $MC = m = 10$. So

$$C(q) = 35{,}000 + 10q.$$

The revenue function is $R(q) = 15q$, so the profit function is

$$\pi(q) = R(q) - C(q) = 15q - 35{,}000 - 10q = 5q - 35{,}000.$$

13. The cost function $C(q) = b + mq$ satisfies $C(0) = 5000$, so $b = 5000$, and $MC = m = 15$. So

$$C(q) = 5000 + 15q.$$

The revenue function is $R(q) = 60q$, so the profit function is

$$\pi(q) = R(q) - C(q) = 60q - 5000 - 15q = 45q - 5000.$$

14. The cost function $C(q) = b + mq$ satisfies $C(0) = 0$, so $b = 0$. There are 32 ounces in a quart, so 160 in 5 quarts, or 20 cups at 8 ounces per cup. Thus each cup costs the operator 20 cents $= 0.20$ dollars, so $MC = m = 0.20$. So

$$C(q) = 0.20q.$$

The revenue function is $R(q) = 0.25q$. So the profit function is

$$\pi(q) = R(q) - C(q) = 0.25q - 0.20q = 0.05q.$$

15. **(a)** We know that the cost function will be of the form

$$C(q) = b + m \cdot q$$

where m is the slope of the graph and b is the vertical intercept. We also know that the fixed cost is the vertical intercept and the variable cost is the slope. Thus, we have

$$C(q) = 5000 + 30q.$$

We know that the revenue function will take on the form

$$R(q) = pq$$

where p is the price charged per unit. In our case the company sells the chairs at $50 a piece so

$$R(q) = 50q.$$

(b) Marginal cost is $30 per chair. Marginal revenue is $50 per chair.

(c)

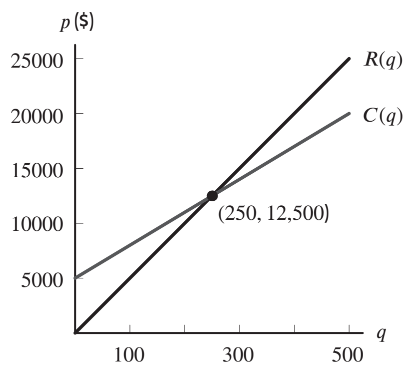


Figure 1.26

(d) We know that the break-even point is the number of chairs that the company has to sell so that the revenue will equal the cost of producing these chairs. In other words, we are looking for q such that

$$C(q) = R(q).$$

Solving we get

$$\begin{aligned} C(q) &= R(q) \\ 5000 + 30q &= 50q \\ 20q &= 5000 \\ q &= 250. \end{aligned}$$

Thus, the break-even point is 250 chairs and $12,500. Graphically, it is the point in Figure 1.26 where the cost function intersects the revenue function.

16. (a) Regardless of the number of rides taken, it costs \$23 to get in. In addition, it costs \$6.00 per ride, so the function $R(n)$ is

$$R(n) = 23 + 6n.$$

(b) Substituting in the values $n = 2$ and $n = 8$ into the formula for $R(n)$, we get

$$R(2) = 23 + 6\cdot = 23 + 12 = \$35.$$

This means that admission and 2 rides costs \$35.

$$R(8) = 23 + 6 \cdot 8 = 23 + 48 = \$71.$$

This means that admission and 8 rides costs \$71.

17. (a) A 20-page hardcover book is \$30 and the price of each additional page is \$1.50. Thus, the price of a hardcover book with q additional pages is

$$P_1(q) = 30 + 1.50q.$$

For the softcover book, the price is \$25 for a 20-page book and \$2 for each additional page. Thus, the price of a softcover book with q additional pages is

$$P_2(q) = 25 + 2q.$$

(b) At 20 additional pages, the price of a hardcover book is

$$P_1(20) = 30 + 1.50(20) = 30 + 30 = \$60.$$

At 20 additional pages, the price of a softcover book is

$$P_2(20) = 25 + 2(20) = 25 + 40 = \$65.$$

Thus, for 20 additional pages, the hardcover book is cheaper.

(c) We find the point q at which

$$P_1(q) = P_2(q).$$

Solving we get

$$\begin{aligned} P_1(q) &= P_2(q) \\ 30 + 1.50q &= 25 + 2q \\ 5 &= 0.50q \\ q &= 10. \end{aligned}$$

Thus, the price of the two formats is the same for 10 additional pages.

We evaluate each price at $q = 10$ to check that they are equal:

$$\begin{aligned} P_1(q) &= 30 + 1.5(10) = \$45 \\ P_2(q) &= 20 + 2(10) = \$45. \end{aligned}$$

18. (a) The fixed cost is the cost that would have to be paid even if nothing was produced. That is, the fixed cost is

$$C(0) = 4000 + 2(0) = \$4000.$$

(b) We know that for every additional item produced, the company must pay another \$2. Thus, the marginal cost is \$2 per chair.

(c) We know that the revenue takes on the form

$$R(q) = p \cdot q$$

where q is the quantity produced and p is the price the company is charging for an item. Thus in our case,

$$p = \frac{R(q)}{q} = \frac{10q}{q} = \$10.$$

So the company is charging \$10 per item.

(d)

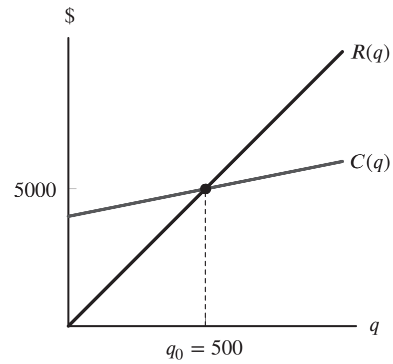


Figure 1.27

We know that the company will make a profit for $q > q_0$ since the line $R(q)$ lies above the line $C(q)$ in that region, so revenues are greater than costs.

(e) Looking at Figure 1.27 we see that the two graphs intersect at the point where

$$q_0 = 500.$$

Thus, the company will break even if it produces 500 units. Algebraically, we know that the company will break even for q_0 such that the cost function is equal to the revenue function at q_0. That is, when

$$C(q_0) = R(q_0).$$

Solving we get

$$\begin{aligned} C(q_0) &= R(q_0) \\ 4000 + 2q_0 &= 10q_0 \\ 8q_0 &= 4000 \\ q_0 &= 500. \end{aligned}$$

To find the total revenue, we evaluate and find:

$$C(500) = R(500) = \$5000.$$

19. (a) We know that the function for the cost of running the theater is of the form

$$C = b + mq$$

where q is the number of customers, m is the variable cost and b is the fixed cost. Thus, the function for the cost is

$$C = 5000 + 11q.$$

We know that the revenue function is of the form

$$R = pq$$

where p is the average revenue per customer per customer. Thus, the revenue function is

$$R = 16q.$$

The theater makes a profit when the revenue is greater than the cost, that is when

$$R > C.$$

Substituting $R = 16q$ and $C = 11q + 5000$, we get

$$\begin{aligned} R &> C \\ 16q &> 11q + 5000 \\ 5q &> 5000 \\ q &> 1000. \end{aligned}$$

Thus, the theater makes a profit when it has more than 1000 customers.

(b) The graph of the two functions is shown in Figure 1.28.

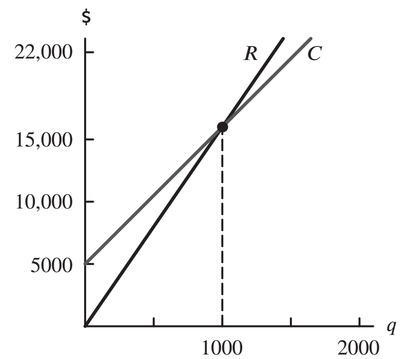


Figure 1.28

20. **(a)** We have $C(q) = 6000 + 2q$ and $R(q) = 5q$ and so

$$\pi(q) = R(q) - C(q) = 5q - (6000 + 2q) = -6000 + 3q.$$

(b) See Figure 1.29. We find the break-even point, q_0, by setting the revenue equal to the cost and solving for q:

$$\begin{aligned} \text{Revenue} &= \text{Cost} \\ 5q &= 6000 + 2q \\ 3q &= 6000 \\ q &= 2000. \end{aligned}$$

The break-even point is $q_0 = 2000$ puzzles. Notice that this is the same answer we get if we set the profit function equal to zero.

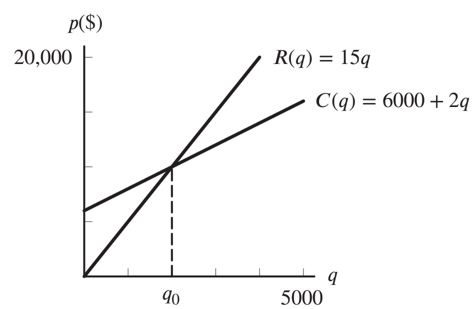


Figure 1.29: Cost and revenue functions for the jigsaw puzzle company

21. The price at any quantity q is the slope of the revenue function at that value of q. To identify the graphs, look at how the slope changes as we move along the line to the right in the direction of increasing q.
Graph (I) corresponds to (c), price increasing with quantity.
Graph (II) corresponds to (b), price increasing for small quantities and decreasing for large quantities.
Graph (III) corresponds to (d), as the quantity increases, the price decreases.
Graph (IV) corresponds to (a), constant price because the curve is a line.

22. **(a)** The cost function is of the form

$$C(q) = b + m \cdot q$$

where m is the variable cost and b is the fixed cost. Since the variable cost is \$20 and the fixed cost is \$650,000, we get

$$C(q) = 650{,}000 + 20q.$$

The revenue function is of the form

$$R(q) = pq$$

where p is the price that the company is charging the buyer for one pair. In our case the company charges \$70 a pair so we get

$$R(q) = 70q.$$

The profit function is the difference between revenue and cost, so

$$\pi(q) = R(q) - C(q) = 70q - (650{,}000 + 20q) = 70q - 650{,}000 - 20q = 50q - 650{,}000.$$

(b) Marginal cost is \$20 per pair. Marginal revenue is \$70 per pair. Marginal profit is \$50 per pair.

(c) We are asked for the number of pairs of shoes that need to be produced and sold so that the profit is larger than zero. That is, we are trying to find q such that

$$\pi(q) > 0.$$

Solving we get

$$\begin{aligned} \pi(q) &> 0 \\ 50q - 650{,}000 &> 0 \\ 50q &> 650{,}000 \\ q &> 13{,}000. \end{aligned}$$

Thus, if the company produces and sells more than 13,000 pairs of shoes, it will make a profit.

23. (a) We know that the function for the value of the robot at time t will be of the form

$$V(t) = m \cdot t + b.$$

We know that at time $t = 0$ the value of the robot is \$15,000. Thus the vertical intercept b is

$$b = 15{,}000.$$

We know that m is the slope of the line. Also at time $t = 10$ the value is \$0. Thus

$$m = \frac{0 - 15{,}000}{10 - 0} = \frac{-15{,}000}{10} = -1500.$$

Thus we get

$$V(t) = -1500t + 15{,}000 \text{ dollars}.$$

(b) The value of the robot in three years is

$$V(3) = -1500(3) + 15{,}000 = -4500 + 15{,}000 = \$10{,}500.$$

24. (a) We know that the function for the value of the tractor will be of the form

$$V(t) = m \cdot t + b$$

where m is the slope and b is the vertical intercept. We know that the vertical intercept is simply the value of the function at time $t = 0$, which is \$50,000. Thus,

$$b = \$50{,}000.$$

Since we know the value of the tractor at time $t = 20$ we know that the slope is

$$m = \frac{V(20) - V(0)}{20 - 0} = \frac{10{,}000 - 50{,}000}{20} = \frac{-40{,}000}{20} = -2000.$$

Thus we get

$$V(t) = -2000t + 50{,}000 \text{ dollars}.$$

(b)

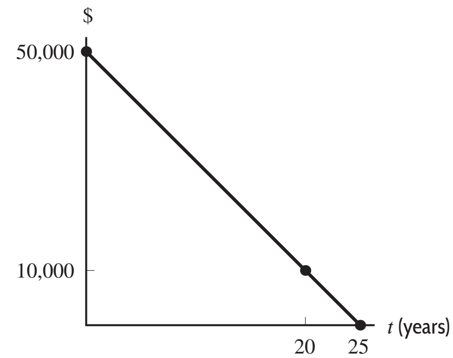


Figure 1.30

(c) Looking at Figure 1.30 we see that the vertical intercept occurs at the point $(0, 50{,}000)$ and the horizontal intercept occurs at $(25, 0)$. The vertical intercept tells us the value of the tractor at time $t = 0$, namely, when it was brand new. The horizontal intercept tells us at what time t the value of the tractor will be \$0. Thus the tractor is worth \$50,000 when it is new, and it is worth nothing after 25 years.

25. **(a)** The slope of the linear function is

$$\text{Slope} = \frac{\Delta V}{\Delta t} = \frac{25{,}000 - 100{,}000}{15 - 0} = -5000.$$

The vertical intercept is $V = 100{,}000$ so the formula is

$$V = 100{,}000 - 5000t.$$

(b) At $t = 5$, we have $V = 100{,}000 - 5000 \cdot 5 = 75{,}000$. In 2020, the bus is worth \$75,000.

(c) The vertical intercept is 100,000 and represents the value of the bus in dollars in 2015. The horizontal intercept is the value of t when $V = 0$. Solving $0 = 100{,}000 - 5000t$ for t, we find $t = 20$. The bus is worth nothing when $t = 20$ years, that is, in the year 2035.

(d) The domain of the function is $0 \le t \le 20$. The bus cannot have negative value.

26. We know that at the point where the price is \$5 per scoop the quantity must be 240. Thus we can fill in the graph as follows:

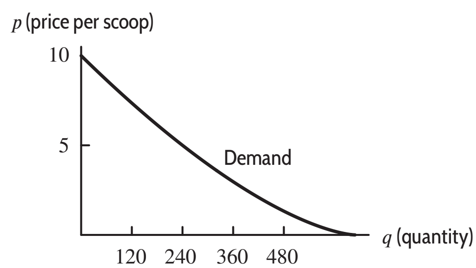


Figure 1.31

(a) Looking at Figure 1.31 we see that when the price per scoop \$2.50, the quantity given by the demand curve is roughly 360 scoops.

(b) Looking at Figure 1.31 we see that when the price per scoop is \$7.50, the quantity given by the demand curve is roughly 120 scoops.

27. **(a)** Since cost is less than revenue for quantities in the table between 20 and 60 units, production appears to be profitable between these values.

(b) Profit = Revenue − Cost is show in Table 1.4. The maximum profit is obtained at a production level of about 40 units.

Table 1.4

Quantity	0	10	20	30	40	50	60	70	80
Cost (\$)	120	400	600	780	1000	1320	1800	2500	3400
Revenue (\$)	0	300	600	900	1200	1500	1800	2100	2400
Profit	−120	−100	0	120	200	180	0	−400	−1000

28. **(a)** We know that as the price per unit increases, the quantity supplied increases, while the quantity demanded decreases. So Table 1.30 of the text is the demand curve (since as the price increases the quantity decreases), while Table 1.31 of the text is the supply curve (since as the price increases the quantity increases.)

(b) Looking at the demand curve data in Table 1.30 we see that a price of \$155 gives a quantity of roughly 14.

(c) Looking at the supply curve data in Table 1.31 we see that a price of \$155 gives a quantity of roughly 24.

(d) Since supply exceeds demand at a price of \$155, the shift would be to a lower price.

(e) Looking at the demand curve data in Table 1.30 we see that if the price is less than or equal to \$143 the consumers would buy at least 20 items.

(f) Looking at the data for the supply curve (Table 1.31) we see that if the price is greater than or equal to \$110 the supplier will produce at least 20 items.

29. **(a)** We know that the equilibrium point is the point where the supply and demand curves intersect. Looking at the figure in the problem, we see that the price at which they intersect is \$10 per unit and the corresponding quantity is 3000 units.

(b) We know that the supply curve climbs upward while the demand curve slopes downward. Thus we see from the figure that at the price of \$12 per unit the suppliers will be willing to produce 3500 units while the consumers will be ready to buy 2500 units. Thus we see that when the price is above the equilibrium point, more items would be produced than the consumers will be willing to buy. Thus the producers end up wasting money by producing that which will not be bought, so the producers are better off lowering the price.

(c) Looking at the point on the rising curve where the price is \$8 per unit, we see that the suppliers will be willing to produce 2500 units, whereas looking at the point on the downward sloping curve where the price is \$8 per unit, we see that the consumers will be willing to buy 3500 units. Thus we see that when the price is less than the equilibrium price, the consumers are willing to buy more products than the suppliers would make and the suppliers can thus make more money by producing more units and raising the price.

30. **(a)** We know that the cost function will be of the form

$$C = b + mq$$

where m is the variable cost and b is the fixed cost. In this case this gives

$$C = 7000 + 10q.$$

We know that the revenue function is of the form

$$R = pq$$

where p is the price per shirt. Thus in this case we have

$$R = 25q.$$

(b) We are given

$$q = 4000 - 40p.$$

We are asked to find the demand when the price is \$25. Substituting $p = 25$ into the equation, we get

$$q = 4000 - 40(25) = 4000 - 1000 = 3000.$$

Given this demand we know that the cost of producing $q = 3000$ shirts is

$$C = 7000 + 10 \cdot 3000 = 7000 + 30000 = \$37{,}000.$$

The revenue from selling $q = 3000$ shirts is

$$R = 25 \cdot 3000 = \$75{,}000.$$

Thus the profit is

$$\pi(25) = R - C$$

or in other words

$$\pi(25) = 75{,}000 - 37{,}000 = \$38{,}000.$$

(c) Since we know that

$$q = 4000 - 40p,$$
$$C = 7000 + 10q,$$

and

$$R = pq,$$

we can write

$$C = 7000_1 0q = 7000 + 10(4000 - 40p) = 7000 + 40{,}000 - 400p = 47{,}000 - 400p$$

and

$$R = pq = p(4000 - 40p) = 4000p - 40p^2.$$

We also know that the profit is the difference between the revenue and the cost so

$$\pi(p) = R - C = 4000p - 40p^2 - (47{,}000 - 400p) = -40p^2 + 3600p - 47{,}000.$$

(d) Looking at Figure 1.32 we see that the maximum profit occurs when the company charges about \$55 per shirt. At this price, the profit is about \$74,000.

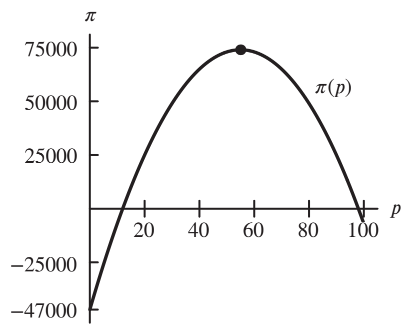


Figure 1.32

31. We know that a formula for passengers versus price will take the form

$$N = mp + b$$

where N is the number of passengers on the boat when the price of a tour is p dollars. We know two points on the line thus we know that the slope is

$$\text{slope} = \frac{650 - 500}{20 - 25} = \frac{150}{-5} = -30.$$

Thus the function will look like

$$N = -30p + b.$$

Plugging in the point $(20, 650)$ we get

$$\begin{aligned} N &= -30p + b \\ 650 &= (-30)(20) + b \\ &= -600 + b \\ b &= 1250. \end{aligned}$$

Thus a formula for the number of passengers as a function of tour price is

$$N = -30p + 1250.$$

32. **(a)** If we think of q as a linear function of p, then q is the dependent variable, p is the independent variable, and the slope $m = \Delta q/\Delta p$. We can use any two points to find the slope. If we use the first two points, we get

$$\text{Slope} = m = \frac{\Delta q}{\Delta p} = \frac{460 - 500}{18 - 16} = \frac{-40}{2} = -20.$$

The units are the units of q over the units of p, or tons per dollar. The slope tells us that, for every dollar increase in price, the number of tons sold every month will decrease by 20.

To write q as a linear function of p, we need to find the vertical intercept, b. Since q is a linear function of p, we have $q = b + mp$. We know that $m = -20$ and we can use any of the points in the table, such as $p = 16$, $q = 500$, to find b. Substituting gives

$$\begin{aligned} q &= b + mp \\ 500 &= b + (-20)(16) \\ 500 &= b - 320 \\ 820 &= b. \end{aligned}$$

Therefore, the vertical intercept is 820 and the equation of the line is

$$q = 820 - 20p.$$

(b) If we now consider p as a linear function of q, we have

$$\text{Slope} = m = \frac{\Delta p}{\Delta q} = \frac{18 - 16}{460 - 500} = \frac{2}{-40} = -\frac{1}{20} = -0.05.$$

The units of the slope are dollars per ton. The slope tells us that, if we want to sell one more ton of the product every month, we should reduce the price by \$0.05.

Since p is a linear function of q, we have $p = b + mq$ and $m = -0.05$. To find b, we substitute any point from the table such as $p = 16$, $q = 500$ into this equation:

$$\begin{aligned} p &= b + mq \\ 16 &= b + (-0.05)(500) \\ 16 &= b - 25 \\ 41 &= b. \end{aligned}$$

The equation of the line is

$$p = 41 - 0.05q.$$

Alternatively, notice that we could have taken our answer to part (a), that is $q = 820 - 20p$, and solved for p.

33. (a) The quantity demanded at a price of \$100 is $q = 120{,}000 - 500(100) = 70{,}000$ units. The quantity supplied at a price of \$100 is $q = 1000(100) = 100{,}000$ units. At a price of \$100, the supply is larger than the demand, so some goods remain unsold and we expect the market to push prices down.

(b) The supply and demand curves are shown in Figure 1.33. The equilibrium price is about $p^* = \$80$ and the equilibrium quantity is about $q^* = 80{,}000$ units. The market will push prices downward from \$100, toward the equilibrium price of \$80. This agrees with the conclusion to part (a) which says that prices will drop.

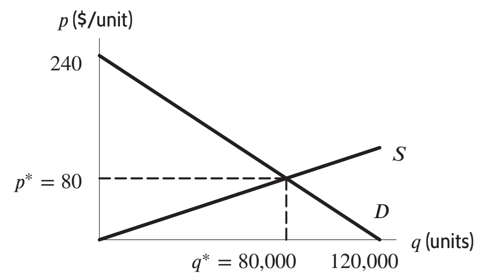


Figure 1.33: Demand and supply curves for a product

34. One possible demand curve is shown in Figure 1.34. Many other answers are possible.

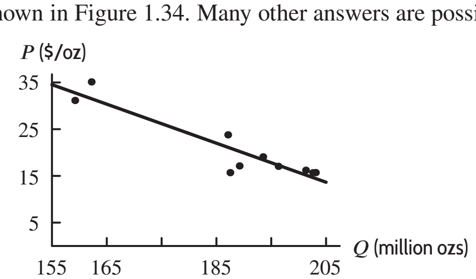


Figure 1.34

35. One possible supply curve is shown in Figure 1.35. Many other answers are possible.

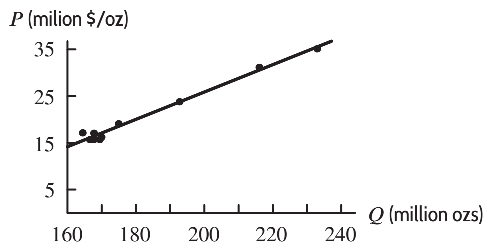


Figure 1.35

36. (a) The total amount spent on compact cars is $20{,}000c$ and the total amount spent on SUV's is $30{,}000s$, so the budget constraint is

$$20{,}000c + 30{,}000s = 720{,}000.$$

(b) When $c = 0$, we have $30{,}000s = 720{,}000$ which gives $s = 24$ SUVs. If the company does not replace any compact cars, it can replace 24 SUVs.

When $s = 0$, we have $20{,}000c = 720{,}000$ which gives $c = 36$ compact cars. If the company does not replace any SUVs, it can replace 36 compact cars.

37. (a) We know that the total amount the company will spend on raw materials will be

$$\text{price for raw materials} = \$100 \cdot m$$

where m is the number of units of raw materials the company will buy. We know that the total amount the company will spend on paying employees will be

$$\text{total employee expenditure} = \$25{,}000 \cdot r$$

where r is the number of employees the company will hire. Since the total amount the company spends is \$500,000, we get

$$25{,}000r + 100m = 500{,}000.$$

(b) Solving for m we get

$$\begin{aligned} 25{,}000r + 100m &= 500{,}000 \\ 100m &= 500{,}000 - 25{,}000r \\ m &= 5000 - 250r. \end{aligned}$$

(c) Solving for r we get

$$\begin{aligned} 25{,}000r + 100m &= 500{,}000 \\ 25{,}000r &= 500{,}000 - 100m \\ r &= 20 - \frac{100}{25{,}000}m \\ &= 20 - \frac{1}{250}m. \end{aligned}$$

38. (a) The amount spent on books will be

$$\text{Amount for books} = \$80 \cdot b$$

where b is the number of books bought. The amount of money spent on outings is

$$\text{money spent on outings} = \$20 \cdot s$$

where s is the number of social outings. Since we want to spend all of the \$2000 budget we end up with

$$80b + 20s = 2000.$$

(b)

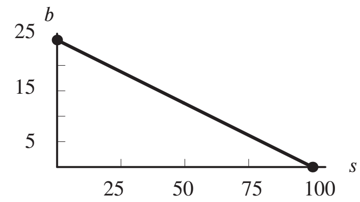


Figure 1.36

(c) Looking at Figure 1.36 we see that the vertical intercept occurs at the point

$$(0, 25)$$

and the horizontal intercept occurs at

$$(100, 0).$$

The vertical intercept tells us how many books we would be able to buy if we wanted to spend all of the budget on books. That is, we could buy at most 25 books. The horizontal intercept tells how many social outings we could afford if we wanted to spend all of the budget on outings. That is, we would be able to go on at most 100 outings.

39. **(a)** If the bakery owner decreases the price, the customers want to buy more. Thus, the slope of $d(q)$ is negative. If the owner increases the price, she is willing to make more cakes. Thus the slope of $s(q)$ is positive.

(b) To determine whether an ordered pair (q, p) is a solution to the inequality, we substitute the values of q and p into the inequality and see whether the resulting statement is true. Substituting the two ordered pairs gives

$$(60, 18): \quad 18 \leq 20 - 60/20, \text{ or } 18 \leq 17. \quad \text{This is false, so } (60, 18) \text{ is not a solution.}$$
$$(120, 12): \quad 12 \leq 20 - 120/20, \text{ or } 12 \leq 14. \quad \text{This is true, so } (120, 12) \text{ is a solution.}$$

The pair (60, 18) is not a solution to the inequality $p \leq 20 - q/20$. This means that the price \$18 is higher than the unit price at which customers would be willing to buy a total of 60 cakes. So customers are not willing to buy 60 cakes at \$18. The pair (120, 12) is a solution, meaning that \$12 is not more than the price at which customers would be willing to buy 120 cakes. Thus customers are willing to buy a total of 120 (and more) cakes at \$12. Each solution (q, p) represents a quantity of cakes q that customers would be willing to buy at the unit price p.

(c) In order to be a solution to both of the given inequalities, a point (q, p) must lie on or below the line $p = 20 - q/20$ and on or above the line $p = 11 + q/40$. Thus, the solution set of the given system of inequalities is the region shaded in Figure 1.37.

A point (q, p) in this region represents a quantity q of cakes that customers would be willing to buy, and that the bakery-owner would be willing to make and sell, at the price p.

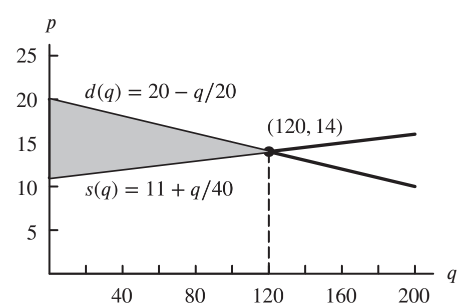


Figure 1.37: Possible cake sales at different prices and quantities

(d) To find the rightmost point of this region, we need to find the intersection point of the lines $p = 20 - q/20$ and $p = 11 + q/40$. At this point, p is equal to both $20 - q/20$ and $11 + q/40$, so these two expressions are equal to each other:

$$\begin{aligned} 20 - \frac{q}{20} &= 11 + \frac{q}{40} \\ 9 &= \frac{q}{20} + \frac{q}{40} \\ 9 &= \frac{3q}{40} \\ q &= 120. \end{aligned}$$

Therefore, $q = 120$ is the maximum number of cakes that can be sold at a price at which customers are willing to buy them all, and the owner of the bakery is willing to make them all. The price at this point is $p = 20 - 120/20 = 14$ dollars. (In economics, this price is called the *equilibrium price*, since at this point there is no incentive for the owner of the bakery to raise or lower the price of the cakes.)

40. **(a)** See Figure 1.38.

(b) If the slope of the supply curve increases then the supply curve will intersect the demand curve sooner, resulting in a higher equilibrium price p_1 and lower equilibrium quantity q_1. Intuitively, this makes sense since if the slope of the supply curve increases. The amount produced at a given price decreases. See Figure 1.39.

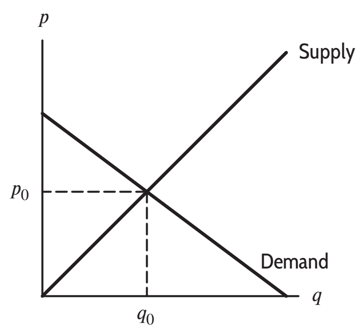


Figure 1.38

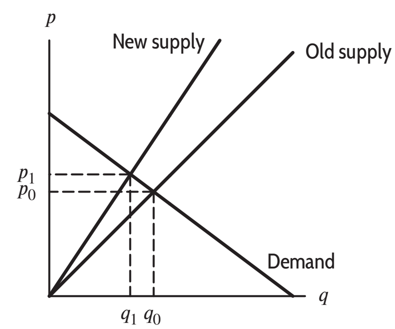


Figure 1.39

(c) When the slope of the demand curve becomes more negative, the demand function will decrease more rapidly and will intersect the supply curve at a lower value of q_1. This will also result in a lower value of p_1 and so the equilibrium price p_1 and equilibrium quantity q_1 will decrease. This follows our intuition, since if demand for a product lessens, the price and quantity purchased of the product will go down. See Figure 1.40.

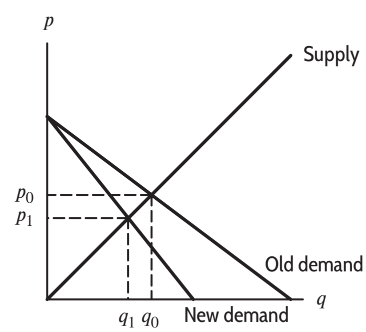


Figure 1.40

41. The quantity demanded for a product is $q = 4$ when the price is $p = 50$. The demand for the product is $q = 3.9$ when the price is $p = 51$. The relative change in demand is

$$\frac{3.9 - 4}{4} = -0.025$$

and the relative change in price is

$$\frac{51 - 50}{50} = 0.02.$$

Hence

$$\left|\frac{\text{Relative change in demand}}{\text{Relative change in price}}\right| = \left|\frac{-0.025}{0.02}\right| = 1.25.$$

A 1% increase in the price resulted in a 1.25% decrease in the demand.

42. The original demand equation, $q = 100 - 5p$, tells us that

$$\text{Quantity demanded} = 100 - 5\left(\begin{array}{c}\text{Amount per unit}\\ \text{paid by consumers}\end{array}\right).$$

The consumers pay $p + 2$ dollars per unit because they pay the price p plus \$2 tax. Thus, the new demand equation is

$$q = 100 - 5(p + 2) = 90 - 5p.$$

See Figure 1.41.

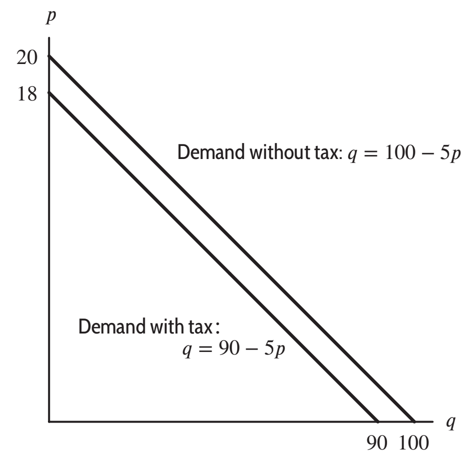


Figure 1.41

43. The original supply equation, $q = 4p - 20$, tells us that

$$\text{Quantity supplied} = 4\left(\begin{array}{c}\text{Amount per unit}\\ \text{received by suppliers}\end{array}\right) - 20.$$

The suppliers receive only $p-2$ dollars per unit because \$2 goes to the government as taxes. Thus, the new supply equation is

$$q = 4(p - 2) - 20 = 4p - 28.$$

See Figure 1.42.

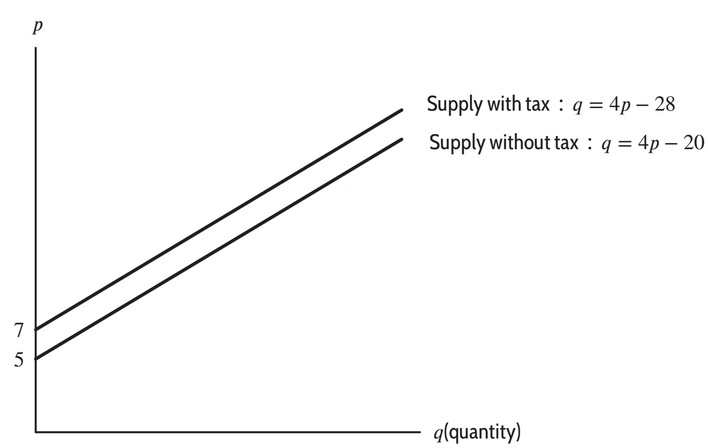


Figure 1.42

44. Before the tax is imposed, the equilibrium is found by solving the equations $q = 0.5p - 25$ and $q = 165 - 0.5p$. Setting the values of q equal, we have

$$\begin{aligned} 0.5p - 25 &= 165 - 0.5p \\ p &= 190 \text{ dollars} \end{aligned}$$

Substituting into one of the equations for q, we find $q = 0.5(190) - 25 = 70$ units. Thus, the pre-tax equilibrium is

$$p = \$190, \quad q = 70 \text{ units}.$$

The original supply equation, $q = 0.5p - 25$, tells us that

$$\text{Quantity supplied } = 0.5\left(\begin{array}{c}\text{Amount per unit}\\ \text{received by suppliers}\end{array}\right) - 25.$$

When the tax is imposed, the suppliers receive only $p - 8$ dollars per unit because \$8 goes to the government as taxes. Thus, the new supply curve is

$$q = 0.5(p - 8) - 25 = 0.5p - 29.$$

The demand curve is still

$$q = 165 - 0.5p.$$

To find the equilibrium, we solve the equations $q = 0.5p - 29$ and $q = 165 - 0.5p$. Setting the values of q equal, we have

$$\begin{aligned} 0.5p - 29 &= 165 - 0.5p \\ p &= 194 \text{ dollars} \end{aligned}$$

Substituting into one of the equations for q, we find that $q = 0.5(194) - 29 = 68$ units. Thus, the post-tax equilibrium is

$$p = \$194, q = 68 \text{ units}.$$

45. **(a)** The equilibrium price and quantity occur when demand equals supply. If we graph these functions on the same axes, we get Figure 1.43.

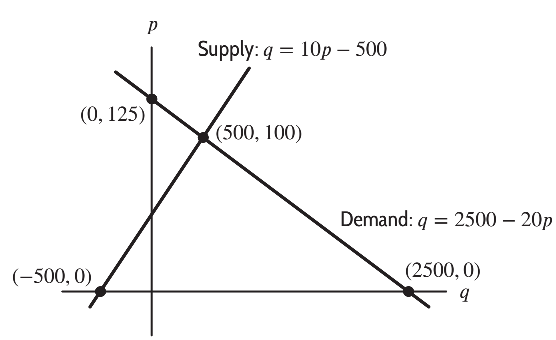


Figure 1.43

We can see from the graph in Figure 1.43 that the supply and demand curves intersect at the point (500, 100). The equilibrium price is \$100 and the equilibrium quantity is 500. This answer can also be obtained algebraically, by setting the values of q equal and solving

$$\begin{aligned} 2500 - 20p &= 10p - 500 \\ 3000 &= 30p \\ p &= 100. \\ q &= 2500 - 20(100) = 500. \end{aligned}$$

(b) The \$6 tax imposed on suppliers has the effect of moving the supply curve upward by \$6.

The original supply equation, $q = 10p - 500$, tells us that

$$\text{Quantity supplied} = 10\begin{pmatrix}\text{Amount received per}\\ \text{unit by suppliers}\end{pmatrix} - 500.$$

The suppliers receive only $p - 6$ dollars per unit because \$6 goes to the government as taxes. Thus, the new supply equation is

$$q = 10(p - 6) - 500 = 10p - 560.$$

The demand equation $q = 2500 - 20p$ is unchanged.

We can find the new equilibrium price and quantity by graphing the new supply equation with the demand equation.

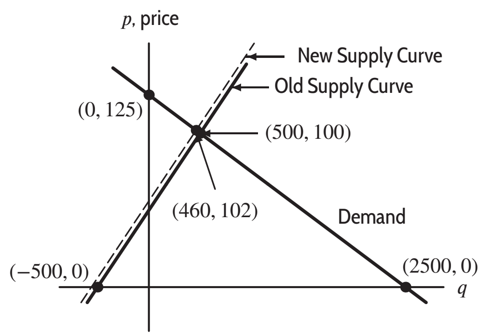


Figure 1.44: Specific tax shifts supply curve

From Figure 1.44, we see that the new equilibrium price (including tax) is \$102 and the new equilibrium quantity is 460 units. We can also obtain these results algebraically:

$$\begin{aligned}2500 - 20p &= 10p - 560\\ 3060 &= 30p\\ p &= 102,\end{aligned}$$

so $q = 10(102) - 560 = 460$.

(c) The tax paid by the consumer is \$2, since the new equilibrium price of \$102 is \$2 more than the old equilibrium price of \$100. Since the tax is \$6, the producer pays \$4 of the tax and receives \$102 − \$6 = \$96 per item after taxes.

(d) The tax received by the government per unit product is \$6. Thus, the total revenue received by the government is equal to the tax per unit times the number of units sold, which is just the equilibrium quantity. Thus,

$$\text{Government revenue} = \text{Tax} \cdot \text{Quantity} = 6 \cdot 460 = \$2760.$$

46. **(a)** The original demand equation, $q = 100 - 2p$, tells us that

$$\text{Quantity demanded} = 100 - 2\begin{pmatrix}\text{Amount per unit}\\ \text{paid by consumers}\end{pmatrix}.$$

The consumers pay $p + 0.05p = 1.05p$ dollars per unit because they pay the price p plus 5% tax. Thus, the new demand equation is

$$q = 100 - 2(1.05p) = 100 - 2.1p.$$

The supply equation remains the same.

(b) To find the new equilibrium price and quantity, we find the intersection of the demand curve and the supply curve $q = 3p - 50$.

$$\begin{aligned}100 - 2.1p &= 3p - 50\\ 150 &= 5.1p\\ p &= \$29.41.\end{aligned}$$

The equilibrium price is \$29.41.
The equilibrium quantity q is given by

$$q = 3(29.41) - 50 = 88.23 - 50 = 38.23 \text{ units.}$$

(c) Since the pre-tax price was \$30 and the suppliers' new price is \$29.41 per unit,

$$\text{Tax paid by supplier} = \$30 - \$29.41 = \$0.59.$$

The consumers' new price is $1.05p = 1.05(29.41) = \$30.88$ per unit and the pre-tax price was \$30, so

$$\text{Tax paid by consumer} = 30.88 - 30 = \$0.88.$$

The total tax paid per unit by suppliers and consumers together is $0.59 + 0.88 = \$1.47$ per unit.
(d) The government receives \$1.47 per unit on 38.23 units. The total tax collected is $(1.47)(38.23) = \$56.20$.

47. (a) The original supply equation, $q = 3p - 50$, tells us that

$$\text{Quantity supplied } = 3\left(\begin{array}{c}\text{Amount received per}\\ \text{unit by suppliers}\end{array}\right) - 50.$$

The suppliers receive only $p - 0.05p = 0.95p$ dollars per unit because 5% of the price goes to the government. Thus, the new supply equation is

$$q = 3(0.95p) - 50 = 2.85p - 50.$$

The demand equation, $q = 100 - 2p$ is unchanged.
(b) At the equilibrium, supply equals demand:

$$\begin{aligned} 2.85p - 50 &= 100 - 2p \\ 4.85p &= 150 \\ p &= \$30.93. \end{aligned}$$

The equilibrium price is \$30.93. The equilibrium quantity q is the quantity demanded at the equilibrium price:

$$q = 100 - 2(30.93) = 38.14 \text{ units.}$$

(c) Since the pretax price was \$30 and consumers' new price is \$30.93,

$$\text{Tax paid by consumers} = 30.93 - 30 = \$0.93$$

The supplier keeps $0.95p = 0.95(30.93) = \$29.38$ per unit, so

$$\text{Tax paid by suppliers} = 30 - 29.38 = \$0.62.$$

The total tax per unit paid by consumers and suppliers together is $0.93 + 0.62 = \$1.55$ per unit.
(d) The government receives \$1.55 per unit on 38.14 units. The total tax collected is $(1.55)(38.14) = \$59.12$.

Solutions for Section 1.5

1. (a) Town (i) has the largest percent growth rate, at 12%.
(b) Town (ii) has the largest initial population, at 1000.
(c) Yes, town (iv) is decreasing in size, since the decay factor is 0.9, which is less than 1.

2. (a) Initial amount = 100; exponential growth; growth rate = 7% = 0.07.
(b) Initial amount = 5.3; exponential growth; growth rate = 5.4% = 0.054.
(c) Initial amount = 3500; exponential decay; decay rate = −7% = −0.07.
(d) Initial amount = 12; exponential decay; decay rate = −12% = −0.12.

3. Graph (I) and (II) are of increasing functions, therefore the growth factor for both functions is greater than 1. Since graph (I) increases faster than graph (II), graph (I) corresponds to the larger growth factor. Therefore graph (I) matches $Q = 50(1.4)^t$ and graph (II) is $Q = 50(1.2)^t$.

Similarly, since graphs (III) and (IV) are both of decreasing functions, they have growth factors between 0 and 1. Since graph (IV) decreases faster than graph (III), graph (IV) has the smaller decay factor. Thus graph (III) is $Q = 50(0.8)^t$ and graph (IV) is $Q = 50(0.6)^t$.

4. **(a)** The city's population is increasing at 5% per year. It is growing exponentially. Thus, it must be either I or II. We notice that part (b) has a higher rate of growth and thus we choose the exponential curve that grows more slowly, II.

(b) Using the same reasoning as part (a), we know that the graph of the population is an exponential growth curve. Comparing it to the previous city, part (b) has a faster rate of growth and thus we choose the faster growing exponential curve, I.

(c) The population is growing by a constant number of people each year. Thus, the graph of the population of the city is a line. Since the population is growing, we know that the line must have positive slope. Thus, the city's population is best described by graph III.

(d) The city's population does not change. That means that the population is a constant. It is thus a line with 0 slope, or graph V.

The remaining curves are IV and VI. Curve IV shows a population decreasing at an exponential rate. Comparison of the slope of curve IV at the points where it intersects curves I and II shows that the slope of IV is less steep than the slope of I or II. From this we know that the city's population is decreasing at a rate less than I or II's is increasing. It is decreasing at a rate of perhaps 4% per year.

Graph VI shows a population decreasing linearly—by the same number of people each year. By noticing that graph VI is more downward sloping than III is upward sloping, we can determine that the annual population decrease of a city described by VI is greater than the annual population increase of a city described by graph III. Thus, the population of city VI is decreasing by more than 5000 people per year.

5. **(a)** Since the y-intercept of each graph is above the x-axis, we know that a, c, and p must all be positive.

(b) Since the graph of $y = ab^x$ is increasing and goes through $(1, 1)$, the y-intercept, a, must be less than 1. Since the y-intercept is positive, we know that $0 < a < 1$. Since $y = cd^x$ and $y = pq^x$ are both decreasing, both d and q must be between 0 and 1.

(c) The graphs of $y = cd^x$ and $y = pq^x$ have the same y-intercepts, therefore we have $c = p$.

(d) Since the point $(1, 1)$ lies on the graph of $y = ab^x$, we know that $1 = ab^1$. Therefore, a and b are reciprocals of each other. Similarly, p and q are reciprocals of each other.

6. This looks like an exponential function $y = Ca^t$. The y-intercept is 500 so we have $y = 500a^t$. We use the point $(3, 2000)$ to find a:

$$\begin{aligned} y &= 500a^t \\ 2000 &= 500a^3 \\ 4 &= a^3 \\ a &= 4^{1/3} = 1.59. \end{aligned}$$

The formula is $y = 500(1.59)^t$.

7. This looks like an exponential decay function $y = Ca^t$. The y-intercept is 30 so we have $y = 30a^t$. We use the point $(25, 6)$ to find a:

$$\begin{aligned} y &= 30a^t \\ 6 &= 30a^{25} \\ 0.2 &= a^{25} \\ a &= (0.2)^{1/25} = 0.94. \end{aligned}$$

The formula is $y = 30(0.94)^t$.

8. We look for an equation of the form $y = y_0a^x$ since the graph looks exponential. The points $(0, 3)$ and $(2, 12)$ are on the graph, so

$$3 = y_0a^0 = y_0$$

and

$$12 = y_0 \cdot a^2 = 3 \cdot a^2, \quad \text{giving} \quad a = \pm 2.$$

Since $a > 0$, our equation is $y = 3(2^x)$.

9. We look for an equation of the form $y = y_0 a^x$ since the graph looks exponential. The points $(0, 18)$ and $(2, 8)$ are on the graph, so

$$18 = y_0 a^0 = y_0$$

and

$$8 = y_0 \cdot a^2 = 18 \cdot a^2, \quad \text{giving} \quad a = \pm\sqrt{\frac{4}{9}}.$$

Since $a > 0$, our equation is

$$y = 18\left(\frac{2}{3}\right)^x.$$

10. **(a)** If G is increasing at a constant percent rate, G is an exponential function of t. The formula is $G = 678.97(1.0109)^t$.
(b) If G is increasing at a constant rate, G is a linear function of t. The formula is $G = 678.97 + 7t$.

11. **(a)** The function is linear with initial population of 1000 and slope of 50, so $P = 1000 + 50t$.
(b) This function is exponential with initial population of 1000 and growth rate of 5%, so $P = 1000(1.05)^t$.

12. **(a)** Since the price is decreasing at a constant absolute rate, the price of the product is a linear function of t. In t days, the product will cost $80 - 4t$ dollars.
(b) Since the price is decreasing at a constant relative rate, the price of the product is an exponential function of t. In t days, the product will cost $80(0.95)^t$.

13. **(a)** This is a linear function with slope -2 grams per day and intercept 30 grams. The function is $Q = 30 - 2t$, and the graph is shown in Figure 1.45.

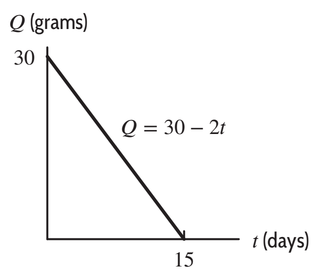


Figure 1.45

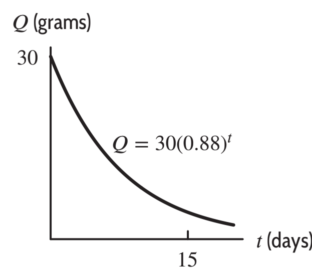


Figure 1.46

(b) Since the quantity is decreasing by a constant percent change, this is an exponential function with base $1-0.12 = 0.88$. The function is $Q = 30(0.88)^t$, and the graph is shown in Figure 1.46.

14. **(a)** We see from the base 1.0103 that the percent rate of growth is 1.03% per year.
(b) The initial population (in the year 2020) is 7.68 billion people. When $t = 5$, we have $P = 7.68(1.0103)^5 = 8.084$. The predicted world population in the year 2025 is 8.084 billion people.
(c) We have

$$\text{Average rate of change} = \frac{\Delta P}{\Delta t} = \frac{8.084 - 7.68}{2025 - 2020} = 0.081 \text{ billion people per year.}$$

The population of the world is projected to increase by about 81 million people per year from 2020 to 2025.

15. **(a)** Since the percent rate of change is constant, A is an exponential function of t. The initial quantity is 50 and the base is $1 - 0.06 = 0.94$. The formula is

$$A = 50(0.94)^t.$$

(b) When $t = 24$, we have $A = 50(0.94)^{24} = 11.33$. After one day, 11.33 mg of quinine remains.
(c) See Figure 1.47. The graph is decreasing and concave up, as we would expect with an exponential decay function.
(d) In Figure 1.47, it appears that when $A = 5$, we have $t \approx 37$. It takes about 37 hours until 5 mg remains.

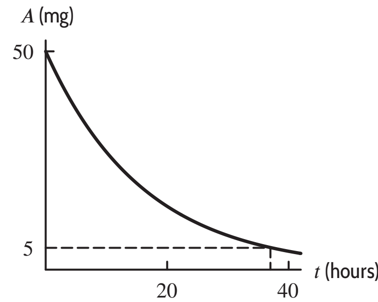


Figure 1.47

16. This function is exponential with initial value 258.8 and growth rate of 2.5%, so CPI $= 258.8(1.025)^t$.

17. Since the decay rate is 0.33%, the base of the exponential function is $1 - 0.0033 = 0.9967$. If P represents the percent of forested land in 2010 remaining t years after 2010, when $t = 0$ we have $P = 100$. The formula is $P = 100(0.9967)^t$. The percent remaining in 2030 will be $P = 100(0.9967)^{20} = 93.603$. About 93.603% of the land will be covered in forests 20 years later if the rate continues.

18. **(a)** The total number of confirmed cases can be modeled by

$$P = 2979(1.051)^t.$$

(b) May 10, 2020 corresponds to $t = 26$, so the expected number of confirmed cases in Colombia is

$$2979(1.051)^{26} = 10{,}858.$$

(c) We compute

$$\frac{11{,}063 - 10{,}858}{11{,}063} = 0.01853 = 1.853\%,$$

which tells us that the value predicted by the model is 1.853% less than the actual value.

19. See Figure 1.48. The graph is decreasing and concave up. It looks like an exponential decay function with y-intercept 100.

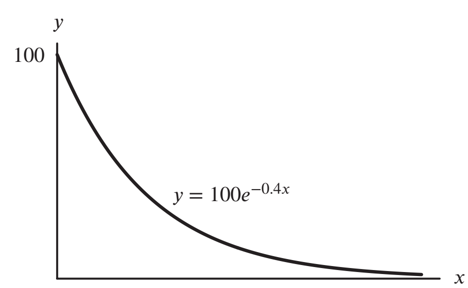


Figure 1.48

20. **(a)** See Table 1.5.

Table 1.5

x	0	1	2	3
e^x	1	2.72	7.39	20.09

(b) See Figure 1.49. The function $y = e^x$ is an exponential growth function.

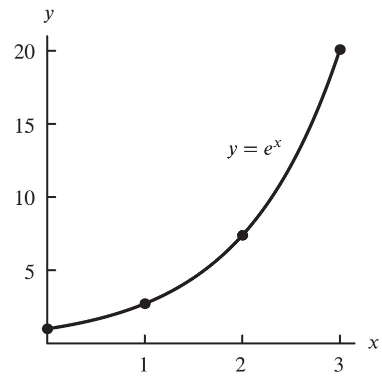


Figure 1.49

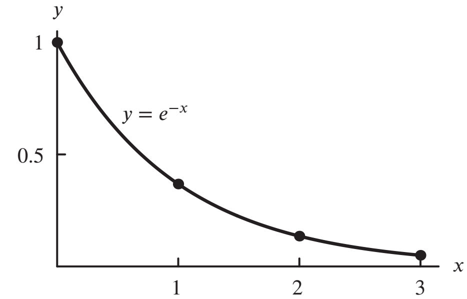


Figure 1.50

(c) See Table 1.6.

Table 1.6

x	0	1	2	3
e^{-x}	1	0.37	0.14	0.05

(d) See Figure 1.50. The function $y = e^{-x}$ is an exponential decay function.

21. The values of $f(x)$ given seem to increase by a factor of 1.4 for each increase of 1 in x, so we expect an exponential function with base 1.4. To assure that $f(0) = 4.30$, we multiply by the constant, obtaining

$$f(x) = 4.30(1.4)^x.$$

22. Each increase of 1 in t seems to cause $g(t)$ to decrease by a factor of 0.8, so we expect an exponential function with base 0.8. To make our solution agree with the data at $t = 0$, we need a coefficient of 5.50, so our completed equation is

$$g(t) = 5.50(0.8)^t.$$

23. Table A and Table B could represent linear functions of x. Table A could represent the constant linear function $y = 2.2$ because all y values are the same. Table B could represent a linear function of x with slope equal to $11/4$. This is because x values that differ by 4 have corresponding y values that differ by 11, and x values that differ by 8 have corresponding y values that differ by 22. In Table C, y decreases and then increases as x increases, so the table cannot represent a linear function. Table D does not show a constant rate of change, so it cannot represent a linear function.

24. Table D is the only table that could represent an exponential function of x. This is because, in Table D, the ratio of y values is the same for all equally spaced x values. Thus, the y values in the table have a constant percent rate of decrease:

$$\frac{9}{18} = \frac{4.5}{9} = \frac{2.25}{4.5} = 0.5.$$

Table A represents a constant function of x, so it cannot represent an exponential function. In Table B, the ratio between y values corresponding to equally spaced x values is not the same. In Table C, y decreases and then increases as x increases. So neither Table B nor Table C can represent exponential functions.

25. **(a)** See Figure 1.51.

(b) No. The points in the plot do not lie even approximately on a straight line, so a linear model is a poor choice.

(c) No. The plot shows that H is a decreasing function of Y that might be leveling off to an asymptote at $H = 0$. These are features of an exponential decay function, but their presence does not show that H is an exponential function of Y. We can do a more precise check by dividing each value of H by the previous year's H.

$$\frac{\text{Houses in 2011}}{\text{Houses in 2010}} = \frac{13 \text{ million}}{18.3 \text{ million}} = 0.710$$

$$\frac{\text{Houses in 2012}}{\text{Houses in 2011}} = \frac{7.8 \text{ million}}{13 \text{ million}} = 0.600$$
$$\frac{\text{Houses in 2013}}{\text{Houses in 2012}} = \frac{3.9 \text{ million}}{7.8 \text{ million}} = 0.500$$
$$\frac{\text{Houses in 2014}}{\text{Houses in 2013}} = \frac{1 \text{ million}}{3.9 \text{ million}} = 0.256$$
$$\frac{\text{Houses in 2015}}{\text{Houses in 2014}} = \frac{0.5 \text{ million}}{1 \text{ million}} = 0.500.$$

If H were an exponential function of Y, the ratios would be approximately constant. Since they are not approximately constant, we see that an exponential model is a poor choice.

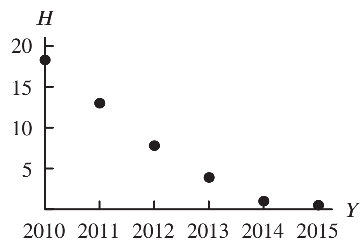


Figure 1.51

26. **(a)** We divide the two equations to solve for a:

$$\frac{P_0 a^3}{P_0 a^2} = \frac{75}{50}$$
$$a = 1.5$$

Now that we know the value of a, we can use either of the equations to solve for P_0. Using the first equation, we have:

$$P_0 a^3 = 75$$
$$P_0(1.5^3) = 75$$
$$P_0 = \frac{75}{1.5^3} = 22.222$$

We see that $a = 1.5$ and $P_0 = 22.222$.

(b) The initial quantity is 22.222 and the quantity is growing at a rate of 50% per unit time.

27. **(a)** We divide the two equations to solve for a:

$$\frac{P_0 a^4}{P_0 a^3} = \frac{18}{20}$$
$$a = 0.9$$

Now that we know the value of a, we can use either of the equations to solve for P_0. Using the first equation, we have:

$$P_0 a^4 = 18$$
$$P_0(0.9^4) = 18$$
$$P_0 = \frac{18}{0.9^4} = 27.435$$

We see that $a = 0.9$ and $P_0 = 27.435$.

(b) The initial quantity is 27.435 and the quantity is decaying at a rate of 10% per unit time.

28. From the statements given, we have the equations: $P_0 a^5 = 320$ and $P_0 a^3 = 500$.

(a) We divide the two equations to solve for a:

$$\frac{P_0 a^5}{P_0 a^3} = \frac{320}{500}$$
$$a^2 = 0.64$$
$$a = 0.64^{1/2} = 0.8$$

Now that we know the value of a, we can use either of the equations to solve for P_0. Using the first equation, we have:

$$\begin{aligned} P_0 a^5 &= 320 \\ P_0(0.8^5) &= 320 \\ P_0 &= \frac{320}{0.8^5} = 976.563 \end{aligned}$$

We see that $a = 0.8$ and $P_0 = 976.563$.

(b) The initial quantity is 976.563 and the quantity is decaying at a rate of 20% per unit time.

29. From the statements given, we have the equations: $P_0 a^3 = 1600$ and $P_0 a^1 = 1000$.

(a) We divide the two equations to solve for a:

$$\begin{aligned} \frac{P_0 a^3}{P_0 a^1} &= \frac{1600}{1000} \\ a^2 &= 1.6 \\ a &= 1.6^{1/2} = 1.2649111 \approx 1.265. \end{aligned}$$

Now that we know the value of a, we can use either of the equations to solve for P_0. From the first equation, we have:

$$\begin{aligned} P_0 a^3 &= 1600 \\ P_0(1.2649111^3) &= 1600 \\ P_0 &= \frac{1600}{1.2649111^3} = 790.569. \end{aligned}$$

We see that $a = 1.265$ and $P_0 = 790.569$.

(b) The initial quantity is 790.569 and the quantity is growing at a rate of 26.5% per unit time.

30. If the population increases exponentially, the formula for the population at time t can be written in the form

$$P(t) = P_0 e^{kt}$$

where P_0 is the population at time $t = 0$ and k is the continuous growth rate; we measure time in years. If we let 2000 be the initial time $t = 0$, we have $P_0 = 6.081$, so

$$P(t) = 6.081e^{kt}.$$

In 2018, time $t = 18$, so we have $P(18) = 7.504$. Thus we get

$$\begin{aligned} 7.504 &= 6.081e^{18k} \\ e^{18k} &= \frac{7.504}{6.081} \\ \ln e^{18k} &= \ln\left(\frac{7.504}{6.081}\right) \\ k &= \frac{\ln(7.504/6.081)}{18} = 0.0117. \end{aligned}$$

Thus, the formula for the population, assuming exponential growth, is

$$P(t) = 6.081e^{0.0117t} \text{ billion.}$$

Using our formula for the year 2020, time $t = 20$, we get

$$P(20) = 6.081e^{0.0117(20)} = 6.081e^{0.2340} = 7.68 \text{ billion.}$$

Thus, we see that we were not too far off the mark when we approximated the population by an exponential function.

31. (a) The revenue is modeled by the exponential function $R = 1.9(1.545)^t$ where t is years since 2010. In 2017 the revenue was

$$R = 1.9(1.545)^7 = 39.926 \text{ billion dollars.}$$

(b) If growth continued exponential at the same rate, then in 2020 the revenue would be

$$R = 1.9(1.545)^{10} = 147.244 \text{ billion dollars.}$$

This is much more than the actual revenue, which was $80.9 billion. The growth did not continue at 54.5% per year.

32. **(a)** Since the population was 2.7 million in 2010, and t is measured since 2010 so $t = 0$ in 2010, we have $N_0 = 2.7$. Then $N = 3.0$ when $t = 8$, so

$$3 = 2.7a^8$$
$$a = \left(\frac{3}{2.7}\right)^{1/8} = 1.0133.$$

Thus,

$$N = 2.7(1.0133)^t.$$

(b) In 2000, we have $t = -10$, so

$$N = 2.7\left(\left(\frac{3}{2.7}\right)^{1/8}\right)^{-10} = 2.367 \text{ million}.$$

33. Use an exponential function of the from $P = P_0 a^t$ for the number of passengers. At $t = 0$, the number of passengers is $P_0 = 190{,}205$, so

$$P = 190{,}205a^t.$$

After $t = 5$ years, the number of passengers is 174,989, so

$$174{,}989 = 190{,}205a^5$$
$$a^5 = \frac{174{,}989}{190{,}205} = 0.920002$$
$$a = (0.920002)^{1/5} = 0.983.$$

Then

$$P = 190{,}205(0.932)^t,$$

which means the annual percentage decrease over this period is $1 - 0.983 = 0.017$ or 1.7%.

34. Since we are looking for an annual percent increase, we use an exponential function. If V represents the value of the company, and t represent the number of years since it was started, we have $V = Ca^t$. The initial value of the company was \$4000, so $V = 4000a^t$. To find a, we use the fact that $V = 14{,}000{,}000$ when $t = 40$:

$$V = 4000a^t$$
$$14{,}000{,}000 = 4000a^{40}$$
$$3500 = a^{40}$$
$$a = (3500)^{1/40} = 1.226.$$

The value of the company increased at an average rate of 22.6% per year.

35. Let y represent the number of zebra mussels t years since 2014.

(a) The slope of the line is $(3186 - 2700)/1 = 486$ and the vertical intercept is 2700. We have $y = 2700 + 486t$. The slope of the line, 486 zebra mussels per year, represents the rate at which the zebra mussel population is growing.

(b) When $t = 0$, we have $y = 2700$, so the exponential function is in the form $y = 2700a^t$. When $t = 1$ we have $y = 3186$, and we substitute to get $3186 = 2700a^1$ so $a = 1.18$. The exponential function is $y = 2700(1.18)^t$ and the zebra mussel population is growing at 18% per year.

36. **(a)** We have $(1.03)^{120} = 34.711$, so infections in Arizona grew by a factor of about 35. Similarly, $(1.0175)^{120} = 8.019$, so infections in West Virginia grew by a factor of about 8.

(b) Note that 35 is about 4 times 8. So, although the growth rates differ by a factor of two, the four month growth differs by a factor of more that 4.

37. **(a)** The slope of the line is

$$\text{Slope} = \frac{\Delta W}{\Delta t} = \frac{591.1 - 371.4}{2018 - 2014} = 54.925.$$

The initial value (when $t = 0$) is 371.4, so the linear formula is

$$W = 371.4 + 54.925t.$$

The rate of growth of wind generating capacity is 54.925 gigawatts per year.

(b) If the function is exponential, we have $W = W_0 a^t$. The initial value (when $t = 0$) is 371.4, so we have $W = 371.4a^t$. We use the fact that $W = 591.1$ when $t = 4$ to find a:

$$591.1 = 371.4a^4$$
$$1.591546 = a^4$$
$$a = 1.591546^{1/4} = 1.123194.$$

The exponential formula is

$$W = 371.4(1.123194)^t.$$

The percent rate of growth of wind generating capacity is 12.319% per year. (Alternatively, if we used the form $W = W_0 e^{kt}$, the formula would be $W = 371.4e^{0.11618t}$, giving a continuous rate of 11.618% per year.)

(c) See Figure 1.52.

(d) The linear function, $371.4+54.925t$, gives a predicted generating capacity in 2019, when $t = 5$, of $371.4+54.925\cdot 5 = 646.025$ gigawatts, 4.775 less than the real power generating capacity of 650.8 gigawatts. The exponential model predicts the capacity in 2019 to be $W = 371.4(1.123194)^5 = 663.920$, which is about 13.12 gigawatts more than the real figure, so the linear model is better.

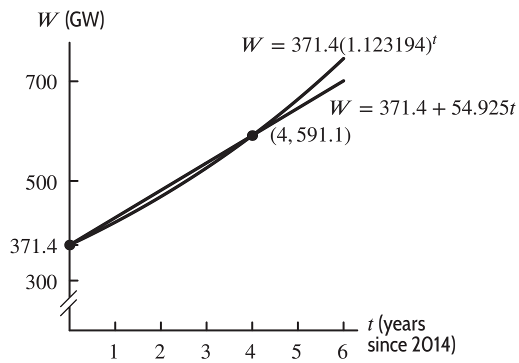


Figure 1.52

38. (a) We see that f cannot be a linear function, since $f(x)$ increases by different amounts ($24 - 16 = 8$ and $36 - 24 = 12$) as x increases by one. Could f be an exponential function? We look at the ratios of successive $f(x)$ values:

$$\frac{24}{16} = 1.5 \qquad \frac{36}{24} = 1.5 \qquad \frac{54}{36} = 1.5 \qquad \frac{81}{54} = 1.5.$$

Since the ratios are all equal to 1.5, this table of values could correspond to an exponential function with a base of 1.5. Since $f(0) = 16$, a formula for $f(x)$ is

$$f(x) = 16(1.5)^x.$$

Check by substituting $x = 0, 1, 2, 3, 4$ into this formula; you get the values given for $f(x)$.

(b) As x increases by one, $g(x)$ increases by 6 (from 14 to 20), then 4 (from 20 to 24), so g is not linear. We check to see if g could be exponential:

$$\frac{20}{14} = 1.43 \quad \text{and} \quad \frac{24}{20} = 1.2.$$

Since these ratios (1.43 and 1.2) are different, g is not exponential.

(c) For h, notice that as x increases by one, the value of $h(x)$ increases by 1.2 each time. So h could be a linear function with a slope of 1.2. Since $h(0) = 5.3$, a formula for $h(x)$ is

$$h(x) = 5.3 + 1.2x.$$

39. (a) A linear function must change by exactly the same amount whenever x changes by some fixed quantity. While $h(x)$ decreases by 3 whenever x increases by 1, $f(x)$ and $g(x)$ fail this test, since both change by different amounts between $x = -2$ and $x = -1$ and between $x = -1$ and $x = 0$. So the only possible linear function is $h(x)$, so it will be given by a formula of the type: $h(x) = mx + b$. As noted, $m = -3$. Since the y-intercept of h is 31, the formula for $h(x)$ is $h(x) = 31 - 3x$.

(b) An exponential function must grow by exactly the same factor whenever x changes by some fixed quantity. Here, $g(x)$ increases by a factor of 1.5 whenever x increases by 1. Since the y-intercept of $g(x)$ is 36, $g(x)$ has the formula $g(x) = 36(1.5)^x$. The other two functions are not exponential; $h(x)$ is not because it is a linear function, and $f(x)$ is not because it both increases and decreases.

40. **(a)** In this case we know that

$$f(1) - f(0) = 12.7 - 10.5 = 2.2$$

while

$$f(2) - f(1) = 18.9 - 12.7 = 6.2.$$

Thus, the function described by these data is not a linear one. Next we check if this function is exponential.

$$\frac{f(1)}{f(0)} = \frac{12.7}{10.5} \approx 1.21$$

while

$$\frac{f(2)}{f(1)} = \frac{18.9}{12.7} \approx 1.49$$

thus $f(x)$ is not an exponential function either.

(b) In this case we know that

$$s(0) - s(-1) = 30.12 - 50.2 = -20.08$$

while

$$s(1) - s(0) = 18.072 - 30.12 = -12.048.$$

Thus, the function described by these data is not a linear one. Next we check if this function is exponential.

$$\frac{s(0)}{s(-1)} = \frac{30.12}{50.2} = 0.6,$$
$$\frac{s(1)}{s(0)} = \frac{18.072}{30.12} = 0.6,$$

and

$$\frac{s(2)}{s(1)} = \frac{10.8432}{18.072} = 0.6.$$

Thus, $s(t)$ is an exponential function. We know that $s(t)$ will be of the form

$$s(t) = P_0 a^t$$

where P_0 is the initial value and $a = 0.6$ is the base. We know that

$$P_0 = s(0) = 30.12.$$

Thus,

$$s(t) = 30.12a^t.$$

Since $a = 0.6$, we have

$$s(t) = 30.12(0.6)^t.$$

(c) In this case we know that

$$\frac{g(2) - g(0)}{2 - 0} = \frac{24 - 27}{2} = \frac{-3}{2} = -1.5,$$
$$\frac{g(4) - g(2)}{4 - 2} = \frac{21 - 24}{2} = \frac{-3}{2} = -1.5,$$

and

$$\frac{g(6) - g(4)}{6 - 4} = \frac{18 - 21}{2} = \frac{-3}{2} = -1.5.$$

Thus, $g(u)$ is a linear function. We know that

$$g(u) = m \cdot u + b$$

where m is the slope and b is the vertical intercept, or the value of the function at zero. So

$$b = g(0) = 27$$

and from the above calculations we know that

$$m = -1.5.$$

Thus,

$$g(u) = -1.5u + 27.$$

41. Assuming that, on average, the minimum wage has increased exponentially from its value of \$1.60 in 1968, we have

$$m = 1.60b^t,$$

where m is the minimum wage t years after 1968.

Since $t = 2021 - 1968 = 53$ in 2021, we have

$$m = 1.60(1.0392)^{53} = \$12.28.$$

Thus, a \$15 minimum wage would be growth faster than the inflation rate.

42. **(a)** Using $Q = Q_0(1 - r)^t$ for loss, we have

$$Q = 10{,}000(1 - 0.1)^{10} = 10{,}000(0.9)^{10} = 3486.78.$$

The investment was worth \$3486.78 after 10 years.

(b) Measuring time from the moment at which the stock begins to gain value and letting $Q_0 = 3486.78$, the value after t years is

$$Q = 3486.78(1 + 0.1)^t = 3486.78(1.1)^t.$$

We can estimate the value of t when $Q = 10{,}000$ by tracing along a graph of Q, giving $t \approx 11$. It will take about 11 years to get the investment back to \$10,000.

43. **(a)** We look at the ratios of successive total cases to estimate the daily growth rate. We have

$$\text{Ratio of total number of cases, March 11 and March 12} = \frac{813}{606} = 1.3416.$$

This means that the total number of cases grew by about 34% between March 11 and March 12. Similarly,

$$\text{Ratio of total number of cases, March 12 and March 13} = \frac{1086}{813} = 1.3358.$$

This means that the total number of cases grew by about 34% between March 12 and March 13 as well.

(b) Since we estimate the daily growth rate to be 34%, an exponential function that models the number of confirmed cases t days after March 10, 2020 is

$$f(t) = 444(1.34)^t.$$

(c) March 27 corresponds to $t = 17$, so (rounding)

$$\text{Expected number of cases on March 27} = f(17) = 444(1.34)^{17} = 64{,}293.126 \approx 64{,}293.$$

This estimate is close to the total number of confirmed cases on March 27 (about 10% error).

(d) April 30 corresponds to $t = 51$, so

$$\text{Expected number of cases on April 30} = f(51) = 444(1.34)^{51} = 1{,}348{,}117{,}315 \approx 1348 \text{ million}.$$

This is more than 1300 times larger than the total number of cases, 988,487. In fact, it is over four times the reported population of the US[1] in 2020 of about 330 million.

44. **(a)** We know $w = 1248$ when $t = 1$ and $w = 25{,}827$ when $t = 24$. One algorithm for exponential regression gives

$$w = 1093.965(1.1408)^t.$$

(b) The annual growth rate is $0.1408 = 14.08\%$.

(c) The model predicts that in 2000 the number of cases would be

$$w = 1093.965(1.1408)^{20} \approx 15{,}247.$$

If the number of cases had doubled between 2000 and 2004, there would have been $15{,}247 \cdot 2 = 30{,}494$ cases in 2004. However, there were 25,827 cases in 2004. Thus the model does not confirm this report.

The reason is that the number of cases has been increasing at an average annual rate of 14.08% between 1980 and 2004, but at a higher rate between 2000 and 2004.

[1] Data from https://www.census.gov/popclock , accessed on June 27, 2020

45. Since the price dropped 20% every year, the price P per kilowatt-hour, where t is in years since 2013 and P_0 is the price per kilowatt-hour in 2013, is given by

$$P(t) = P_0(0.8)^t.$$

When the price is half the price in 2013, it is $P_0/2$, so we solve

$$\begin{aligned} \frac{P_0}{2} &= P_0(0.8)^t \\ \frac{1}{2} &= (0.8)^t \\ \ln\frac{1}{2} &= \ln(0.8)^t \\ \ln\frac{1}{2} &= t\ln(0.8) \\ t &= \frac{\ln(1/2)}{\ln(0.8)} = 3.106. \end{aligned}$$

It took about three years for the price to go down by 50%.

46. Direct calculation reveals that each 1000 foot increase in altitude results in a longer takeoff roll by a factor of about 1.096. Since the value of d when $h = 0$ (sea level) is $d = 670$, we are led to the formula

$$d = 670(1.096)^{h/1000},$$

where d is the takeoff roll, in feet, and h is the airport's elevation, in feet.

Alternatively, we can write

$$d = d_0 a^h,$$

where d_0 is the sea level value of d, $d_0 = 670$. In addition, when $h = 1000$, $d = 734$, so

$$734 = 670a^{1000}.$$

Solving for a gives

$$a = \left(\frac{734}{670}\right)^{1/1000} = 1.00009124,$$

so

$$d = 670(1.00009124)^h.$$

47. **(a)** Let P represent the total number of confirmed cases in Africa with $P_0 = 1$ at time $t = 0$. We model the total number of cases by $P = a^t$. Since $P = 100{,}000$ when $t = 98$, we have

$$\begin{aligned} 100{,}000 &= a^{98} \\ a &= 100{,}000^{1/98} \\ a &= 1.125. \end{aligned}$$

So $P = 1.125^t$ and the number of confirmed cases, P, was growing at a 12.5% daily rate.

(b) After 116 days, the predicted number of cases is much higher than the 200,000 cases:

$$P = (100{,}000^{1/98})^{116} = 828{,}642.773.$$

48. **(a)** Since the annual growth factor from 2012 to 2013 was $1 + (58.95/100) = 1.5895$ and $899(1.5895) = 1428.961$, the US consumed about 1429 million gallons of biodiesel in 2013. Since the annual growth factor from 2013 to 2014 was $1 + (-0.0084) = 0.9916$ and $1428.961(0.9916) = 1416.96$, the US consumed approximately 1417 million gallons of biodiesel in 2014.

(b) Completing the table of annual consumption of biodiesel and plotting the data gives Figure 1.53.

Table 1.7

Year	2012	2013	2014	2015	2016	2017	2018
Consumption of biodiesel (mn gal)	899	1429	1417	1494	2085	1985	1896

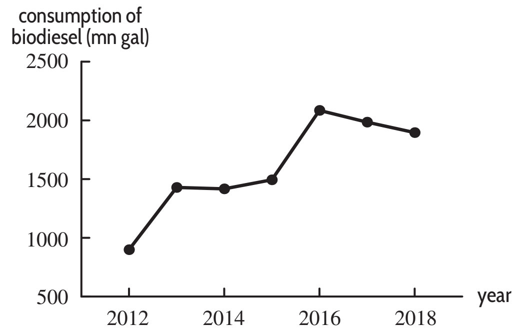


Figure 1.53

49. (a) False, because the annual percent growth is not constant over this interval.

(b) (i) The US consumption of biodiesel grew by more that 58% in 2013.

(ii) The US consumption of biodiesel fell by 4.8% in 2017.

50. (a) Since the annual growth factor from 2013 to 2014 was $1 - 0.0342 = 0.9658$ and $2.686 \cdot 0.9658 = 2.5941$, the US generated approximately 2.6 quadrillion BTUs of hydroelectric power in 2014. Since the annual growth factor from 2014 to 2015 was $1 - 0.0397 = 0.9603$ and $2.5941(1 - 0.0397) = 2.4911$, the US generated about 2.5 quadrillion BTUs of hydroelectric power in 2015.

(b) Completing the table of annual consumption of hydroelectric power and plotting the data gives Figure 1.54.

Year	2013	2014	2015	2016	2017	2018
Generation of hydro. power (quadrillion BTU)	2.686	2.594	2.491	2.678	3.004	2.918

(c) The largest increase in the US generation of hydroelectric power occurred in 2017. In this year, the US generation of hydroelectric power increased by about 326 trillion BTUs to 3.004 quadrillion BTUs, up from 2.678 quadrillion BTUs in 2016.

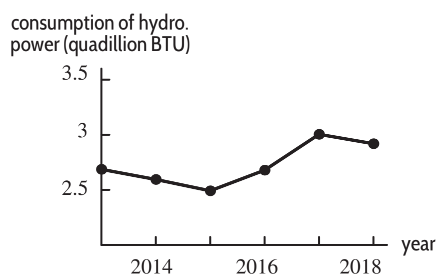


Figure 1.54

51. (a) From Figure 1.81 in the text, we can see the approximate percent growth for each year over the previous year:

Year	2013	2014	2015	2016	2017	2018
% growth over previous yr	20	8	3	18	12	8

Since the annual growth factor from 2013 to 2014 was $1 + 0.08 = 1.08$ and $1601(1 + 0.08) = 1729.08$, the US consumed approximately 1729 trillion BTUs of wind power energy in 2014. Since the annual growth factor from 2014 to 2015 was $1 + 0.03 = 1.03$ and $1729.08(1 + 0.03) = 1780.95$, the US consumed about 1781 trillion BTUs of wind power energy in 2015.

(b) Completing the table of annual consumption of wind power and plotting the data gives Figure 1.55.

Year	2013	2014	2015	2016	2017	2018
Consumption of wind power (trillion BTU)	1601	1729	1781	2102	2354	2542

(c) The largest increase in the US consumption of wind power energy occurred in 2016. In this year the US consumption of wind power energy rose by about 321 trillion BTUs to 2102 trillion BTUs, up from 1781 trillion BTUs in 2015.

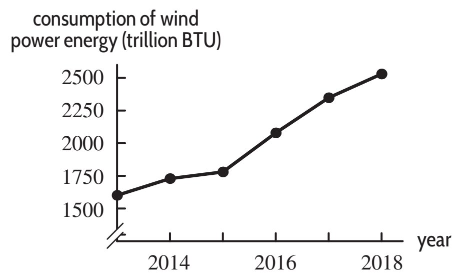


Figure 1.55

52. (a) The US consumption of wind power energy increased by at least 10% in 2013, 2016 and in 2017, relative to the previous year. Consumption did not decrease during the time period shown because all the annual percent growth values are positive, indicating a steady increase in the US consumption of wind power energy between 2013 and 2018.

(b) True. From the figure we see that consumption increased by about 18% during 2016, by about 12% during 2017, and by about 12% during 2018. This means that $x(1+0.18)$ units of wind power energy were consumed in 2016 if x had been consumed in 2015. Similarly, $x(1+0.18)(1+0.12)$ units of wind power energy were consumed in 2017 if x had been consumed in 2015, and

$$x(1+0.18)(1+0.12)(1+0.08)$$

units of wind power energy were consumed in 2018 if x had been consumed in 2015. Since

$$x(1+0.18)(1+0.12)(1+0.08) = x(1.4273) = x(1+0.4273),$$

wind power consumption was about 42.73% greater in 2018 than in 2015. Thus, consumption of wind power energy grew by about 42.73% from the beginning of 2015 to the end of 2018.

Note that adding these three percentages would be only 38%, however there is a multiplier effect which makes the change more than 40%.

Solutions for Section 1.6

1. Taking natural logs of both sides we get

$$\ln 10 = \ln(2^t).$$

This gives

$$t \ln 2 = \ln 10$$

or in other words

$$t = \frac{\ln 10}{\ln 2} \approx 3.3219$$

2. Taking natural logs of both sides we get

$$\ln(5^t) = \ln 7.$$

This gives

$$t \ln 5 = \ln 7$$

or in other words

$$t = \frac{\ln 7}{\ln 5} \approx 1.209.$$

3. Taking natural logs of both sides we get

$$\ln 2 = \ln(1.02^t).$$

This gives

$$t \ln 1.02 = \ln 2$$

or in other words

$$t = \frac{\ln 2}{\ln 1.02} \approx 35.003.$$

4. Taking natural logs of both sides we get

$$\ln 130 = \ln(10^t).$$

This gives

$$t \ln 10 = \ln 130$$

or in other words

$$t = \frac{\ln 130}{\ln 10} \approx 2.1139.$$

5. Taking natural logs of both sides we get

$$\ln(e^t) = \ln 10$$

which gives

$$t = \ln 10 \approx 2.3026.$$

6. Dividing both sides by 25 we get

$$4 = 1.5^t.$$

Taking natural logs of both side gives

$$\ln(1.5^t) = \ln 4.$$

This gives

$$t \ln 1.5 = \ln 4$$

or in other words

$$t = \frac{\ln 4}{\ln 1.5} \approx 3.419.$$

7. Dividing both sides by 10 we get

$$5 = 3^t.$$

Taking natural logs of both sides gives

$$\ln(3^t) = \ln 5.$$

This gives

$$t \ln 3 = \ln 5$$

or in other words

$$t = \frac{\ln 5}{\ln 3} \approx 1.465.$$

8. Dividing both sides by 2 we get

$$2.5 = e^t.$$

Taking the natural log of both sides gives

$$\ln(e^t) = \ln 2.5.$$

This gives

$$t = \ln 2.5 \approx 0.9163.$$

9. Taking natural logs of both sides we get

$$\ln(e^{3t}) = \ln 100.$$

This gives

$$3t = \ln 100$$

or in other words

$$t = \frac{\ln 100}{3} \approx 1.535.$$

10. Dividing both sides by 6 gives

$$e^{0.5t} = \frac{10}{6} = \frac{5}{3}.$$

Taking natural logs of both sides we get

$$\ln(e^{0.5t}) = \ln\left(\frac{5}{3}\right).$$

This gives

$$0.5t = \ln\left(\frac{5}{3}\right) = \ln 5 - \ln 3$$

or in other words

$$t = 2(\ln 5 - \ln 3) \approx 1.0217.$$

11. We divide both sides by 100 and then take the natural logarithm of both sides and solve for t:

$$\begin{aligned} 40 &= 100e^{-0.03t} \\ 0.4 &= e^{-0.03t} \\ \ln(0.4) &= \ln(e^{-0.03t}) \\ \ln(0.4) &= -0.03t \\ t &= \frac{\ln(0.4)}{-0.03} = 30.54. \end{aligned}$$

12. Taking natural logs of both sides (and assuming $a > 0$, $b > 0$, and $b \neq 1$) gives

$$\ln(b^t) = \ln a.$$

This gives

$$t \ln b = \ln a$$

or in other words

$$t = \frac{\ln a}{\ln b}.$$

13. Dividing both sides by P we get

$$\frac{B}{P} = e^{rt}.$$

Taking the natural log of both sides gives

$$\ln(e^{rt}) = \ln\left(\frac{B}{P}\right).$$

This gives

$$rt = \ln\left(\frac{B}{P}\right) = \ln B - \ln P.$$

Dividing by r gives

$$t = \frac{\ln B - \ln P}{r}.$$

14. Dividing both sides by P we get

$$2 = e^{0.3t}.$$

Taking the natural log of both sides gives

$$\ln(e^{0.3t}) = \ln 2.$$

This gives

$$0.3t = \ln 2$$

or in other words

$$t = \frac{\ln 2}{0.3} \approx 2.3105.$$

15. Taking natural logs of both sides we get

$$\ln(7 \cdot 3^t) = \ln(5 \cdot 2^t)$$

which gives

$$\ln 7 + \ln(3^t) = \ln 5 + \ln(2^t)$$

or in other words

$$\ln 7 - \ln 5 = \ln(2^t) - \ln(3^t).$$

This gives

$$\ln 7 - \ln 5 = t \ln 2 - t \ln 3 = t(\ln 2 - \ln 3).$$

Thus, we get

$$t = \frac{\ln 7 - \ln 5}{\ln 2 - \ln 3} \approx -0.8298.$$

16. Taking natural logs of both sides we get

$$\ln(5e^{3t}) = \ln(8e^{2t}).$$

This gives

$$\ln 5 + \ln(e^{3t}) = \ln 8 + \ln(e^{2t})$$

or in other words

$$\ln(e^{3t}) - \ln(e^{2t}) = \ln 8 - \ln 5.$$

This gives

$$3t - 2t = \ln 8 - \ln 5$$

or in other words

$$t = \ln 8 - \ln 5 \approx 0.47.$$

17. Taking natural logs of both sides, we get

$$\ln(Ae^{2t}) = \ln(Be^t)$$

so

$$\ln A + \ln(e^{2t}) = \ln B + \ln(e^t)$$

or in other words

$$\ln A + 2t \ln e = \ln B + t \ln e.$$

Since $\ln e = 1$, this gives

$$\ln A + 2t = \ln B + t.$$

Thus, combining ts on the left side of the equation, we get

$$t = \ln B - \ln A = \ln(B/A).$$

18. We cannot take the log of 0. Moving the 5 to the other side of the equation, we have

$$2e^t = 5.$$

Then we take natural logs of both sides, giving

$$\ln(2e^t) = \ln 5$$

so

$$\ln 2 + \ln(e^t) = \ln 5$$

or in other words

$$\ln 2 + t \ln e = \ln 5.$$

Since $\ln e = 1$, this gives

$$\ln 2 + t = \ln 5.$$

Thus, we get

$$t = \ln 5 - \ln 2 = \ln(5/2) \approx 0.9163.$$

19. We cannot take the log of 0. Moving the $3e^t$ to the other side of the equation, we have

$$3e^t = 7.$$

Then we take natural logs of both sides, giving

$$\ln(3e^t) = \ln 7$$

so

$$\ln 3 + \ln(e^t) = \ln 7$$

or in other words

$$\ln 3 + t \ln e = \ln 7.$$

Since $\ln e = 1$, this gives

$$\ln 3 + t = \ln 7.$$

Thus, we get

$$t = \ln 7 - \ln 3 = \ln(7/3) \approx 0.8473.$$

20. Since we cannot take the log of 0, we move Qe^{-t} to the other side of the equation

$$Pe^{4t} = Qe^{-t}.$$

Then, taking natural logs of both sides, we get

$$\ln(Pe^{4t}) = \ln(Qe^{-t})$$

so

$$\ln P + \ln(e^{4t}) = \ln Q + \ln(e^{-t})$$

or in other words

$$\ln P + 4t \ln e = \ln Q - t \ln e.$$

Since $\ln e = 1$, this gives

$$\ln P + 4t = \ln Q - t.$$

Thus, combining ts on the left side of the equation, we get

$$\begin{aligned} 5t &= \ln Q - \ln P \\ t &= \frac{1}{5}(\ln Q - \ln P) = \frac{\ln(Q/P)}{5}. \end{aligned}$$

21. Initial quantity = 5; growth rate = 0.07 = 7%.

22. Initial quantity = 7.7; growth rate = −0.08 = −8% (decay).

23. Initial quantity = 15; growth rate = −0.06 = −6% (continuous decay).

24. Initial quantity = 3.2; growth rate = 0.03 = 3% (continuous).

25. Since $e^{0.25t} = \left(e^{0.25}\right)^t \approx (1.2840)^t$, we have $P = 15(1.2840)^t$. This is exponential growth since 0.25 is positive. We can also see that this is growth because $1.2840 > 1$.

26. Since $e^{-0.5t} = (e^{-0.5})^t \approx (0.6065)^t$, we have $P = 2(0.6065)^t$. This is exponential decay since −0.5 is negative. We can also see that this is decay because $0.6065 < 1$.

27. $P = P_0(e^{0.2})^t = P_0(1.2214)^t$. Exponential growth because $0.2 > 0$ or $1.2214 > 1$.

28. $P = 7(e^{-\pi})^t = 7(0.0432)^t$. Exponential decay because $-\pi < 0$ or $0.0432 < 1$.

29. Since we want $(1.5)^t = e^{kt} = (e^k)^t$, so $1.5 = e^k$, and $k = \ln 1.5 = 0.4055$. Thus, $P = 15e^{0.4055t}$. Since 0.4055 is positive, this is exponential growth.

30. We want $1.7^t = e^{kt}$ so $1.7 = e^k$ and $k = \ln 1.7 = 0.5306$. Thus $P = 10e^{0.5306t}$.

31. We want $0.9^t = e^{kt}$ so $0.9 = e^k$ and $k = \ln 0.9 = -0.1054$. Thus $P = 174e^{-0.1054t}$.

32. Since we want $(0.55)^t = e^{kt} = (e^k)^t$, so $0.55 = e^k$, and $k = \ln 0.55 = -0.5978$. Thus $P = 4e^{-0.5978t}$. Since −0.5978 is negative, this represents exponential decay.

33. We use the information to create two equations: $P_0e^{3k} = 140$ and $P_0e^{1k} = 100$.

(a) We divide the two equations to solve for k:

$$\begin{aligned}\frac{P_0e^{3k}}{P_0e^k} &= \frac{140}{100}\\ e^{2k} &= 1.4\\ 2k &= \ln 1.4\\ k &= \frac{\ln 1.4}{2} = 0.168\end{aligned}$$

Now that we know the value of k, we can use either of the equations to solve for P_0. Using the first equation, we have:

$$\begin{aligned}P_0e^{3k} &= 140\\ P_0e^{3(0.168)} &= 140\\ P_0 &= \frac{140}{e^{0.504}} = 84.575\end{aligned}$$

We see that $a = 0.168$ and $P_0 = 84.575$.

(b) The initial quantity is 84.575 and the quantity is growing at a continuous rate of 16.8% per unit time.

34. We use the information to create two equations: $P_0e^{4k} = 40$ and $P_0e^{3k} = 50$.

(a) We divide the two equations to solve for k:

$$\begin{aligned}\frac{P_0e^{4k}}{P_0e^{3k}} &= \frac{40}{50}\\ e^k &= 0.8\\ k &= \ln 0.8 = -0.223144.\end{aligned}$$

Now that we know the value of k, we can use either of the equations to solve for P_0. Using the first equation, we have:

$$\begin{aligned}P_0e^{4k} &= 40\\ P_0e^{4(-0.223144)} &= 40\\ P_0 &= \frac{40}{e^{-0.892576}} = 97.656.\end{aligned}$$

We see that $k = -0.22314$ and $P_0 = 97.656$.

(b) The initial quantity is 97.656 and the quantity is decaying at a continuous rate of 22.314% per unit time.

35. **(a)** The continuous percent growth rate is 6%.
(b) We want $P_0a^t = 100e^{0.06t}$, so we have $P_0 = 100$ and $a = e^{0.06} = 1.0618$. The corresponding function is

$$P = 100(1.0618)^t.$$

The annual percent growth rate is 6.18%. An annual rate of 6.18% is equivalent to a continuous rate of 6%.

36. **(a)** Since $0.88 = 1 - 0.12$, the annual percent decay rate is 12%.
(b) We want $P_0e^{kt} = 25(0.88)^t$, so we have $P_0 = 25$ and $e^k = 0.88$. Since $e^k = 0.88$, we have $k = \ln(0.88) = -0.128$. The corresponding function is

$$P = 25e^{-0.128t}.$$

The continuous percent decay rate is 12.8%. A continuous rate of 12.8% is equivalent to an annual rate of 12%.

37. We find a with $a^t = e^{0.08t}$. Thus, $a = e^{0.08} = 1.0833$. The corresponding annual percent growth rate is 8.33%.

38. We find k with $e^{kt} = (1.10)^t$. We have $e^k = 1.10$ so $k = \ln(1.10) = 0.0953$. The corresponding continuous percent growth rate is 9.53%.

39. **(a)** Town D is growing fastest, since the rate of growth $k = 0.12$ is the largest.
(b) Town C is largest now, since $P_0 = 1200$ is the largest.
(c) Town B is decreasing in size, since k is negative ($k = -0.02$).

40. If production continues to decay exponentially at a continuous rate of 9.1% per year, the production $f(t)$ at time t years after 2008 is

$$f(t) = 84e^{-0.091t}.$$

In 2025, production is predicted to be

$$f(17) = 84e^{-0.091(17)} = 17.882 \text{ million barrels per day.}$$

41. **(a)** (i) $P = 1000(1.05)^t$; (ii) $P = 1000e^{0.05t}$
(b) (i) 1629; (ii) 1649

42. **(a)** $P = 5.97(1.0099)^t$
(b) Since $P = 5.97e^{kt} = 5.97(1.0099)^t$, we have

$$\begin{aligned} e^{kt} &= (1.0099)^t \\ kt &= t\ln(1.0099) \\ k &= 0.00985. \end{aligned}$$

Thus, $P = 5.97e^{0.00985t}$.
(c) The annual growth rate is 0.99%, while the continuous growth rate is 0.985%. Thus the growth rates are not equal, though for small growth rates (such as these), they are close. The annual growth rate is larger.

43. We find k with $e^{kt} = (1.032)^t$. We have $e^k = 1.032$ so $k = \ln(1.032) = 0.0315$. An equivalent formula for gross world product as a function of time is

$$W = 123.3e^{0.0315t}.$$

Notice that the annual growth rate of 3.2% is equivalent to a continuous growth rate of 3.15%.

44. We find k with $e^{kt} = (1.0103)^t$. We have $e^k = 1.0103$ so $k = \ln(1.0103) = 0.01025$. An equivalent formula for the population of the world as a function of time is

$$P = 7.68e^{0.01025t}.$$

Notice that the annual growth rate of 1.03% is equivalent to a continuous growth rate of 1.025%.

45. **(a)** Substituting $t = 0.5$ in $P(t)$ gives

$$\begin{aligned} P(0.5) &= 1000e^{-0.5(0.5)} \\ &= 1000e^{-0.25} \\ &\approx 1000(0.7788) \\ &= 778.8 \approx 779 \text{ trout.} \end{aligned}$$

Substituting $t = 1$ in $P(t)$ gives

$$\begin{aligned} P(1) &= 1000e^{-0.5} \\ &\approx 1000(0.6065) \\ &= 606.5 \approx 607 \text{ trout.} \end{aligned}$$

(b) Substituting $t = 3$ in $P(t)$ gives

$$\begin{aligned} P(3) &= 1000e^{-0.5(3)} \\ &= 1000e^{-1.5} \\ &\approx 1000(0.2231) \\ &= 223.1 \approx 223 \text{ trout.} \end{aligned}$$

This tells us that after 3 years the fish population has gone down to about 22% of the initial population.

(c) We are asked to find the value of t such that $P(t) = 100$. That is, we are asked to find the value of t such that

$$100 = 1000e^{-0.5t}.$$

Solving this gives

$$\begin{aligned} 100 &= 1000e^{-0.5t} \\ 0.1 &= e^{-0.5t} \\ \ln 0.1 &= \ln e^{-0.5t} = -0.5t \\ t &= \frac{\ln 0.1}{-0.5} \\ &\approx 4.6. \end{aligned}$$

Thus, after roughly 4.6 years, the trout population will have decreased to 100.

(d) A graph of the population is given in Figure 1.56.

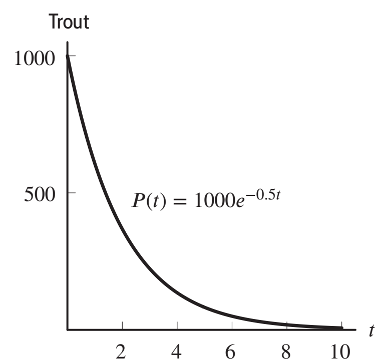


Figure 1.56

Looking at the graph, we see that as time goes on there are fewer and fewer trout in the pond. A possible cause for this may be the presence of predators, such as fishermen.The rate at which the fish die decreases due to the fact that there are fewer fish in the pond, which means that there are fewer fish that have to try to survive. Notice that the survival chances of any given fish remains the same, it is just the overall mortality that is decreasing.

46. (a) Since the percent rate of growth is constant and given as a continuous rate, we use the exponential function $S = 7.986e^{0.025t}$.

(b) In 2022, we have $t = 3$ and $S = 7.986e^{0.025\cdot 3} = 8.608$ billion dollars.

(c) Using the graph in Figure 1.57, we see that $S = 10$ at approximately $t = 9$. Solving $7.986e^{0.025t} = 10$, we have

$$\begin{aligned} e^{0.025t} &= \frac{10}{7.986} \\ 0.025t &= \ln\left(\frac{10}{7.986}\right) \\ t &= \frac{\ln(10/7.986)}{0.025} = 8.996 \text{ years.} \end{aligned}$$

In other words, assuming this growth rate continues, annual net sales are predicted to pass 10 billion dollars in 2028.

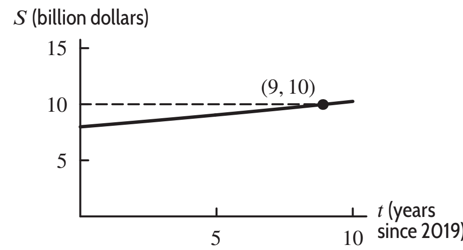


Figure 1.57

47. **(a)** The current revenue is $R(0) = 5e^{-0} = 5$ million dollars. In two years time the revenue will be $R(2) = 5e^{-0.15(2)} = 3.704$ million dollars.

(b) To find the time t at which the revenue has fallen to \$2.7 million we solve the equation $5e^{-0.15t} = 2.7$. Divide both sides by 5 giving
$$e^{-0.15t} = 0.54.$$
Now take natural logs of both sides:
$$-0.15t = -0.6162,$$
so $t = 4.108$. The revenue will have fallen to \$2.7 million early in the fifth year.

48. **(a)** We use the formula $P = P_0e^{kt}$, so we have
$$P = 50{,}000e^{0.045t},$$
where t is the number of years since 2014. See Figure 1.58.

(b) The year 2020 is when $t = 6$, so we have
$$P = 50{,}000e^{0.045(6)} = 65{,}498.2.$$
We can predict that the population will be about 65,500 in the year 2020.

(c) We see in Figure 1.58 that $P = 100{,}000$ approximately when $t = 15$, so the doubling time is about 15 years.

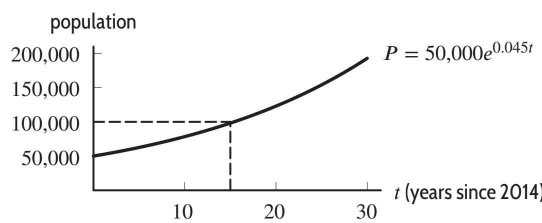


Figure 1.58: The doubling time is about 15 years

To find the doubling time more precisely, we use logarithms. Solving for the value of t for which $P = 100{,}000$:
$$\begin{aligned}100{,}000 &= 50{,}000e^{0.045t}\\ 2 &= e^{0.045t}\\ \ln 2 &= 0.045t\\ t &= \frac{\ln 2}{0.045} = 15.403.\end{aligned}$$
The doubling time is 15.403 years.

49. **(a)** Since the percent increase in deaths during a year is constant for constant increase in pollution, the number of deaths per year is an exponential function of the quantity of pollution. If Q_0 is the number of deaths per year without pollution, then the number of deaths per year, Q, when the quantity of pollution is x micrograms per cubic meter of air is
$$Q = Q_0(1.0033)^x.$$

(b) We want to find the value of x making $Q = 2Q_0$, that is,

$$Q_0(1.0033)^x = 2Q_0.$$

Dividing by Q_0 and then taking natural logs yields

$$\ln((1.0033)^x) = x\ln 1.0033 = \ln 2,$$

so

$$x = \frac{\ln 2}{\ln 1.0033} = 210.391.$$

When there are 210.391 micrograms of pollutants per cubic meter of air, respiratory deaths per year are double what they would be in the absence of air pollution.

50. If C_0 is the concentration of NO_2 on the road, then the concentration x meters from the road is

$$C = C_0 e^{-0.0254x}.$$

We want to find the value of x making $C = C_0/2$, that is,

$$C_0 e^{-0.0254x} = \frac{C_0}{2}.$$

Dividing by C_0 and then taking natural logs yields

$$\ln\left(e^{-0.0254x}\right) = -0.0254x = \ln\left(\frac{1}{2}\right) = -0.6931,$$

so

$$x = 27 \text{ meters}.$$

At 27 meters from the road the concentration of NO_2 in the air is half the concentration on the road.

51. **(a)** $B(t) = B_0 e^{0.067t}$

(b) $P(t) = P_0 e^{0.033t}$

(c) If the initial price is \$50, then

$$B(t) = 50e^{0.067t}$$
$$P(t) = 50e^{0.033t}.$$

We want the value of t such that

$$\begin{aligned} B(t) &= 2P(t) \\ 50e^{0.067t} &= 2\cdot 50e^{0.033t} \\ \frac{e^{0.067t}}{e^{0.033t}} &= e^{0.034t} = 2 \\ t &= \frac{\ln 2}{0.034} = 20.387 \text{ years}. \end{aligned}$$

Thus, when $t = 20.387$ the price of the textbook was predicted to be double what it would have been had the price risen by inflation only. This occurred in the year 2000.

52. The population of China, C, in billions, is given by

$$C = 1.379(1.0041)^t$$

where t is time in years from 2017, and the population of India, I, in billions, is given by

$$I = 1.282(1.0117)^t.$$

The two populations will be equal when $C = I$, thus, we must solve the equation:

$$1.379(1.0041)^t = 1.282(1.0117)^t$$

for t, which leads to

$$\frac{1.379}{1.282} = \frac{(1.0117)^t}{(1.0041)^t} = \left(\frac{1.0117}{1.0041}\right)^t.$$

Taking logs on both sides, we get

$$t\ln\frac{1.0117}{1.0041} = \ln\frac{1.379}{1.282},$$

so

$$t = \frac{\ln(1.379/1.282)}{\ln(1.0117/1.0041)} = 9.67 \text{ years}.$$

This model predicts the population of India will exceed that of China in 2026.

53. If t is time in decades, then the number of vehicles, V, in millions, is given by

$$V = 246(1.155)^t.$$

For time t in decades, the number of people, P, in millions, is given by

$$P = 308.7(1.097)^t.$$

There is an average of one vehicle per person when $\frac{V}{P} = 1$, or $V = P$. Thus, we solve for t in the equation:

$$246(1.155)^t = 308.7(1.097)^t,$$

which leads to

$$\left(\frac{1.155}{1.097}\right)^t = \frac{(1.155)^t}{(1.097)^t} = \frac{308.7}{246}$$

Taking logs on both sides, we get

$$t \ln \frac{1.155}{1.097} = \ln \frac{308.7}{246},$$

so

$$t = \frac{\ln(308.7/246)}{\ln(1.155/1.097)} = 4.41 \text{ decades.}$$

This model predicts one vehicle per person in 2054.

54. **(a)** Using the properties of the natural logarithm, we have

$$\begin{aligned} \ln P &= \ln(P_0 e^{kt}) \\ &= \ln P_0 + \ln(e^{kt}) \\ &= \ln P_0 + kt \end{aligned}$$

Thus, $y = \ln P$ is of the form $y = b + mt$, so it is a line.

(b) The slope of $y = \ln P = \ln P_0 + kt$ is k.

(c) The vertical intercept of $y = \ln P = \ln P_0 + kt$ is $\ln P_0$.

55. **(a)** We see in Figure 1.85 that $\ln(P(t))$ is a straight line. From Problem 54 we know the slope of this line is the continuous growth rate k and the vertical intercept is $\ln(P_0)$.

We use the points $(t, \ln(P)) = (10, 13.75)$ and $(t, \ln(P)) = (60, 14.2)$ to estimate the slope:

$$k = \frac{14.2 - 13.75}{60 - 10} = 0.009.$$

This tells us that daily new cases in Florida were growing approximately exponentially with a continuous growth rate $k = 0.009 = 0.9\%$ per day.

(b) We estimate the vertical intercept to be 13.65, so $13.65 = \ln(P_0)$ which gives $P_0 = e^{13.65} = 847{,}460.916 \approx 850{,}000$ cases.

56. **(a)** If the graph of $\ln(P)$ is a straight line with slope m and intercept b, then $\ln(P) = b + mt$, so

$$\begin{aligned} \ln(P) &= b + mt \\ P &= e^{b+mt} = e^b e^{mt} \\ P &= Ce^{mt} \qquad \text{where } C = e^b \text{ is a constant.} \end{aligned}$$

This tells us that the slope m of the line is the continuous growth rate of P.

On the first straight segment in Figure 1.86, we estimate

$$\text{Slope } = m_1 = \frac{3}{10} = 0.3,$$

so the number of confirmed cases was growing at a continuous rate of about 30% per day before day $t = 10$.

On the second straight segment in Figure 1.86, we estimate

$$\text{Slope } = m_2 = \frac{10 - 8}{70 - 35} = 0.057,$$

so the number of confirmed cases was growing at a continuous rate of about 5.7% per day after day $t = 10$.

(b) On March 27, 2020, the South African government imposed a strict lockdown.[2] This date corresponds to $t = 9$.

[2] https://en.wikipedia.org /wiki/COVID-19_pandemic_in_South_Africa, accessed on January 19, 2021.

Solutions for Section 1.7

1. At the doubling time, $t = 10$, we have $p = 2p_0$. Thus

$$\begin{aligned} p_0 e^{10k} &= 2p_0 \\ e^{10k} &= 2 \\ 10k &= \ln 2 \\ k &= \frac{1}{10}\ln 2 = 0.0693. \end{aligned}$$

The function $p = p_0 e^{0.0693t}$ has doubling time equal to 10.

2. At the doubling time, $t = 0.4$, the we have $p = 2p_0$. Thus

$$\begin{aligned} p_0 e^{0.4k} &= 2p_0 \\ e^{0.4k} &= 2 \\ 0.4k &= \ln 2 \\ k &= \frac{1}{0.4}\ln 2 = 1.733. \end{aligned}$$

The function $p = p_0 e^{1.733t}$ has doubling time equal to 0.4.

3. In both cases the initial deposit was \$20. Compounding continuously earns more interest than compounding annually at the same interest rate. Therefore, curve A corresponds to the account which compounds interest continuously and curve B corresponds to the account which compounds interest annually. We know that this is the case because curve A is higher than curve B over the interval, implying that bank account A is growing faster, and thus is earning more money over the same time period.

4. Since $y(0) = Ce^0 = C$ we have that $C = 2$. Similarly, substituting $x = 1$ gives $y(1) = 2e$ so

$$2e \quad = 1.$$

Rearranging gives $e \quad = 1/2$. Taking logarithms we get $\quad = \ln(1/2) = -\ln 2 = -0.693$. Finally,

$$y(2) = 2e^{2(-\ln 2)} = 2e^{-2\ln 2} = \frac{1}{2}.$$

5. **(a)** If the interest is added only once a year (i.e. annually), then at the end of a year we have $1.015x$ where x is the amount we had at the beginning of the year. After two years, we'll have $1.015(1.015x)$ and after eight years, we'll have $(1.015)^8 x$. Since we started with \$1000, after eight years we'll have $(1.015)^8(1000) \approx \$1126.49$.

(b) If an initial deposit of $\$P_0$ is compounded continuously at interest rate r then the account will have $P = P_0 e^{rt}$ after t years. In this case, $P = P_0 e^{rt} = 1000e^{(0.015)(8)} \approx \$1127.50$.

6. We use the equation $B = Pe^{rt}$, where P is the initial principal, t is the time in years since the deposit is made to the account, r is the annual interest rate and B is the balance. After $t = 5$ years, we will have

$$B = (10{,}000)e^{(0.08)5} = (10{,}000)e^{0.4} \approx \$14{,}918.25.$$

7. We use the equation $B = Pe^{rt}$. We want to have a balance of $B = \$20{,}000$ in $t = 6$ years, with an annual interest rate of 10%.

$$\begin{aligned} 20{,}000 &= Pe^{(0.1)6} \\ P &= \frac{20{,}000}{e^{0.6}} \\ &\approx \$10{,}976.23. \end{aligned}$$

8. We know that the formula for the balance in an account after t years is

$$P(t) = P_0 e^{rt}$$

where P_0 is the initial deposit and r is the nominal rate. In our case the initial deposit is \$15,000 that is

$$P_0 = 15{,}000$$

and the nominal rate is

$$r = 0.015.$$

Thus we get

$$P(t) = 15{,}000e^{0.015t}.$$

We are asked to find t such that $P(t) = 20{,}000$, that is we are asked to solve

$$\begin{aligned} 20{,}000 &= 15{,}000e^{0.015t} \\ e^{0.015t} &= \frac{20{,}000}{15{,}000} = \frac{4}{3} \\ \ln e^{0.015t} &= \ln(4/3) \\ 0.015t &= \ln 4 - \ln 3 \\ t &= \frac{\ln 4 - \ln 3}{0.015} \approx 19.18. \end{aligned}$$

Thus after roughly 19.18 years there will be \$20,000 in the account.

9. The formula which models compounding interest continuously for t years is $P_0 e^{rt}$ where P_0 is the initial deposit and r is the interest rate. So if we want to double our money while getting 1.25% interest, we want to solve the following for t:

$$\begin{aligned} P_0 e^{0.0125t} &= 2P_0 \\ e^{0.0125t} &= 2 \\ 0.0125t &= \ln(2) \\ t &= \frac{\ln 2}{0.0125} \approx 55.5 \text{ years} \end{aligned}$$

Alternatively, by the Rule of 70, we have

$$\text{Doubling time} \approx \frac{70}{1.25} = 56 \text{ years.}$$

10. We know that the formula for the total process is

$$P(t) = P_0 a^t$$

where P_0 is the initial size of the process and $a = 1 + 0.07 = 1.07$. We are asked to find the doubling time. Thus, we solve

$$\begin{aligned} 2P_0 &= P_0(1.07)^t \\ 1.07^t &= 2 \\ \ln(1.07^t) &= \ln 2 \\ t \ln 1.07 &= \ln 2 \\ t &= \frac{\ln 2}{\ln 1.07} \approx 10.24. \end{aligned}$$

The doubling time is roughly 10.24 years.

Alternatively we could have used the "rule of 70" to estimate that the doubling time is approximately

$$\frac{70}{7} = 10 \text{ years.}$$

11. **(a)** The concentration decreases from 100 to 50 during the time from $t = 0$ to $t = 5$, a period of 5 years.

(b) The concentration decreases from 50 to 25 during the time from $t = 5$ to $t = 10$, a period of 5 years.

(c) The decay is exponential if the half-life of the concentration is the same beginning at any concentration level. We only checked at two levels, 100 and 50, but for those the half-lives were both 5 years. This is some evidence that the decay might be exponential.

Also, if we look at any other 5-year period, we can check that the concentration appears to decreases by half.

(d) Since the initial value of concentration is 100, an exponential model is of the form $C = 100e^{kt}$. We can evaluate k by using the value $C = 25$ at $t = 10$.

$$\begin{aligned} 25 &= 100e^{10k} \\ 0.25 &= e^{10k} \\ \ln(0.25) &= 10k \\ k &= -0.139. \end{aligned}$$

An exponential model is given by

$$C = 100e^{-0.139t}.$$

Alternative solution. Using the half-life $T = 5$ years, we have

$$C = 100 \cdot 2^{-t/T} = 100 \cdot 2^{-t/5}.$$

12. Every 2 hours, the amount of nicotine remaining is reduced by $1/2$. Thus, after 4 hours, the amount is $1/2$ of the amount present after 2 hours.

Table 1.8

t (hours)	0	2	4	6	8	10
Nicotine (mg)	0.4	0.2	0.1	0.05	0.025	0.0125

From the table it appears that it will take just over 6 hours for the amount of nicotine to reduce to 0.04 mg.

13. **(a)** We have $W = 591e^{0.092t}$.

(b) We substitute $W = 1000$ and solve for t:

$$\begin{aligned} 1000 &= 591e^{0.092t} \\ \frac{1000}{591} &= e^{0.092t} \\ \ln\frac{1000}{591} &= 0.092t \\ t &= \frac{\ln\frac{1000}{591}}{0.092} = 5.72. \end{aligned}$$

Wind capacity is expected to pass 1000 gigawatts in 2024.

14. Let $f(t)$ be oil consumption t years after 2015. Since consumption is exponential, and $f(0) = 11.986$ million barrels per day, we have

$$f(t) = 11.986e^{kt}.$$

In 2018, when $t = 3$, consumption was 13.525, so

$$\begin{aligned} f(3) &= 13.525 \\ 11.986e^{3t} &= 13.525 \\ 3t &= \ln\left(\frac{13.525}{11.986}\right) \\ k &= 0.040267. \end{aligned}$$

Our model predicts Chinese oil consumption in 2025, when $t = 10$, to be

$$f(10) = 11.986e^{0.040267(10)} = 17.929 \text{ million barrels per day.}$$

15. The doubling time for $N(t)$ is the time t such that $N(t) = 2N(0)$. Since $N(t) = N_0e^{0.054t}$ and $N(0) = N_0$ we have

$$\begin{aligned} N_0e^{0.054t} &= 2N_0 \\ e^{0.054t} &= 2 \\ 0.054t &= \ln 2 \\ t &= \frac{1}{0.054}\ln 2 = 12.84 \end{aligned}$$

During the 5 years from January 2014 to January 2019, the doubling time for the number of Wikipedia articles was about 12.84 years.

16. **(a)** If the CD pays 9% interest over a 10-year period, then $r = 0.09$ and $t = 10$. We must find the initial amount P_0 if the balance after 10 years is $P = \$12{,}000$. Since the compounding is annual, we use

$$\begin{aligned} P &= P_0(1+r)^t \\ 12{,}000 &= P_0(1.09)^{10}, \end{aligned}$$

and solve for P_0:

$$P_0 = \frac{12{,}000}{(1.09)^{10}} \approx \frac{12{,}000}{2.36736} = 5068.93.$$

The initial deposit must be \$5068.93 if interest is compounded annually.

(b) On the other hand, if the CD interest is compounded continuously, we have

$$\begin{aligned} P &= P_0e^{rt} \\ 12{,}000 &= P_0e^{(0.09)(10)}. \end{aligned}$$

Solving for P_0 gives

$$P_0 = \frac{12{,}000}{e^{(0.09)(10)}} = \frac{12{,}000}{e^{0.9}} \approx \frac{12{,}000}{2.45960} = 4878.84.$$

The initial deposit must be \$4878.84 if the interest is compounded continuously. Notice that to achieve the same result, continuous compounding requires a smaller initial investment than annual compounding. This is to be expected, since continuous compounding yields more money.

17. **(a)** Since sales of electronic devices doubled in 12 years, sales in 1997 were $438/2 = 219$ million devices. Because part (b) asks for the annual percentage growth rate (not a continuous growth rate), we use an exponential function of the form $S(t) = S_0b^t$, where b is the annual growth factor. We have

$$438 = 219b^{12},$$

so $2 = b^{12}$ and $b = 2^{1/12} = 1.05946$. Thus,

$$S(t) = 219(1.05946)^t.$$

(b) The annual growth rate was 5.946%.

18. **(a)** Let P represent the population of the world (in billions), and let t represent the number of years since 2017. Then we have $P = 7.41(1.011)^t$.

(b) According to this formula, the population of the world in the year 2025 (at $t = 8$) will be $P = 7.41(1.011)^8 = 8.09$ billion people.

(c) The graph is shown in Figure 1.59. The population of the world has doubled when $P = 14.82$; we see on the graph that this occurs at approximately $t = 63.4$. Under these assumptions, the doubling time of the world's population is about 63.4 years.

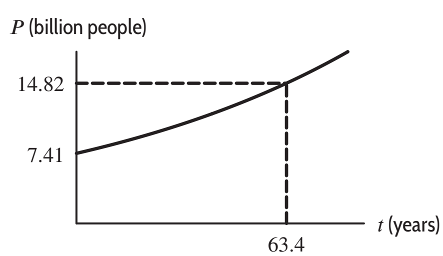


Figure 1.59

19. (a) Since the initial amount of caffeine is 100 mg and the exponential decay rate is -0.17, we have $A = 100e^{-0.17t}$.

(b) See Figure 1.60. We estimate the half-life by estimating t when the caffeine is reduced by half (so $A = 50$); this occurs at approximately $t = 4$ hours.

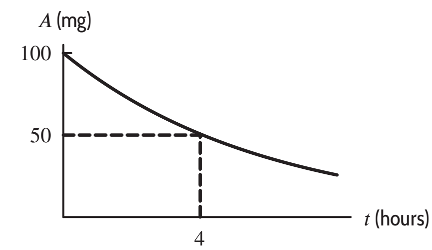


Figure 1.60

(c) We want to find the value of t when $A = 50$:

$$\begin{aligned} 50 &= 100e^{-0.17t} \\ 0.5 &= e^{-0.17t} \\ \ln 0.5 &= -0.17t \\ t &= 4.077. \end{aligned}$$

The half-life of caffeine is about 4.077 hours. This agrees with what we saw in Figure 1.60.

20. (a) Since $P(t)$ is an exponential function, it will be of the form $P(t) = P_0 a^t$, where P_0 is the initial population and a is the base. $P_0 = 200$, and a 5% growth rate means $a = 1.05$. Thus, we get

$$P(t) = 200(1.05)^t.$$

(b) The graph is shown in Figure 1.61.

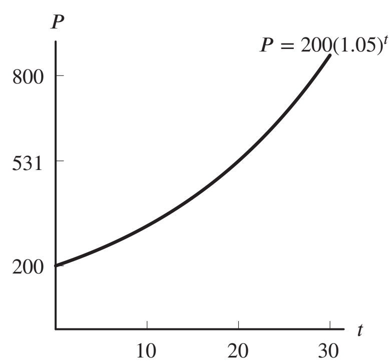


Figure 1.61

(c) Evaluating gives that $P(10) = 200(1.05)^{10} \approx 326$.

(d) From the graph we see that the population is 400 at about $t = 15$, so the doubling time appears to be about 15 years.

21. (a) The pressure P at 6194 meters is given in terms of the pressure P_0 at sea level to be

$$\begin{aligned} P &= P_0 e^{-0.00012h} \\ &= P_0 e^{(-0.00012)6194} \\ &= P_0 e^{-0.74328} \\ &\approx 0.4756 P_0 \quad \text{or about 47.6\% of sea level pressure.} \end{aligned}$$

(b) At $h = 12{,}000$ meters, we have

$$\begin{aligned} P &= P_0 e^{-0.00012h} \\ &= P_0 e^{(-0.00012)12{,}000} \\ &= P_0 e^{-1.44} \\ &\approx 0.2369 P_0 \quad \text{or about 23.7\% of sea level pressure.} \end{aligned}$$

22. We use $P = P_0 e^{kt}$. Since the half-life is 3 days, we can find k:

$$\begin{aligned} P &= P_0 e^{kt} \\ 0.50P_0 &= P_0 e^{k(3)} \\ 0.50 &= e^{3k} \\ \ln(0.50) &= 3k \\ k &= \frac{\ln(0.50)}{3} \approx -0.231. \end{aligned}$$

After one day,

$$P = P_0 e^{-0.231(1)} = P_0(0.79).$$

About 79% of the original dose is in the body one day later. After one week (at $t = 7$), we have

$$P = P_0 e^{-0.231(7)} = P_0(0.20).$$

After one week, 20% of the dose is still in the body.

23. If the rate of increase is $r\%$ then each year the output will increase by $1 + r/100$ so that after five years the level of output is

$$20{,}000\left(1 + \frac{r}{100}\right)^5.$$

To achieve a final output of 30,000, the value of r must satisfy the equation

$$20{,}000\left(1 + \frac{r}{100}\right)^5 = 30{,}000.$$

Dividing both sides by 20,000 gives

$$\left(1 + \frac{r}{100}\right)^5 = 1.5,$$

so $(1 + \frac{r}{100}) = 1.5^{1/5} = 1.0845$. Solving for r gives an annual growth rate of 8.45%.

24. **(a)** Using the formula for exponential decay, $A = A_0 e^{-kt}$, with $A_0 = 10.32$. Since the half-life is 12 days, when $t = 12$ we have $A = \frac{10.32}{2} = 5.16$. Substituting this into the formula, we have

$$\begin{aligned} 5.16 &= 10.32e^{-12k} \\ 0.5 &= e^{-12k}, \quad \text{and, taking ln of both sides} \\ \ln 0.5 &= -12k \\ k &= -\frac{\ln(0.5)}{12} \approx 0.057762. \end{aligned}$$

The full equation is

$$A = 10.32e^{-0.057762t}.$$

(b) We want to solve for t when $A = 1$. Substituting into the equation from part (a) yields

$$\begin{aligned} 1 &= 10.32e^{-0.057762t} \\ \frac{1}{10.32} \approx 0.096899 &= e^{-0.057762t} \\ \ln 0.096899 &= -0.057762t \\ t &\approx \frac{-2.33408}{-0.057762} = 40.41 \text{ days}. \end{aligned}$$

25. **(a)** Since $P(t)$ is an exponential function, it will be of the form $P(t) = P_0 a^t$. We have $P_0 = 1$, since 100% is present at time $t = 0$, and $a = 0.976$, because each year 97.6% of the contaminant remains. Thus,

$$P(t) = (0.976)^t.$$

(b) The graph is shown in Figure 1.62.

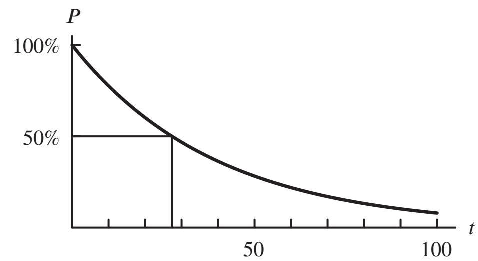


Figure 1.62

(c) The half-life is about 29 years, since $(0.976)^{29} \approx 0.5$.
(d) At time $t = 100$ there is about 9% remaining, since $(0.976)^{100} \approx 0.09$.

26. (a) We use an exponential function in the form $H = H_0e^{kt}$. Since t is in years since 1985, the initial value of H is 2.5 and we have $H = 2.5e^{kt}$. We use the fact that $H = 36.7$ when $t = 30$ to find k:

$$\begin{aligned} H &= 2.5e^{kt} \\ 36.7 &= 2.5e^{k\cdot 30} \\ 14.68 &= e^{30k} \\ \ln(14.68) &= 30k \\ k &= \frac{\ln(14.68)}{30} = 0.0895. \end{aligned}$$

The formula is $H = 2.5e^{0.0895t}$. See Figure 1.63.

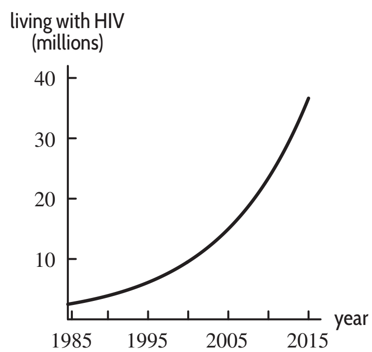


Figure 1.63: HIV infections worldwide

(b) The annual continuous percent change is an increase of approximately 8.95% per year.

27. (a) At $t = 0$, the population is 1000. The population doubles (reaches 2000) at about $t = 4$, so the population doubled in about 4 years.
(b) At $t = 3$, the population is about 1700. The population reaches 3400 at about $t = 7$. The population doubled in about 4 years.
(c) No matter when you start, the population doubles in 4 years.

28. We have $P_0 = 500$, so $P = 500e^{kt}$. We can use the fact that $P = 1{,}500$ when $t = 2$ to find k:

$$\begin{aligned} 1{,}500 &= 500e^{kt} \\ 3 &= e^{2k} \\ \ln 3 &= 2k \\ k &= \frac{\ln 3}{2} \approx 0.5493. \end{aligned}$$

The size of the population is given by

$$P = 500e^{0.5493t}.$$

At $t = 5$, we have

$$P = 500e^{0.5493(5)} \approx 7794.$$

29. We know that a function modeling the quantity of substance at time t will be an exponential decay function satisfying the fact that at $t = 10$ hours, 4% of the substance has decayed, or in other words 96% of the substance is left. Thus, a formula for the quantity of substance left at time t is

$$Q(t) = Q_0 e^{kt},$$

where Q_0 is the initial quantity. We solve for k:

$$\begin{aligned} 0.96Q_0 &= Q_0 e^{k\cdot 10} \\ 0.96 &= e^{k\cdot 10} \\ \ln 0.96 &= 10k \\ k &= \frac{\ln 0.96}{10} \approx -0.004. \end{aligned}$$

We are asked to find the time t at which the quantity of material will be half of the initial quantity. Thus, we are asked to solve

$$\begin{aligned} 0.5Q_0 &= Q_0 e^{-0.004t} \\ 0.5 &= e^{-0.004t} \\ \ln 0.5 &= -0.004t \\ t &= \frac{\ln 0.5}{-0.004} \approx 173. \end{aligned}$$

Thus, the half life of the material is roughly 173 hours.

30. We use $Q = Q_0 e^{kt}$ and let $t = 0$ represent 8 am. Then $Q_0 = 100$, so

$$Q = 100e^{kt}.$$

For the husband, the half-life is four hours. We solve for k:

$$\begin{aligned} 50 &= 100e^{k\cdot 4} \\ 0.5 &= e^{4k} \\ \ln 0.5 &= 4k \\ k &= \frac{\ln 0.5}{4} \approx -0.173. \end{aligned}$$

We have $Q = 100e^{-0.173t}$. At 10 pm, we have $t = 14$ so the amount of caffeine for the husband is

$$Q = 100e^{-0.173(14)} = 8.87 \text{ mg} \approx 9 \text{ mg}.$$

For the pregnant woman, the half-life is 10. Solving for k as above, we obtain

$$k = \frac{\ln 0.5}{10} = -0.069.$$

At 10 pm, the amount of caffeine for the pregnant woman is

$$Q = 100e^{-0.069(14)} \approx 38 \text{ mg}.$$

31. **(a)** Since the population of koalas is growing exponentially, we have $P = P_0 e^{kt}$. Since the initial population is 5000, we have $P_0 = 5000$, so

$$P = 5000e^{kt}.$$

We use the fact that $P = 27{,}000$ in 2005 (when $t = 9$) to find k:

$$27{,}000 = 5000e^{k(9)}$$

Divide both sides by 5000:

$$\frac{27{,}000}{5000} = e^{9k}$$

Take logs of both sides:

$$\ln\left(\frac{27{,}000}{5000}\right) = 9k$$

Divide both sides by 9:

$$k = \frac{\ln(27{,}000/5000)}{9} = 0.187.$$

The koala population increased at a continuous rate of 18.7% per year.

(b) The population, for t in years since 1996, is given by

$$P = 5000e^{0.187t}.$$

(c) In the year 2020, we have $t = 24$, and using the exact value of $k = (\ln(27/5))/9$, we have

$$P = 5000e^{24\ln(27/5)/9} = 448{,}766 \text{ koalas}.$$

Thus, if growth continues at the same rate, there will be nearly half a million koalas in 2020.

32. **(a)** A daily growth rate of 3.4% corresponds to a daily growth factor of 1.034. If this is constant through December 31, 2020, we predict

$$\text{Total number of cases on December 31, 2020 } = P(60) = 337{,}245 \cdot (1.034)^{60} = 2{,}507{,}135.723.$$

That is, around 2.5 million cases.

(b) We want to find a new growth factor a so that

$$\begin{aligned} 337{,}245 \cdot a^{60} &= \frac{2{,}507{,}135.723}{2} \\ a^{60} &= \frac{2{,}507{,}135.723}{2(337{,}245)} = 3.717 \\ a &= (3.717)^{1/60} = 1.022. \end{aligned}$$

If the daily growth factor is reduced to 1.022 for the period from November 1 to December 31, 2020, then the number of total cases by December 31, 2020 will be half of what continuation with the growth factor of 1.034 predicts. In other words, if the growth rate can be cut from 3.4% per day to 2.2% per day, then the number of total cases by December 31, 2020 would be cut in half.

33. Let $t = 0$ be April 14. The total number of confirmed cases can be modeled by

$$P = 2979(1.051)^t.$$

To find when $P = 50$ million cases, we set up the equation and solve for t.

$$\begin{aligned} 2979(1.051)^t &= 50{,}000{,}000 \\ (1.051)^t &= 16{,}784.15576 \\ t &= \frac{\ln 16{,}784.15576}{\ln 1.051} \\ t &= 195.573 \text{ days} \end{aligned}$$

Thus, the total number of cases would have reached the population of Colombia in about 196 days; that is, six and a half months.

34. With t in days since April 14 and P_0 the total number of cases on April 14, the total number of cases at time t is given by

$$P = P_0(1.051)^t,$$

(a) The doubling time can be found by setting P equal to $2P_0$ and solving for t:

$$\begin{aligned} 2 \cdot P_0 &= P_0(1.051)^t \\ 2 &= (1.051)^t \\ \ln 2 &= t \ln 1.051 \\ t &= \frac{\ln 2}{\ln 1.051} = 13.935 \text{ days}. \end{aligned}$$

Thus, at this rate, it takes about two weeks for the number of confirmed cases to double.

(b) We set up an equation similar to in part (a) but with 10 instead of 2, and solve for t:

$$\begin{aligned} 10 \cdot P_0 &= P_0(1.051)^t \\ 10 &= (1.051)^t \\ \ln 10 &= t \ln 1.051 \\ t &= \frac{\ln 10}{\ln 1.051} = 46.29 \text{ days}. \end{aligned}$$

Thus, at this rate, it takes about 1.5 months for the number of cases to increase by a factor of 10. (Note, this is less than 5 times 2 weeks.)

35. **(a)** If the daily growth rate is 33.33%, the daily growth factor is $1 + 0.3333 = 1.3333$. We see that

$$\text{Growth over 3 days } = (1.3333)^3 = 2.37 = 1 + 1.37.$$

So a 33.33% daily growth corresponds to a 137% increase over 3 days, which is larger than a 100% increase.

(b) We want a so that $2P_0 = P_0 a^3$. We take the cube root:

$$\begin{aligned} 2P_0 &= P_0 a^3 \\ 2 &= a^3 \\ 2^{1/3} &= a \\ a &= 1.2599. \end{aligned}$$

Thus, $P = P_0 a^t = P_0(1.2599)^t$, and the daily growth rate is about 26%.

36. Since P doubles every 10 days we know P is exponential, so $P = P_0 a^t$ for some a. The doubling time of 10 days tells us that

$$2P_0 = P_0 a^{10}.$$

We find a by taking 10$^{\text{th}}$ roots:

$$\begin{aligned} 2P_0 &= P_0 a^{10} \\ 2 &= a^{10} \\ 2^{1/10} &= a \\ a &= 1.0718. \end{aligned}$$

Thus, $P = P_0(1.0718)^t$. To find the time it takes to multiply by a factor of 10 we set up the equation $10P_0 = P$ and solve for t:

$$\begin{aligned} 10P_0 &= P_0(1.0718)^t \\ 10 &= (1.0718)^t \\ \ln 10 &= \ln((1.0718)^t) \\ \ln 10 &= t \ln(1.0718) \\ t &= \ln 10/\ln(1.0718) \\ t &= 33.2074 \end{aligned}$$

This tells us that in $t = 33.207$ days, the number of cases of the new variant P will have multiplied by 10.

37. **(a)** Assuming the US population grows exponentially, we have population $P(t) = 282.2e^{kt}$ at time t years after 2000. Using the 2018 population, which corresponds to $t = 18$, we have

$$\begin{aligned} 327.2 &= 282.2e^{18k} \\ k &= \frac{\ln(327.2/282.2)}{18} = 0.0082198. \end{aligned}$$

We want to find the time t in which

$$\begin{aligned} 350 &= 282.2e^{0.0082198t} \\ t &= \frac{\ln(350/282.2)}{0.0082198} = 26.195 \text{ years.} \end{aligned}$$

This model predicts the population to go over 350 million 26.195 years after 2000, in the year 2026.

(b) Evaluate $P = 282.2e^{0.0082198t}$ for $t = 20$ to find $P = 332.6$ million people.

38. We know that the world population is an exponential function over time. Thus the function for world population will be of the form

$$P(t) = P_0 a^t$$

where P_0 is the initial population, t is measured in years after 2020 and a is the growth factor. We know that the initial population is the population in the year 2020 so

$$P_0 = 7.68 \text{ billion.}$$

We are also told that in the year 2037 the population will be 9 billion, that is

$$P(17) = 9 \text{ billion}.$$

Solving for a we get

$$\begin{aligned} 9 &= 7.68a^{17} \\ \frac{9}{7.68} &= a^{17} \\ \left(\frac{9}{7.68}\right)^{1/17} &= a \\ a &\approx 1.009 \end{aligned}$$

Since the growth factor is 1.009, we know the annual growth rate is projected to be 0.9%.

39. We assume exponential decay and solve for k using the half-life:

$$e^{-k(5730)} = 0.5 \quad \text{so} \quad k = 1.21 \cdot 10^{-4}.$$

Now find t, the age of the painting:

$$e^{-1.21\cdot 10^{-4}t} = 0.995, \quad \text{so} \quad t = \frac{\ln 0.995}{-1.21 \cdot 10^{-4}} = 41.43 \text{ years}.$$

Since Vermeer died in 1675, the painting is a fake.

40. **(a)** The 1.109 represents the number of food parcels, in millions, when $t = 0$. Approximately 1,109,000 food parcels were supplied in 2015.

(b) The continuous growth rate is 0.12, or 12%, per year.

(c) Since $e^{0.12t} = (e^{0.12})^t = 1.127^t$, we see that the annual growth factor is 1.127. Thus, the annual growth rate is $1.127 - 1 = 0.127 = 12.7\%$ per year.

(d) Since the annual growth factor is only 1.127, the number of food parcels does not double each year. The doubling time is more than one year.

41. **(a)** We assume $f(t) = Ae^{-kt}$, where A is the initial population, so $A = 100{,}000$. When $t = 110$, there were 3200 tigers, so

$$3200 = 100{,}000e^{-k\cdot 110}$$

Solving for k gives

$$e^{-k\cdot 110} = \frac{3200}{100{,}000} = 0.032$$

$$k = -\frac{1}{110}\ln(0.032) = 0.0313 = 3.13\%$$

so

$$f(t) = 100{,}000e^{-0.0313t}.$$

(b) In 2000, the predicted number of tigers was

$$f(100) = 100{,}000e^{-0.0313(100)} = 4372.$$

In 2010, we know the number of tigers was 3200. The predicted percent reduction is

$$\frac{3200 - 4372}{4372} = -0.268 = -26.8\%.$$

Thus the actual decrease is larger than the predicted decrease.

42. For an exponentially growing population, $p = p_0e^{kt}$, that doubles in n days, we find the continuous growth rate k in terms of n. Since $p = 2p_0$ when $t = n$, we have

$$\begin{aligned} 2p_0 &= p_0e^{kn} \\ e^{kn} &= 2 \\ kn &= \ln 2 \\ k &= \frac{1}{n}\ln 2 \\ p &= p_0e^{(\ln 2)t/n}. \end{aligned}$$

Doubling time for the phytoplankton population, P, is 0.5 days, and for the foraminifera population, F, it is 5 days. If they both initially have the same population C, then after t days we have

$$P = Ce^{(\ln 2)t/0.5} = Ce^{1.3863t}$$
$$F = Ce^{(\ln 2)t/5} = Ce^{0.13863t}.$$

(a) We have

$$\begin{aligned} P &= 2F \\ Ce^{1.3863t} &= 2Ce^{0.13863t} \\ e^{1.3863t} &= 2e^{0.13863t} \\ 1.3863t &= \ln 2 + 0.13863t \\ 1.24767t &= \ln 2 \\ t &= 0.556 \text{ days.} \end{aligned}$$

The phytoplankton population is double the foraminifera population after 0.556 days, a little over 13 hours.

(b) We have

$$\begin{aligned} P &= 1000F \\ Ce^{1.3863t} &= 1000Ce^{0.13863t} \\ e^{1.3863t} &= 1000e^{0.13863t} \\ 1.3863t &= \ln 1000 + 0.13863t \\ 1.24767t &= \ln 1000 \\ t &= 5.537 \text{ days.} \end{aligned}$$

The phytoplankton population is 1000 times the foraminifera population after 5.537 days, about five and a half days.

43. Let $P(t)$ be the world population in billions t years after 2010.

(a) Assuming exponential growth, we have

$$P(t) = 6.9e^{kt}.$$

In 2050, we have $t = 40$ and we expect the population then to be 9 billion, so

$$9 = 6.9e^{k\cdot 40}.$$

Solving for k, we have

$$e^{k\cdot 40} = \frac{9}{6.9}$$
$$k = \frac{1}{40}\ln\left(\frac{9}{6.9}\right) = 0.00664 = 0.664\% \text{ per year.}$$

(b) The "Day of 8 Billion" should occur when

$$8 = 6.9e^{0.00664t}.$$

Solving for t gives

$$e^{0.00664t} = \frac{8}{6.9}$$
$$t = \frac{\ln(8/6.9)}{0.00664} = 22.277 \text{ years.}$$

So the "Day of 8 Billion" should be 22.277 years after the end of 2010. This is 22 years and $0.277 \cdot 365 \approx 101$ days; so 101 days into 2032. That is, April 11, 2032. This solution does not take leap years into account.

44. (a) From the table, we can see that the number of E85 vehicles increased by $393{,}553 - 302{,}341 = 91{,}212$ between 2012 to 2017. Thus the percent increase is $91{,}212/302{,}341 = 0.302 = 30.2\%$. Alternatively, we see that $393{,}553/30{,}2341 = 1.302$, again showing an increase of 30.2%.

(b) To find A, we solve when $t = 0$:

$$\begin{aligned} Ae^{r\cdot 0} &= 302{,}341 \\ A \cdot 1 &= 302{,}341 \\ A &= 302{,}341. \end{aligned}$$

Since t is the number of years since 2012, we know that $t = 5$ in 2017 and get:

$$393{,}553 = 302{,}341e^{5r}.$$

Solving for r, we get

$$\begin{aligned} \frac{393{,}553}{302{,}341} &= e^{5r} \\ \ln\left(\frac{393{,}553}{302{,}341}\right) &= 5r \\ r &= 0.052732. \end{aligned}$$

Substituting $t = 1, 2, 3, 4$ into

$$302{,}341e^{(0.052732)t},$$

we find the remaining table values:

Year	2012	2013	2014	2015	2016	2017
E85 vehicles	302,341	318,712	335,969	354,161	373,338	393,553

(c) The year 2011 is the year $t = -1$. Therefore, If N is the number of E85-powered vehicles in 2011, we have

$$N = 302{,}341e^{0.052732(-1)} = 286{,}811.$$

45. We have

$$\text{Future value } = 10{,}000e^{0.03\cdot 8} = \$12{,}712.49.$$

46. We have

$$\text{Future value } = 20{,}000e^{0.038\cdot 15} = \$35{,}365.34.$$

47. We have

$$\begin{aligned} \text{Future value } &= \text{Present value } \cdot e^{0.04\cdot 5} \\ 8000 &= \text{Present value } \cdot e^{0.04\cdot 5} \\ \text{Present value } &= \frac{8000}{e^{0.04\cdot 5}} = \$6549.85. \end{aligned}$$

48. We have

$$\begin{aligned} \text{Future value } &= \text{Present value } \cdot e^{0.032\cdot 10} \\ 20{,}000 &= \text{Present value } \cdot e^{0.032\cdot 10} \\ \text{Present value } &= \frac{20{,}000}{e^{0.032\cdot 10}} = \$14{,}522.98. \end{aligned}$$

49. **(a)** The total present value for each of the two choices are in the following table. Choice 1 is the preferred choice since it has the larger present value.

Choice 1			Choice 2	
Year	Payment	Present value	Payment	Present value
0	1500	1500	1900	1900
1	3000	$3000/(1.05) = 2857.14$	2500	$2500/(1.05) = 2380.95$
	Total	4357.14	Total	4280.95

(b) The difference between the choices is an extra \$400 now (\$1900 in Choice 2 instead of \$1500 in Choice 1) versus an extra \$500 one year from now (\$3000 in Choice 1 instead of \$2500 in Choice 2). Since $400 \times 1.25 = 500$, Choice 2 is better at interest rates above 25%.

50. **(a)** (i) We know that the formula for the account balance at time t is given by

$$P(t) = P_0 e^{rt}$$

(since the account is being compounded continuously), where P_0 is the initial deposit and r is the annual rate. In our case the initial deposit is

$$P_0 = 24$$

and the annual rate is

$$r = 0.05.$$

Thus, the formula for the balance is

$$P(t) = 24e^{0.05t}.$$

We are asked for the amount of money in the account in the year 2012, that is we are asked for the amount of money in the account 386 years after the initial deposit. This gives

$$\begin{aligned}\text{amount of money in the account in the year 2012} &= P(386)\\ &= 24e^{0.05(386)}\\ &= 24e^{19.3}\\ &\approx 24(240{,}925{,}906)\\ &= 5{,}782{,}217{,}428.\end{aligned}$$

Thus, in the year 2012, there would be about 5.78 billion dollars in the account.

(ii) Here the annual rate is

$$r = 0.07.$$

Thus, the formula for the balance is

$$P(t) = 24e^{0.07t}.$$

Again, we want $P(386)$. This gives

$$\begin{aligned}\text{Amount of money in the account in the year 2012} &= P(386)\\ &= 24e^{0.07(386)}\\ &= 24e^{27.02}\\ &\approx 24(332{,}531{,}537{,}200)\\ &= 13{,}027{,}111{,}872{,}500.\end{aligned}$$

Thus, in the year 2012, there would be about 13.02 trillion dollars in the account.

(b) Here the annual rate is

$$r = 0.06.$$

Thus, the formula for the balance is

$$P(t) = 24e^{0.06t}.$$

We are asked to solve for t such that

$$P(t) = 1{,}000{,}000{,}000.$$

Solving we get

$$\begin{aligned}
1{,}000{,}000{,}000 &= B(t) = 24e^{0.06t} \\
\frac{1{,}000{,}000{,}000}{24} &= e^{0.06t} \\
e^{0.06t} &\approx 41{,}666{,}666.7 \\
\ln e^{0.06t} &\approx \ln 41{,}666{,}666.7 \\
0.06t &\approx \ln 41{,}666{,}666.7 \\
t &\approx \frac{\ln 41{,}666{,}666.7}{0.06} \approx 292.4.
\end{aligned}$$

Thus, there would be one million dollars in the account 292.4 years after the initial deposit, that is roughly in 1871.

51. (a) Using the Rule of 70, we get that the doubling time of the investment is

$$\frac{70}{8} = 8.75.$$

That is, it would take about 8.75 years for the investment to double.

(b) We know that the formula for the balance at the end of t years is

$$B(t) = Pa^t$$

where P is the initial investment and a is 1+(interest per year). We are asked to solve for the doubling time, which amounts to asking for the time t at which

$$B(t) = 2P.$$

Solving, we get

$$\begin{aligned}
2P &= P(1+0.08)^t \\
2 &= 1.08^t \\
\ln 2 &= \ln 1.08^t \\
t \ln 1.08 &= \ln 2 \\
t &= \frac{\ln 2}{\ln 1.08} \approx 9.01.
\end{aligned}$$

Thus, the actual doubling time is 9.01 years. And so our estimation by the "rule of 70" was off by a quarter of a year.

52. You should choose the payment which gives you the highest present value. The immediate lump-sum payment of \$2800 obviously has a present value of exactly \$2800, since you are getting it now. We can calculate the present value of the installment plan as:

$$\begin{aligned}
\text{PV} &= 1000e^{-0.03(0)} + 1000e^{-0.03(1)} + 1000e^{-0.03(2)} \\
&\approx \$2912.21.
\end{aligned}$$

Since the installment payments offer a higher present value, you should accept the installment option.

53. (a) Option 1 is the best option since money received now can earn interest.

(b) In Option 1, the entire \$2000 earns 5% interest for 1 year, so we have:

$$\text{Future value of Option 1} = 2000e^{0.05\cdot 1} = \$2102.54.$$

In Option 2, the payment in one year does not earn interest, but the payment made now earns 5% interest for one year. We have

$$\text{Future value of Option 2} = 1000 + 1000e^{0.05\cdot 1} = 1000 + 1051.27 = \$2051.27.$$

Since Option 3 is paid all in the future, we have

$$\text{Future value of Option 3} = \$2000.$$

As we expected, Option 1 has the highest future value.

(c) In Option 1, the entire $2000 is paid now, so we have:

$$\text{Present value of Option 1} = \$2000.$$

In Option 2, the payment now has a present value of $1000 but the payment in one year has a lower present value. We have

$$\text{Present value of Option 2} = 1000 + 1000e^{-0.05\cdot 1} = 1000 + 951.23 = \$1951.23.$$

Since Option 3 is paid all in the future, we have

$$\text{Present value of Option 3} = 2000e^{-0.05\cdot 1} = \$1902.46.$$

Again, we see that Option 1 has the highest value.
Alternatively, we could have computed the present values directly from the future values found in part (b).

54. **(a)** The following table gives the present value of the cash flows. The total present value of the cash flows is $116,224.95.

Year	Payment	Present value
1	50,000	$50{,}000/(1.075) = 46,511.63$
2	40,000	$40{,}000/(1.075)^2 = 34,613.30$
3	25,000	$25{,}000/(1.075)^3 = 20,124.01$
4	20,000	$20{,}000/(1.075)^4 = 14,976.01$
	Total	116,224.95

(b) The present value of the cash flows, $116,224.95, is larger than the price of the machine, $97,000, so the recommendation is to buy the machine.

55. **(a)** For Myers, the present value of the first year's payment is equal to the payment, $17 million. The payment the next year is discounted by 4%, so it is $2/(1.04) = \$1.923$ million. Continuing, we get:

$$\text{Present value, Myers contract} = 17 + \frac{2}{1.04} + \frac{3}{(1.04)^2} + \frac{7.4}{(1.04)^3} + \frac{20}{(1.04)^4} + \frac{20}{(1.04)^5} = 61.810 \text{ million dollars.}$$

For Fowler, we calculate similarly:

$$\text{Present value, Fowler contract} = 16.5 + \frac{16.5}{1.04} + \frac{16.5}{(1.04)^2} + \frac{7.4}{(1.04)^3} + \frac{1.8}{(1.04)^4} = 55.738 \text{ million dollars.}$$

We see that Myers's contract is about 6 million dollars more valuable in present value.

(b) We perform the same calculations using a discount rate of 13%:

$$\text{Present value, Myers contract} = 17 + \frac{2}{1.13} + \frac{3}{(1.13)^2} + \frac{7.4}{(1.13)^3} + \frac{20}{(1.13)^4} + \frac{20}{(1.13)^5} = 49.369 \text{ million dollars.}$$

For Fowler, we calculate similarly:

$$\text{Present value, Fowler contract} = 16.5 + \frac{16.5}{1.13} + \frac{16.5}{(1.13)^2} + \frac{7.4}{(1.13)^3} + \frac{1.8}{(1.13)^4} = 50.256 \text{ million dollars.}$$

If investment returns are very high, Fowler's contract becomes more valuable because his payments are not as far in the future.

56. In effect, your friend is offering to give you $17,000 now in return for the $19,000 lottery payment one year from now. Since $19{,}000/17{,}000 = 1.11764\cdots$, your friend is charging you 11.7% interest per year, compounded annually. You can expect to get more by taking out a loan as long as the interest rate is less than 11.7%. In particular, if you take out a loan, you have the first lottery check of $19,000 plus the amount you can borrow to be paid back by a single payment of $19,000 at the end of the year. At 8.25% interest, compounded annually, the present value of 19,000 one year from now is $19{,}000/(1.0825) = 17{,}551.96$. Therefore the amount you can borrow is the total of the first lottery payment and the loan amount, that is, $19{,}000 + 17{,}551.96 = 36{,}551.96$. So you do better by taking out a one-year loan at 8.25% per year, compounded annually, than by accepting your friend's offer.

57. The following table contains the present value of each of the payments, though it does not take into account the resale value if you buy the machine.

Buy			Lease	
Year	Payment	Present Value	Payment	Present Value
0	12000	12000	2650	2650
1	580	$580/(1.0775) = 538.28$	2650	$2650/(1.0775) = 2459.40$
2	464	$464(1.0775)^2 = 399.65$	2650	$2650/(1.0775)^2 = 2282.50$
3	290	$290/(1.0775)^3 = 231.82$	2650	$2650/(1.0775)^3 = 2118.33$
	Total	13,169.75	Total	9510.23

Now we consider the $5000 resale.

$$\text{Present value of resale} = \frac{5000}{(1.0775)^3} = 3996.85.$$

The net present value associated with buying the machine is the present value of the payments minus the present value of the resale price, which is

$$\text{Present value of buying} = 13{,}169.75 - 3996.85 = 9172.90$$

Since the present value of the expenses associated with buying ($9172.90) is smaller than the present value of leasing ($9510.23), you should buy the machine.

58. The following table contains the present value of each of the expenses. Since the total present value of the expenses, $352.01, is less than the price of the extended warranty, it is not worth purchasing the extended warranty.

	Present value of expenses	
Year	Expense	Present value
2	150	$150/(1.05)^2 = 136.05$
3	250	$250/(1.05)^3 = 215.96$
	Total	352.01

59. Let P be the required present value in 2020 in millions of dollars. Then with an interest rate of r per year, compounded annually over a time period of t years, a payment of $\$P$ grows to a future value of $\$B$ where $B = P(1+r)^t$. We have $r = 0.02$ over $t = 10$ years with a required future value of $B = \$222$ million, giving

$$P(1+0.02)^{10} = 222.$$

Solving for P gives

$$P = \frac{222}{1.02^{10}} \approx \$182.117 \text{ million.}$$

60. Let P be the required present value in 2020 in millions of dollars. Then with an interest rate of r per year, compounded continuously over a time period of t years, a payment of $\$P$ grows to a future value of $\$B$ where $B = Pe^{rt}$. We have $r = 0.015$ over $t = 30$ years with a required future value of $B = \$650$ million, giving

$$Pe^{(0.015)30} = 650.$$

Solving for P gives

$$P = \frac{650}{e^{0.45}} \approx \$414.458 \text{ million.}$$

61. The future value of \$150 million invested in 2020 at an interest rate r compounded annually for 10 years is $150(1+r)^{10}$ so we must solve

$$150(1+r)^{10} = 222.$$

Solving, we get

$$\begin{aligned} 150(1+r)^{10} &= 222 \\ (1+r)^{10} &= \frac{222}{150} \\ 1+r &= \left(\frac{222}{150}\right)^{\frac{1}{10}} \\ r &= \left(\frac{222}{150}\right)^{\frac{1}{10}} - 1 \approx 0.03998 \end{aligned}$$

To achieve the required payment would require an interest rate of about 4%.

62. The future value of \$300 million invested in 2020 at an interest rate r compounded continuously for 30 years is $300e^{30r}$ so we must solve

$$300e^{30r} = 650.$$

Solving, we get

$$\begin{aligned} 300e^{30r} &= 650 \\ e^{30r} &= \frac{650}{300} \\ 30r &= \ln\frac{650}{300} \\ r &= \frac{1}{30}\ln\frac{650}{300} \approx 0.0258 \end{aligned}$$

To achieve the required payment would require an interest rate of about 2.6%

63. Let P, be the present value in 2020 in billions of the \$19.5 billion cost of damage attributed to climate change in 2075. Since the interest rate is 2% over the period of 55 years from 2020 to 2075 we get

$$P(1.02)^{55} = 19.5$$

Solving for P gives

$$P = \frac{19.5}{1.02^{55}} = 6.562.$$

The present value required is \$6.562 billion.

64. **(a)** We want the present value of \$1.4 billion in 2020. Since 2070 is 50 years in the future and the interest rate is $1\% = 0.01$, we have

$$\text{Present value in 2020} = \frac{1.4}{(1+0.01)^{50}} = 0.851 \text{ billion dollars.}$$

(b) To find out how much of the \$1.4 billion the \$0.28 billion will cover, we find the future value of the \$0.28 billion:

$$\text{Future value in 2070} = 0.28(1.01)^{50} = 0.460 \text{ billion dollars.}$$

So

$$\text{Amount not covered} = 1.4 - 0.460 = 0.94 \text{ billion dollars.}$$

65. The interest rate is $1.5\% = 0.015$. Thus, the present value of a beach nourishing t years in the future is $12/(1+0.015)^t = 12(1.015)^{-t}$. The beach nourishings are 1, 6, 11, 16 years in the future, so

$$\text{Present value} = 12(1.015)^{-1} + 12(1.015)^{-6} + 12(1.015)^{-11} + 12(1.015)^{-16} = 42.441 \text{ million dollars.}$$

66. **(a)** We find the future value of \$7 million in 2011. Since this is one year in the future and the continuous interest rate is $2\% = 0.02$, we have

$$\text{Future value in 2011} = 7e^{0.02\cdot 1} = 7.141 \text{ million dollars.}$$

Thus, $7.141 - 2.5 = 4.641$ million dollars remain.

(b) To find out how many floods are covered by the \$7 million, we can calculate the present value of the money remaining after each flood or we can use future values. The floods are 1, 6, 11, … years after 2010. We see how many floods occur before the present value remaining is negative—at that point the flood wall has paid off:

$$\begin{aligned}
\text{Present value after 2011 flood} &= 7 - 2.5e^{-0.02\cdot 1} = 4.550 \text{ million dollars.}\\
\text{Present value after 2016 flood} &= 7 - 2.5e^{-0.02\cdot 1} - 2.5e^{-0.02\cdot 6} = 2.332 \text{ million dollars.}\\
\text{Present value after 2021 flood} &= 7 - 2.5e^{-0.02\cdot 1} - 2.5e^{-0.02\cdot 6} - 2.5e^{-0.02\cdot 11} = 0.326 \text{ million dollars.}\\
\text{Present value after 2026 flood} &= 7 - 2.5e^{-0.02\cdot 1} - 2.5e^{-0.02\cdot 6} - 2.5e^{-0.02\cdot 11} - 2.5e^{-0.02\cdot 16} = -1.489 \text{ million dollars.}
\end{aligned}$$

Thus the flood wall has paid off after 4 floods.

Alternatively we can use future value:

$$\text{Future value after 2011 flood} = 7e^{0.02\cdot 1} - 2.5 = 4.6414 \text{ million dollars.}$$

Five years later, at the 2016 flood, we have

$$\text{Future value after 2016 flood} = 4.6414e^{0.02\cdot 5} - 2.5 = 2.63 \text{ million dollars.}$$

And then for the 2021 and 2026 floods:

$$\begin{aligned}
\text{Future value after 2021 flood} &= 2.63e^{0.02\cdot 5} - 2.5 = 0.41 \text{ million dollars.}\\
\text{Future value after 2026 flood} &= 0.41e^{0.02\cdot 5} - 2.5 = -2 \text{ million dollars.}
\end{aligned}$$

As we saw with the present value calculation, the flood wall pays off after 4 floods.

67. The interest rate is $1.2\% = 0.012$. The present value of the repairs is \$150,000, since they are paid at the time of the purchase. Then

$$\begin{aligned}
\text{Present value of the first \$2400} &= \frac{2400}{1.012} = 2400(1.012)^{-1}\\
\text{Present value of the second \$2400} &= \frac{2400}{1.012^2} = 2400(1.012)^{-2}.
\end{aligned}$$

Similarly for the third and fourth payments. Thus the total present value is

$$\text{Present value} = 150{,}000 + 2400(1.012)^{-1} + 2400(1.012)^{-2} + 2400(1.012)^{-3} + 2400(1.012)^{-4} = 159{,}318.770 \text{ dollars.}$$

68. The interest rate is $1\% = 0.01$. Suppose they take in \$P each year for 5 years. Then, over the five years,

$$\begin{aligned}
\text{Present value of earnings} &= Pe^{-0.01\cdot 1} + Pe^{-0.01\cdot 2} + Pe^{-0.01\cdot 3} + Pe^{-0.01\cdot 4} + Pe^{-0.01\cdot 5}\\
&= P\left(e^{-0.01\cdot 1} + e^{-0.01\cdot 2} + e^{-0.01\cdot 3} + e^{-0.01\cdot 4} + e^{-0.01\cdot 5}\right)\\
&= 4.85271P \text{ dollars.}
\end{aligned}$$

To pay off the \$700,000, the present value must satisfy

$$4.85271P = 700{,}000$$

so

$$P = \frac{700{,}000}{4.85271} = 144{,}249 \text{ dollars each year.}$$

Solutions for Section 1.8

1. **(a)** We have $f(g(x)) = f(3x+2) = 5(3x+2) - 1 = 15x + 9$.

(b) We have $g(f(x)) = g(5x-1) = 3(5x-1) + 2 = 15x - 1$.

(c) We have $f(f(x)) = f(5x-1) = 5(5x-1) - 1 = 25x - 6$.

2. **(a)** We have $f(g(x)) = f(x^2+8) = (x^2+8) - 2 = x^2 + 6$.
(b) We have $g(f(x)) = g(x-2) = (x-2)^2 + 8 = x^2 - 4x + 12$.
(c) We have $f(f(x)) = f(x-2) = (x-2) - 2 = x - 4$.

3. **(a)** We have $f(g(x)) = f(e^{2x} = 3e^{2x}$.
(b) We have $g(f(x)) = g(3x) = e^{2(3x)} = e^{6x}$.
(c) We have $f(f(x)) = f(3x) = 3(3x) = 9x$.

4. Notice that $f(2) = 4$ and $g(2) = 5$.

(a) $f(2) + g(2) = 4 + 5 = 9$
(b) $f(2) \cdot g(2) = 4 \cdot 5 = 20$
(c) $f(g(2)) = f(5) = 5^2 = 25$
(d) $g(f(2)) = g(4) = 3(4) - 1 = 11$.

5. **(a)** $g(2+h) = (2+h)^2 + 2(2+h) + 3 = 4 + 4h + h^2 + 4 + 2h + 3 = h^2 + 6h + 11$.
(b) $g(2) = 2^2 + 2(2) + 3 = 4 + 4 + 3 = 11$, which agrees with what we get by substituting $h = 0$ into (a).
(c) $g(2+h) - g(2) = (h^2 + 6h + 11) - (11) = h^2 + 6h$.

6. **(a)** $f(t+1) = (t+1)^2 + 1 = t^2 + 2t + 1 + 1 = t^2 + 2t + 2$.
(b) $f(t^2+1) = (t^2+1)^2 + 1 = t^4 + 2t^2 + 1 + 1 = t^4 + 2t^2 + 2$.
(c) $f(2) = 2^2 + 1 = 5$.
(d) $2f(t) = 2(t^2+1) = 2t^2 + 2$.
(e) $(f(t))^2 + 1 = \left(t^2+1\right)^2 + 1 = t^4 + 2t^2 + 1 + 1 = t^4 + 2t^2 + 2$.

7. **(a)** $f(g(1)) = f(1+1) = f(2) = 2^2 = 4$
(b) $g(f(1)) = g(1^2) = g(1) = 1 + 1 = 2$
(c) $f(g(x)) = f(x+1) = (x+1)^2$
(d) $g(f(x)) = g(x^2) = x^2 + 1$
(e) $f(t)g(t) = t^2(t+1)$

8. **(a)** $f(g(1)) = f(1^2) = f(1) = \sqrt{1+4} = \sqrt{5}$
(b) $g(f(1)) = g(\sqrt{1+4}) = g(\sqrt{5}) = (\sqrt{5})^2 = 5$
(c) $f(g(x)) = f(x^2) = \sqrt{x^2+4}$
(d) $g(f(x)) = g(\sqrt{x+4}) = (\sqrt{x+4})^2 = x + 4$
(e) $f(t)g(t) = (\sqrt{t+4})t^2 = t^2\sqrt{t+4}$

9. **(a)** $f(g(1)) = f(1^2) = f(1) = e^1 = e$
(b) $g(f(1)) = g(e^1) = g(e) = e^2$
(c) $f(g(x)) = f(x^2) = e^{x^2}$
(d) $g(f(x)) = g(e^x) = (e^x)^2 = e^{2x}$
(e) $f(t)g(t) = e^t t^2$

10. **(a)** $f(g(1)) = f(3 \cdot 1 + 4) = f(7) = \dfrac{1}{7}$
(b) $g(f(1)) = g(1/1) = g(1) = 7$
(c) $f(g(x)) = f(3x+4) = \dfrac{1}{3x+4}$
(d) $g(f(x)) = g\left(\dfrac{1}{x}\right) = 3\left(\dfrac{1}{x}\right) + 4 = \dfrac{3}{x} + 4$
(e) $f(t)g(t) = \dfrac{1}{t}(3t+4) = 3 + \dfrac{4}{t}$

11. **(a)** We see in the tables that $g(1) = 6$ and $f(6) = 5$, so $f(g(1)) = f(6) = 5$.
(b) We see in the tables that $f(1) = 5$ and $g(5) = 2$, so $g(f(1)) = g(5) = 2$.
(c) We see in the tables that $g(4) = 3$ and $f(3) = 3$, so $f(g(4)) = f(3) = 3$.
(d) We see in the tables that $f(4) = 3$ and $g(3) = 4$, so $g(f(4)) = g(3) = 4$.
(e) We see in the tables that $g(6) = 1$ and $f(1) = 5$, so $f(g(6)) = f(1) = 5$.
(f) We see in the tables that $f(6) = 5$ and $g(5) = 2$, so $g(f(6)) = g(5) = 2$.

12. **(a)** We see in the tables that $g(0) = 2$ and $f(2) = 3$, so $f(g(0)) = f(2) = 3$.
(b) We see in the tables that $g(1) = 3$ and $f(3) = 4$, so $f(g(1)) = f(3) = 4$.
(c) We see in the tables that $g(2) = 5$ and $f(5) = 11$, so $f(g(2)) = f(5) = 11$.
(d) We see in the tables that $f(2) = 3$ and $g(3) = 8$, so $g(f(2)) = g(3) = 8$.
(e) We see in the tables that $f(3) = 4$ and $g(4) = 12$, so $g(f(3)) = g(4) = 12$.

13.

(a)

x	$f(x)+3$
0	13
1	9
2	6
3	7
4	10
5	14

(b)

x	$f(x-2)$
2	10
3	6
4	3
5	4
6	7
7	11

(c)

x	$5g(x)$
0	10
1	15
2	25
3	40
4	60
5	75

(d)

x	$-f(x)+2$
0	−8
1	−4
2	−1
3	−2
4	−5
5	−9

(e)

x	$g(x-3)$
3	2
4	3
5	5
6	8
7	12
8	15

(f)

x	$f(x)+g(x)$
0	12
1	9
2	8
3	12
4	19
5	26

14. **(a)** Let $u = 5t^2 - 2$. We now have $y = u^6$.
(b) Let $u = -0.6t$. We now have $P = 12e^u$.
(c) Let $u = q^3 + 1$. We now have $C = 12 \ln u$.

15. **(a)** Let $u = 3x - 1$. We now have $y = 2^u$.
(b) Let $u = 5t^2 + 10$. We now have $P = \sqrt{u}$.
(c) Let $u = 3r + 4$. We now have $w = 2 \ln u$.

16. $m(z+1) - m(z) = (z+1)^2 - z^2 = 2z + 1$.

17. $m(z+h) - m(z) = (z+h)^2 - z^2 = 2zh + h^2$.

18. $m(z) - m(z-h) = z^2 - (z-h)^2 = 2zh - h^2$.

19. $m(z+h) - m(z-h) = (z+h)^2 - (z-h)^2 = z^2 + 2hz + h^2 - (z^2 - 2hz + h^2) = 4hz$.

20. We see in the graph of $g(x)$ that $g(1) = 2$ so we have $f(g(1)) = f(2) = 4$.

21. We see in the graph of $f(x)$ that $f(1) = 5$ so we have $g(f(1)) = g(5) = 6$.

22. We see in the graph of $g(x)$ that $g(4) = 5$ so we have $f(g(4)) = f(5) = 1$.

23. We see in the graph of $f(x)$ that $f(4) = 2$ so we have $g(f(4)) = g(2) = 3$.

24. We see in the graph of $f(x)$ that $f(2) = 4$ so we have $f(f(2)) = f(4) = 2$.

25. We see in the graph of $g(x)$ that $g(2) = 3$ so we have $g(g(2)) = g(3) = 4$.

26. $f(g(1)) = f(2) \approx 0.4$.

27. $g(f(2)) \approx g(0.4) \approx 1.1$.

28. $f(f(1)) \approx f(-0.4) \approx -0.9$.

29. We see in the graph of $g(x)$ that $g(3) \approx -2.5$ so we have $f(g(3)) = f(-2.5) \approx 0$.

30. To find $f(g(-3))$, we notice that $g(-3) = 3$ and $f(3) = 0$ so $f(g(-3)) = f(3) = 0$. We find the other entries in the table similarly.

x	−3	−2	−1	0	1	2	3
$f(g(x))$	0	1	1	3	1	1	0
$g(f(x))$	0	−2	−2	−3	−2	−2	0

31. The tree has $B = y - 1$ branches on average and each branch has $n = 2B^2 - B = 2(y-1)^2 - (y-1)$ leaves on average. Therefore

$$\text{Average number of leaves } = Bn = (y-1)(2(y-1)^2 - (y-1)) = 2(y-1)^3 - (y-1)^2.$$

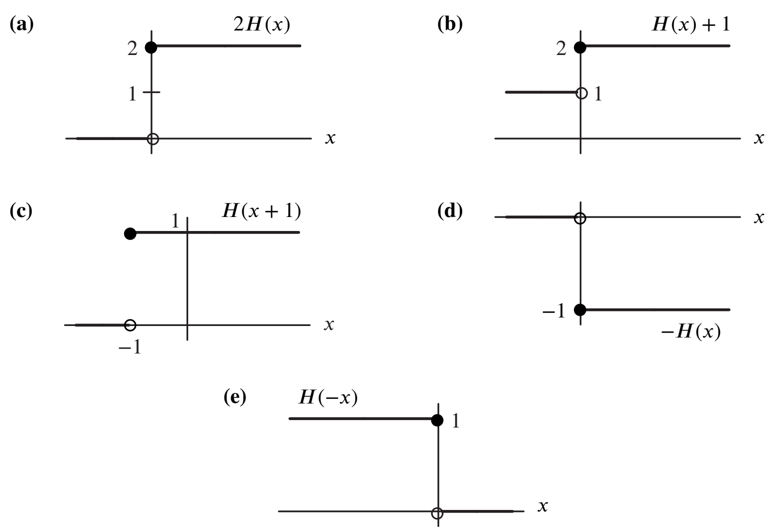
32. We have $v(10) = 65$ but the graph of u only enables us to evaluate $u(x)$ for $0 \le x \le 50$. There is not enough information to evaluate $u(v(10))$.

33. We have approximately $v(40) = 15$ and $u(15) = 18$ so $u(v(40)) = 18$.

34. We have approximately $u(10) = 13$ and $v(13) = 60$ so $v(u(10)) = 60$.

35. We have $u(40) = 60$ but the graph of v only enables us to evaluate $v(x)$ for $0 \le x \le 50$. There is not enough information to evaluate $v(u(40))$.

36.



37. The graph of $y = f(x)$ is shifted vertically upward by 1 unit. See Figure 1.64.

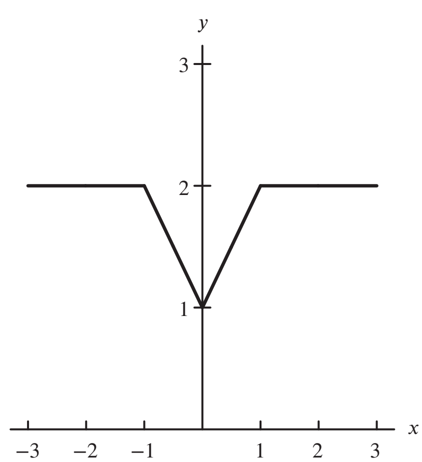


Figure 1.64: Graph of $y = f(x) + 1$

38. The graph of $y = f(x)$ is shifted horizontally to the right by 2 units. See Figure 1.65.

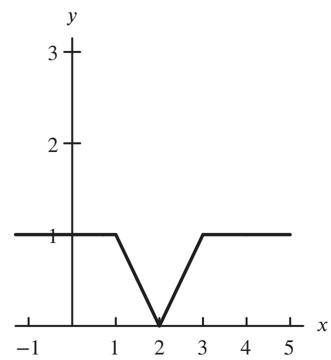


Figure 1.65: Graph of $y = f(x - 2)$

39. The graph of $y = f(x)$ is stretched vertically 3 units. See Figure 1.66.

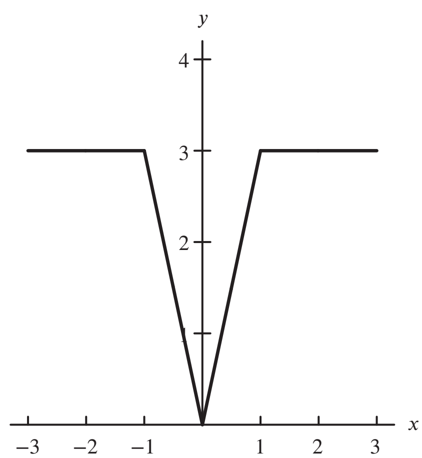


Figure 1.66: Graph of $y = 3f(x)$

40. The graph of $y = f(x)$ is shifted horizontally to the left by 1 unit and is shifted vertically down by 2 units. See Figure 1.67.

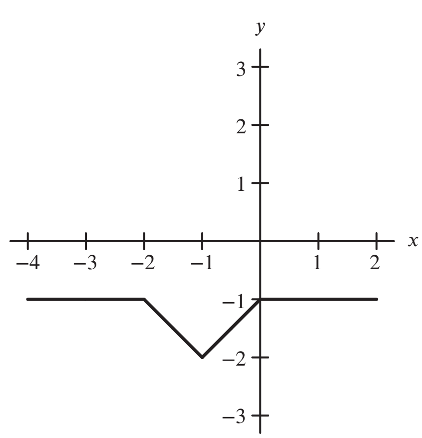


Figure 1.67: Graph of $y = f(x + 1) - 2$

41. The graph of $y = f(x)$ is reflected over the x-axis and shifted vertically up by 3 units. See Figure 1.68.

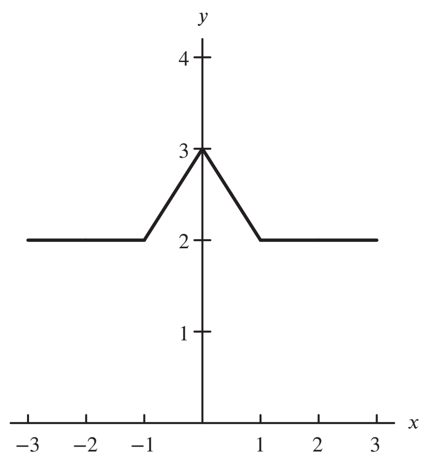


Figure 1.68: Graph of $y = -f(x) + 3$

42. The graph of $y = f(x)$ is reflected over the x-axis, stretched by 2 units, and shifted horizontally to the right by 1 units. See Figure 1.69.

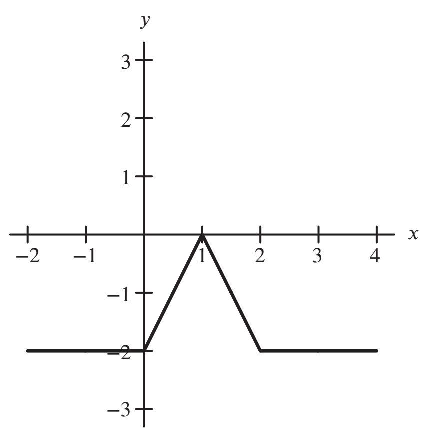
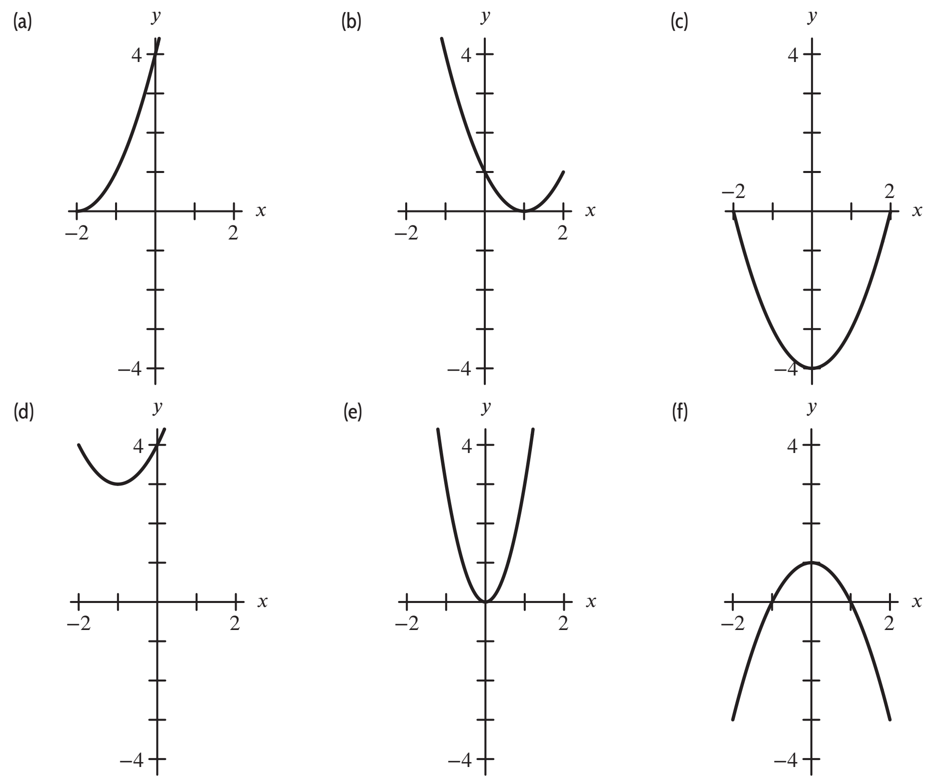
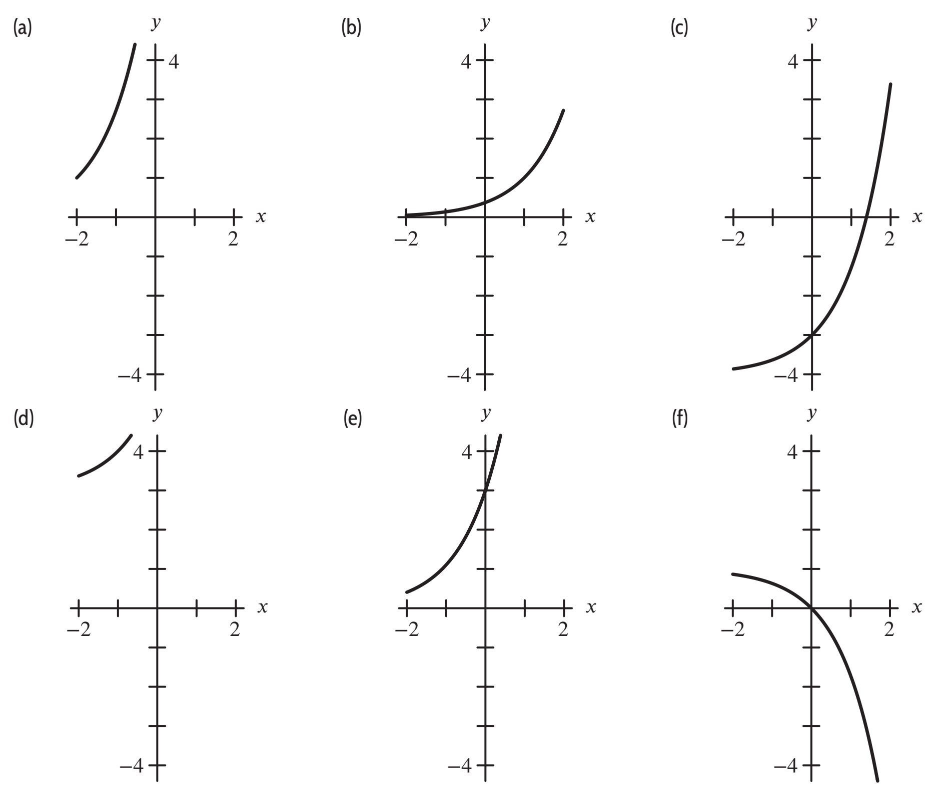


Figure 1.69: Graph of $y = -2f(x - 1)$

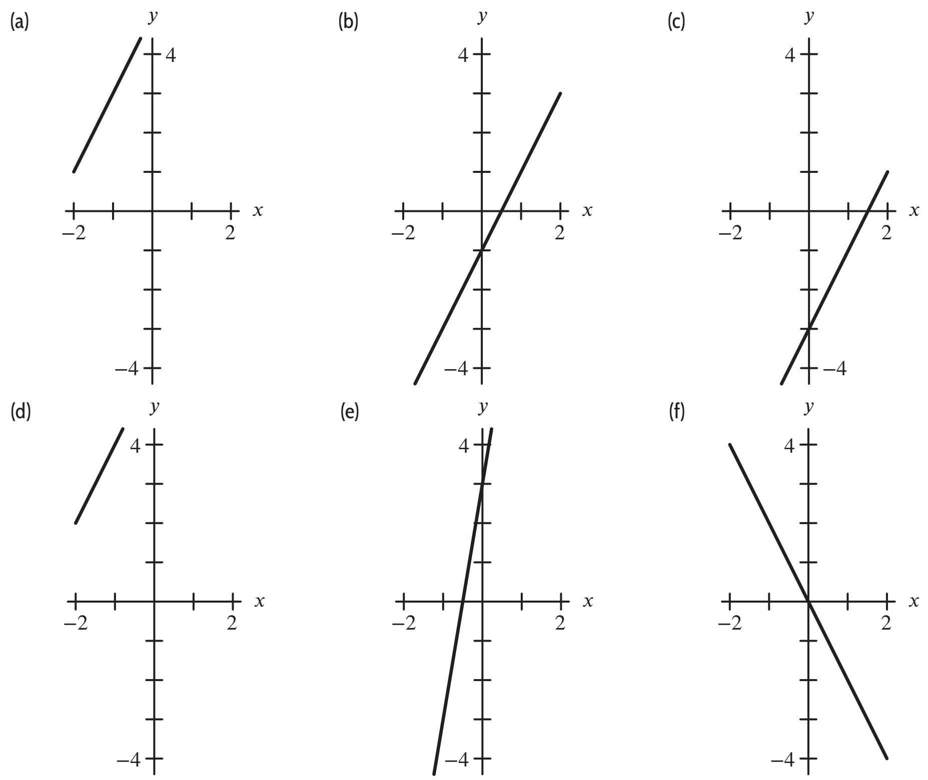
43.



44.



45.



46. See Figure 1.70.

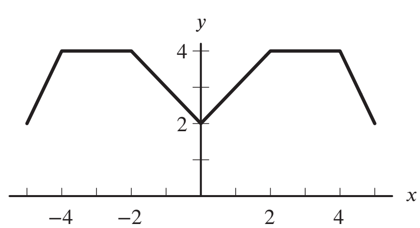


Figure 1.70: Graph of $y = f(x) + 2$

47. See Figure 1.71.

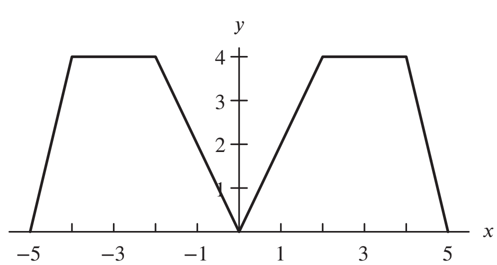


Figure 1.71: Graph of $y = 2f(x)$

48. See Figure 1.72.

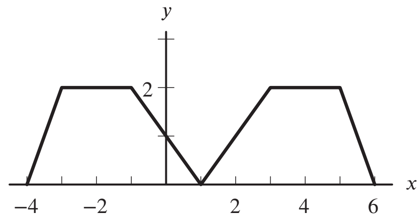


Figure 1.72: Graph of $y = f(x-1)$

49. See Figure 1.73.

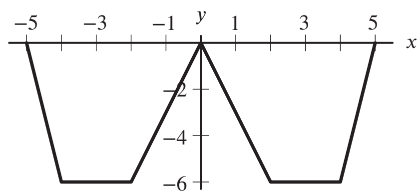


Figure 1.73: Graph of $y = -3f(x)$

50. See Figure 1.74.

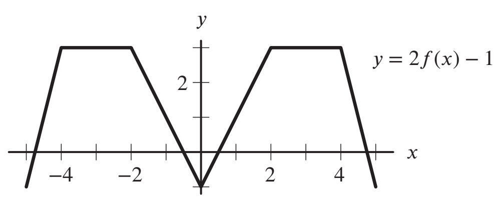


Figure 1.74

51. See Figure 1.75.

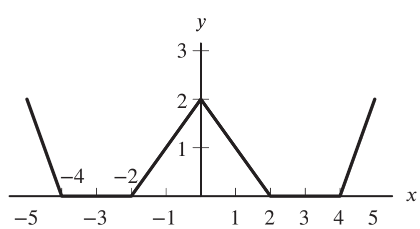


Figure 1.75: Graph of $2 - f(x)$

52. The graph shows the concentration leveling off to a.

(a) The value of a represents the value at which the concentration levels off.
(b) We see a is positive.
(c) At $t = 0$, we have $f(0) = a - be^{k\cdot 0} = a - b$. Since the graph goes through the origin, $f(0) = 0$. Thus $a - b = 0$ so $a = b$.
(d) Since the concentration of f levels off to a and $f(t) = a - be^{kt}$, the term be^{kt} must decay to zero. To make the term be^{kt} decay to zero, k must be negative.

53. **(a)** Since the IV line is started 4 hours later, the drug saturation curve is as shown in Figure 1.76.
(b) The function g is a horizontal shift of f.
(c) Since g is f shifted 4 hours to the right, we have $g(t) = f(t - 4)$ for $t \geq 4$.

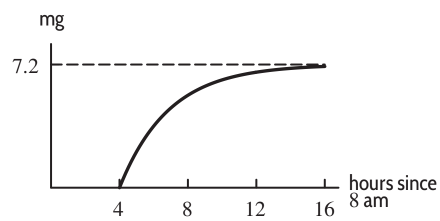


Figure 1.76

54. **(a)** The equation is $y = 2x^2 + 1$. Note that its graph is narrower than the graph of $y = x^2$ which appears in gray. See Figure 1.77.

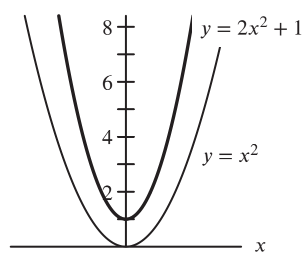


Figure 1.77

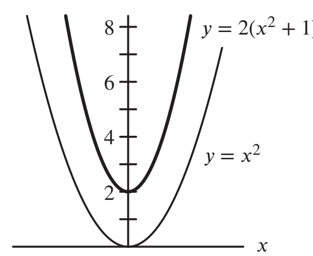


Figure 1.78

(b) $y = 2(x^2 + 1)$ moves the graph up one unit and *then* stretches it by a factor of two. See Figure 1.78.
(c) No, the graphs are not the same. Since $2(x^2 + 1) = (2x^2 + 1) + 1$, the second graph is always one unit higher than the first.

55. The volume of the balloon t minutes after inflation began is: $g(f(t))$ ft^3.

56. The volume of the balloon if its radius were twice as big is: $g(2r)$ ft^3.

57. The time elapsed is: $f^{-1}(30)$ min.

58. The time elapsed is: $f^{-1}(g^{-1}(10{,}000))$ min.

59. **(a)** We find an exponential model in the form $c(t) = c_0 a^t$. Using $c(0) = 25$ and $c(1) = 21.8$, we have

$$c(t) = 25a^t$$
$$21.8 = 25a^1 = 25a.$$

Thus

$$a = \frac{21.8}{25} = 0.872$$
$$c(t) = 25(0.872)^t.$$

Alternately, we could use $c(t) = c_0 e^{kt}$. Then we have $21.8 = 25e^{k\cdot 1} = 25e^k$, giving

$$k = \ln\left(\frac{21.8}{25}\right) = \ln(0.872) = -0.136966$$
$$c(t) = 25e^{-0.136966t}.$$

(b) We find t so that $25(0.872)^t = 10$, giving

$$\begin{aligned}(0.872)^t &= \frac{10}{25} \\ t\ln(0.872) &= \ln\left(\frac{10}{25}\right) \\ t &= \frac{\ln(10/25)}{\ln(0.872)} = 6.6899.\end{aligned}$$

Alternatively, solving $25e^{-0.136966t} = 10$ gives

$$t = -\frac{\ln(10/25)}{0.136966} = 6.6899.$$

Using either method shows that it takes 6.690 years for the cyanide concentration to fall to 10 ppm.

(c) If $D(t) = c(2t)$, then

$$D(t) = 25(0.872)^{2t} = 25\left(0.872^2\right)^t = 25(0.7604)^t.$$

Alternatively,

$$D(t) = 25e^{-0.136966(2t)} = 25e^{-0.273932t}.$$

(d) Starting three years earlier, but with t measured from the original time, we have $E(t) = c(t+3)$, so

$$E(t) = 25(0.872)^{t+3} = 25(0.872)^3(0.872)^t = 16.576(0.872)^t.$$

Alternatively,

$$E(t) = 25e^{-0.136966(t+3)} = 25e^{-0.136966(3)}e^{-0.136966t} = 16.576e^{-0.136966t}.$$

60. **(a)** The patients in quarantine on day t are all the cases that were confirmed up to and including day t but who have not yet left quarantine. Since people stay for exactly 10 days, all cases confirmed before day $t-10$ leave. Thus the number in quarantine is the difference $Q(t) = P(t) - P(t-10)$.

(b) Of the confirmed cases $P(t)$ up to and including day t, only $rP(t)$ have entered quarantine. The $rP(t-s)$ who were in quarantine before day $t-s$ leave by day t. Thus, the number in quarantine on day t is the difference $r\cdot(P(t)-P(t-s))$ or $rP(t) - rP(t-s)$.

61. We have

$$\begin{aligned} h(t+1) &= \text{People spending night } t \text{ in the hospital} \\ &= \text{People spending night } (t-1) \text{ in the hospital} \\ &\quad +(\text{People checking in on day } t) - (\text{People checking out on day } t). \end{aligned}$$

We investigate the three terms of the sum on the right hand side separately.

By definition,

$$h(t) = \text{ People spending night } (t-1) \text{ in the hospital.}$$

There were $N(t)$ new cases on day t. Therefore

$$0.14N(t) = \text{People checking in to the hospital on day } t.$$

The people who check out of the hospital on day t are exactly those who checked in s days earlier. They are the ones who were confirmed on day $t-s$. Therefore

$$0.14N(t-s) = \text{People checking out on day } t.$$

Putting it all together gives

$$h(t+1) = h(t) + 0.14N(t) - 0.14N(t-s).$$

62. Using the definitions given:

(a) $P(t-1)$ is the number of cases confirmed up to and including day $t-1$.

(b) $P(t) = P(t-1) + N(t)$. The number of cases confirmed up to and including day t is the number confirmed up to and including day $t-1$, that is $P(t-1)$, plus the number who are diagnosed on day t, that is $N(t)$.

(c) $N(t-1)$ is the number diagnosed on day $t-1$ (yesterday, if t is today) and $N(t-2)$ is the number diagnosed two days before day t (2 days ago, if t is today).

63. Using the definitions given:

(a) The infectious people on day t is the number of cases minus those who have recovered and those who have passed away:

$$A(t) = P(t) - R(t).$$

(b) The active cases are those that were diagnosed yesterday and for the 13 days before:

$$A(t) = N(t) + N(t-1) + \ldots + N(t-13).$$

Solutions for Section 1.9

1. $y = (1/5)x$; $k = 1/5$, $p = 1$.

2. $y = 5x^{1/2}$; $k = 5$, $p = 1/2$.

3. $y = 8x^{-1}$; $k = 8$, $p = -1$.

4. $y = 3x^{-2}$; $k = 3$, $p = -2$.

5. Not a power function.

6. $y = \frac{3}{8}x^{-1}$; $k = \frac{3}{8}$, $p = -1$.

7. $y = 9x^{10}$; $k = 9$, $p = 10$.

8. $y = \frac{5}{2}x^{-1/2}$; $k = \frac{5}{2}$, $p = -1/2$.

9. Not a power function.

10. $y = 0.2x^2$; $k = 0.2$, $p = 2$.

11. $y = 5^3 \cdot x^3 = 125x^3$; $k = 125$, $p = 3$.

12. Not a power function because of the +4.

13. For some constant k, we have $S = kh^2$.

14. We know that E is proportional to v^3, so $E = kv^3$, for some constant k.

15. If distance is d, then $v = \dfrac{d}{t}$.

16. For some constant k, we have $F = \dfrac{k}{d^2}$.

17. **(a)** This is a graph of $y = x^3$ shifted to the right 2 units and up 1 unit. A possible formula is $y = (x-2)^3 + 1$.
(b) This is a graph of $y = -x^2$ shifted to the left 3 units and down 2 units. A possible formula is $y = -(x+3)^2 - 2$.

18. Since

$$S = kM^{2/3}$$

we have

$$18{,}600 = k(70^{2/3})$$

and so

$$k = 1095.$$

We have $S = 1095M^{2/3}$. If $M = 60$, then

$$S = 1095(60^{2/3}) = 16{,}782 \text{ cm}^2.$$

19. If we let N represent the number of species of lizard on an island and A represent the area of the island, we have

$$N = kA^{1/4}.$$

A graph is shown in Figure 1.79. The function is increasing and concave down. Larger islands have more species of lizards. For small islands, the number of species increases rapidly as the size of the island increases. For larger islands, the increase is much less.

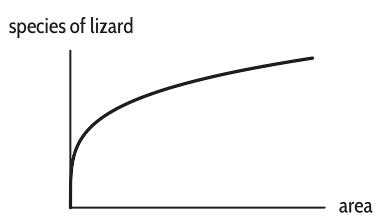


Figure 1.79

20. Let M = blood mass and B = body mass. Then $M = k \cdot B$. Using the fact that $M = 150$ when $B = 3000$, we have

$$\begin{aligned} M &= k \cdot B \\ 150 &= k \cdot 3000 \\ k &= 150/3000 = 0.05. \end{aligned}$$

We have $M = 0.05B$. For a human with $B = 70$, we have $M = 0.05(70) = 3.5$ kilograms of blood.

21. **(a)** Since daily calorie consumption, C, is proportional to the 0.75 power of weight, W, we have

$$C = kW^{0.75}.$$

(b) Since the exponent for this power function is positive and less than one, the graph is increasing and concave down. See Figure 1.80.

(c) We use the information about the human to find the constant of proportionality k. With W in pounds we have

$$\begin{aligned} 1800 &= k(150)^{0.75} \\ k &= \frac{1800}{150^{0.75}} = 42.0. \end{aligned}$$

The function is

$$C = 42.0W^{0.75}.$$

For a horse weighing 700 lbs, the daily calorie requirement is

$$C = 42.0 \cdot 700^{0.75} = 5{,}715.745 \text{ calories.}$$

For a rabbit weighing 9 lbs, the daily calorie requirement is

$$C = 42.0 \cdot 9^{0.75} = 218.238 \text{ calories.}$$

(d) Because the exponent is less than one, the graph is concave down. A mouse has a faster metabolism and needs to consume more calories per pound of weight.

Figure 1.80

22. If $y = k \cdot x^3$, then we have $y/x^3 = k$ with k a constant. In other words, the ratio

$$\frac{y}{x^3} = \frac{\text{Weight}}{(\text{Length})^3}$$

should all be approximately equal (and the ratio will be the constant of proportionality k). For the first fish, we have

$$\frac{y}{x^3} = \frac{332}{(33.5)^3} = 0.0088.$$

If we check all 11 data points, we get the following values of the ratio y/x^3: 0.0088, 0.0088, 0.0087, 0.0086, 0.0086, 0.0088, 0.0087, 0.0086, 0.0087, 0.0088, 0.0088.These numbers are indeed approximately constant with an average of about 0.0087. Thus, $k \approx 0.0087$ and the allometric equation $y = 0.0087x^3$ fits this data well.

23. Since N is inversely proportional to the square of L, we have

$$N = \frac{k}{L^2}.$$

As L increases, N decreases, so there are more species at small lengths.

24. The specific heat is larger when the atomic weight is smaller, so we test to see if the two are inversely proportional to each other by calculating their product. This method works because if y is inversely proportional to x, then

$$y = \frac{k}{x} \quad \text{for some constant } k, \quad \text{so} \quad yx = k.$$

Table 1.9 shows that the product is approximately 6 in each case, so we conjecture that $sw \approx 6$, or $s \approx 6/w$.

Table 1.9

Element	Lithium	Magnesium	Aluminum	Iron	Silver	Lead	Mercury
w	6.9	24.3	27.0	55.8	107.9	207.2	200.6
s (cal/deg-gm)	0.92	0.25	0.21	0.11	0.056	0.031	.033
sw	6.3	6.1	5.7	6.1	6.0	6.4	6.6

25. **(a)** T is proportional to the fourth root of B, and so

$$T = k\sqrt[4]{B} = kB^{1/4}.$$

(b) $148 = k \cdot (5230)^{1/4}$ and so $k = 148/(5230)^{1/4} = 17.4$.

(c) Since $T = 17.4B^{1/4}$, for a human with $B = 70$ we have

$$T = 17.4(70)^{1/4} = 50.3 \text{ seconds}$$

It takes about 50 seconds for all the blood in the body to circulate and return to the heart.

26. **(a)** Since P is inversely proportional to R, we have

$$P = k\frac{1}{R}.$$

(b) If $k = 300{,}000$, we have $P = 300{,}000/R$. If $R = 1$, the population of the city is 300,000 people, since $P = 300{,}000/1$. The population of the second largest city is about 150,000 since $P = 300{,}000/2$. The population of the third largest city is about 100,000 since $P = 300{,}000/3$.

(c) If $k = 6$ million, the population of the largest city is about 6 million, the population of the second largest city is 6 million divided by 2, which is 3 million, and the population of the third largest city is 6 million divided by 3, which is 2 million.

(d) The constant k represents the population of the largest city.

27. **(a)** Since N is proportional to the 0.77 power of P , we have

$$N = kP^{0.77}.$$

(b) Since $P_A = 10 \cdot P_B$, we have

$$N_A = k(P_A)^{0.77} = k(10P_B)^{0.77} = k(10)^{0.77}(P_B)^{0.77} = (10)^{0.77}(k(P_B)^{0.77}) = (10)^{0.77} \cdot (N_B) = 5.888 \cdot N_B.$$

City A has about 5.888 times as many gas stations as city B.

(c) The number of gas stations per person is

$$\frac{N}{P} = \frac{kP^{0.77}}{P} = \frac{k}{P^{0.23}} = kP^{-0.23}.$$

This is a power function with negative exponent, so is decreasing. Therefore, the town of 10,000 people has more gas stations per person.

28. **(a)** We know that the demand function is of the form

$$q = mp + b$$

where m is the slope and b is the vertical intercept. We know that the slope is

$$m = \frac{q(30) - q(25)}{30 - 25} = \frac{460 - 500}{5} = \frac{-40}{5} = -8.$$

Thus, we get

$$q = -8p + b.$$

Substituting in the point (30, 460) we get

$$460 = -8(30) + b = -240 + b,$$

so that

$$b = 700.$$

Thus, the demand function is

$$q = -8p + 700.$$

(b) We know that the revenue is given by

$$R = pq$$

where p is the price and q is the demand. Thus, we get

$$R = p(-8p + 700) = -8p^2 + 700p.$$

(c) Figure 1.81 shows the revenue as a function of price.

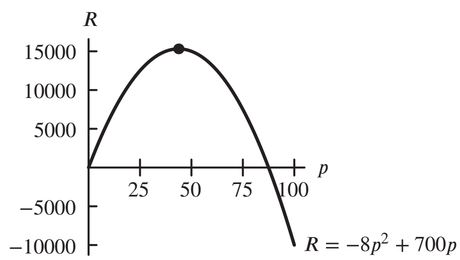


Figure 1.81

Looking at the graph, we see that maximal revenue is attained when the price charged for the product is roughly \$44. At this price the revenue is roughly \$15,300.

29. (a) We know that

$$q = 3000 - 20p,$$
$$C = 10{,}000 + 35q,$$

and

$$R = pq.$$

Thus, we get

$$\begin{aligned} C &= 10{,}000 + 35q \\ &= 10{,}000 + 35(3000 - 20p) \\ &= 10{,}000 + 105{,}000 - 700p \\ &= 115{,}000 - 700p \end{aligned}$$

and

$$\begin{aligned} R &= pq \\ &= p(3000 - 20p) \\ &= 3000p - 20p^2. \end{aligned}$$

(b) The graph of the cost and revenue functions are shown in Figure 1.82.

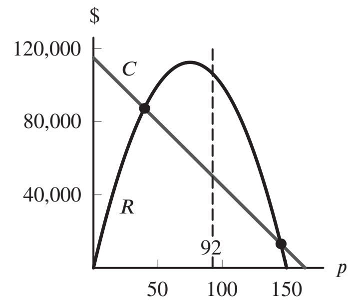


Figure 1.82

(c) It makes sense that the revenue function looks like a parabola, since if the price is too low, although a lot of people will buy the product, each unit will sell for a pittance and thus the total revenue will be low. Similarly, if the price is too high, although each sale will generate a lot of money, few units will be sold because consumers will be repelled by the high prices. In this case as well, total revenue will be low. Thus, it makes sense that the graph should rise to a maximum profit level and then drop.

(d) The club makes a profit whenever the revenue curve is above the cost curve in Figure 1.82. Thus, the club makes a profit when it charges roughly between \$40 and \$145.

(e) We know that the maximal profit occurs when the difference between the revenue and the cost is the greatest. Looking at Figure 1.82, we see that this occurs when the club charges roughly \$92.

Solutions for Section 1.10

1.

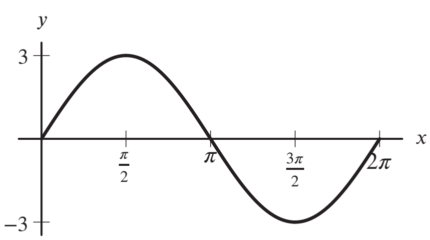


Figure 1.83

See Figure 1.83. The amplitude is 3; the period is 2π.

2.

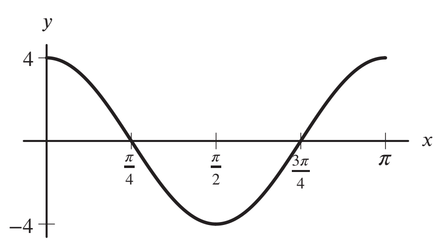


Figure 1.84

See Figure 1.84. The amplitude is 4; the period is π.

3.

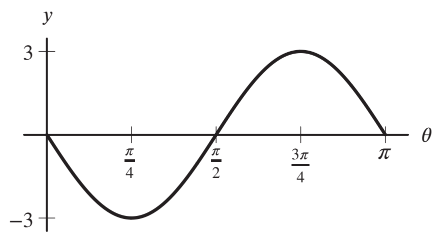


Figure 1.85

See Figure 1.85. The amplitude is 3; the period is π.

4.

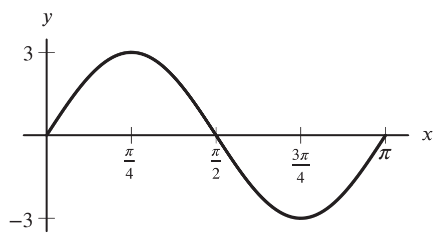


Figure 1.86

See Figure 1.86. The amplitude is 3; the period is π.

5.

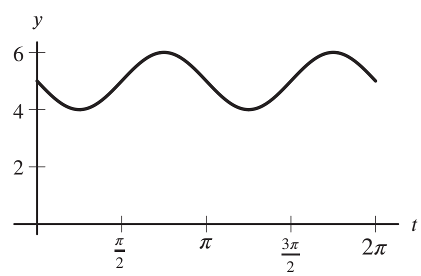


Figure 1.87

See Figure 1.87. The amplitude is 1; the period is π.

6.

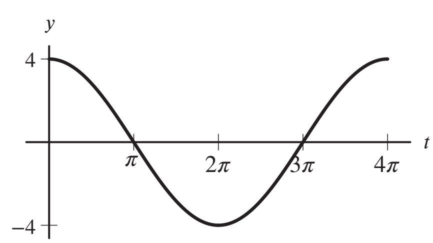


Figure 1.88

See Figure 1.88. The amplitude is 4; the period is 4π.

7. **(a)** The graph *looks* like a periodic function because it appears to repeat itself, reaching approximately the same minimum and maximum each year.

(b) The maximum occurs in the 2nd quarter and the minimum occurs in the 4th quarter. It seems reasonable that people would drink more beer in the summer and less in the winter. Thus, production would be highest the quarter just before summer and lowest the quarter just before winter.

(c) The period is 4 quarters or 1 year.

$$\text{Amplitude} = \frac{\text{max} - \text{min}}{2} \approx \frac{51 - 41}{2} = 5 \text{ million barrels}$$

8. It makes sense that sunscreen sales would be lowest in the winter (say from December through February) and highest in the summer (say from June to August). It also makes sense that sunscreen sales would depend almost completely on time of year, so the function should be periodic with a period of 1 year.

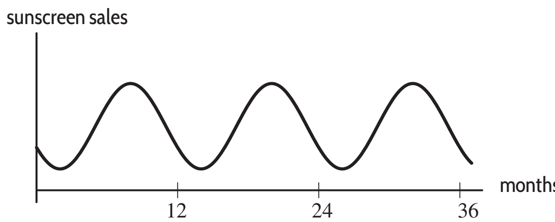


Figure 1.89

9. **(a)** The function appears to vary between 5 and −5, and so the amplitude is 5.
(b) The function begins to repeat itself at $x = 8$, and so the period is 8.
(c) The function is at its highest point at $x = 0$, so we use a cosine function. It is centered at 0 and has amplitude 5, so we have $f(x) = 5\cos(Bx)$. Since the period is 8, we have $8 = 2\pi/B$ and $B = \pi/4$. The formula is

$$f(x) = 5\cos\left(\frac{\pi}{4}x\right).$$

10. **(a)** Since C represents the vertical shift, $C = 27.5°\text{C}$, the average temperature.
(b) The amplitude, A, is the distance above and below the midline at C, which represents the temperature variation above and below the average. Since the data rises $36.7 - 27.5 = 9.2°\text{C}$ above $27.5°\text{C}$, and falls $27.5 - 18.4 = 9.1°\text{C}$. We take A to be the average

$$A = \frac{9.2 + 9.1}{2} = 9.15°\text{C}.$$

(c) Since the period is 12 months,

$$12 = \frac{2\pi}{B}$$

so $B = \pi/6$.
(d) The function is

$$H(n) = 9.15\cos\left(\frac{\pi}{6}n\right) + 27.5.$$

See Figure 1.90.

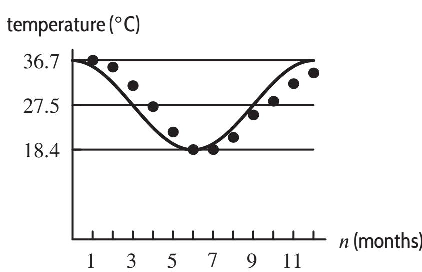


Figure 1.90

11. Since the volume of the function varies between 2 and 4 liters, the amplitude is 1 and the graph is centered at 3. Since the period is 3, we have

$$3 = \frac{2\pi}{B} \quad \text{so} \quad B = \frac{2\pi}{3}.$$

The correct formula is (b). This function is graphed in Figure 1.91 and we see that it has the right characteristics.

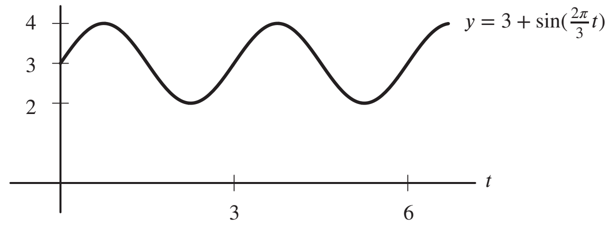


Figure 1.91

12. If we graph the data, it certainly looks like there's a pattern from $t = 20$ to $t = 45$ which begins to repeat itself after $t = 45$. And the fact that $f(20) = f(45)$ helps to reinforce this theory.

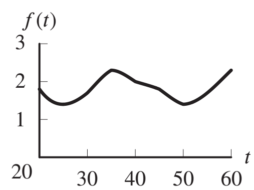


Figure 1.92

Since it looks like the graph repeats after 45, the period is $(45 - 20) = 25$. The amplitude $= \frac{1}{2}(\max - \min) = \frac{1}{2}(2.3 - 1.4) = 0.45$. Assuming periodicity,

$$f(15) = f(15 + 25) = f(40) = 2.0$$
$$f(75) = f(75 - 25) = f(50) = 1.4$$
$$f(135) = f(135 - 25 - 25 - 25) = f(60) = 2.3.$$

13. Since the temperature oscillates over time, we use a trigonometric function. The function is at its lowest point at $t = 0$, so we use a negative cosine curve. The amplitude is about $(26 - (-6))/2 = 16$ and the midline is halfway between -6 and 26, at 10° C. We have $H = -16\cos(Bt) + 10$. The period is 12 months so the coefficient $B = 2\pi/12 = \pi/6$. Thus, the formula is

$$H = -16\cos\left(\frac{\pi}{6}t\right) + 10.$$

14. The levels of both hormones certainly look periodic. In each case, the period seems to be about 28 days. Estrogen appears to peak around the 12th day. Progesterone appears to peak from the 17th through 21st day. (Note: the days given in the answer are approximate so your answer may differ slightly.)

15. Let b be the brightness of the star, and t be the time measured in days from a when the star is at its peak brightness. Because of the periodic behavior of the star,

$$b(t) = A\cos(Bt) + C.$$

The oscillations have amplitude $A = 0.35$, shift $C = 4.0$,and period 5.4, so $5.4B = 2\pi$ and $B = 2\pi/5.4$. This gives

$$b(t) = 0.35\cos\left(\frac{2\pi}{5.4}t\right) + 4.$$

16. **(a)** See Figure 1.93.

(b) The number of species oscillates between a high of 28 and a low of 10. The amplitude is $(28 - 10)/2 = 9$. The period is 12 months or one year.

(c) The graph is at its maximum at $t = 0$ so we use a cosine function. The amplitude is 9 and the graph is shifted up 19 units, so we have $N = 19 + 9\cos(Bt)$. The period is 12 so $B \cdot 12 = 2\pi$ and we have $B = \pi/6$. The formula is

$$N = 19 + 9\cos\left(\frac{\pi}{6}t\right).$$

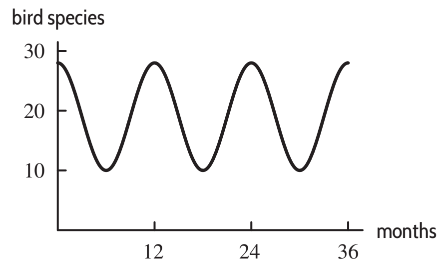


Figure 1.93

17. The graph looks like a sine function of amplitude 7 and period 10. If the equation is of the form

$$y = 7\sin(kt),$$

then $2\pi/k = 10$, so $k = \pi/5$. Therefore, the equation is

$$y = 7\sin\left(\frac{\pi t}{5}\right).$$

18. The graph looks like an upside-down sine function with amplitude $(90 - 10)/2 = 40$ and period π. Since the function is oscillating about the line $x = 50$, the equation is

$$x = 50 - 40\sin(2t).$$

19. This graph is a cosine curve with period 6π and amplitude 5, so it is given by $f(x) = 5\cos\left(\frac{x}{3}\right)$.

20. This graph is a sine curve with period 8π and amplitude 2, so it is given by $f(x) = 2\sin\left(\frac{x}{4}\right)$.

21. This graph is an inverted sine curve with amplitude 4 and period π, so it is given by $f(x) = -4\sin(2x)$.

22. The graph is a sine curve which has been shifted up by 2, so $f(x) = (\sin x) + 2$.

23. This graph is an inverted cosine curve with amplitude 8 and period 20π, so it is given by $f(x) = -8\cos\left(\frac{x}{10}\right)$.

24. This graph has period 5, amplitude 1 and no vertical shift or horizontal shift from $\sin x$, so it is given by

$$f(x) = \sin\left(\frac{2\pi}{5}x\right).$$

25. This graph has period 6, amplitude 5 and no vertical or horizontal shift, so it is given by

$$f(x) = 5\sin\left(\frac{2\pi}{6}x\right) = 5\sin\left(\frac{\pi}{3}x\right).$$

26. The graph is a cosine curve with period $2\pi/5$ and amplitude 2, so it is given by $f(x) = 2\cos(5x)$.

27. This graph has period 8, amplitude 3, and a vertical shift of 3 with no horizontal shift. It is given by

$$f(x) = 3 + 3\sin\left(\frac{2\pi}{8}x\right) = 3 + 3\sin\left(\frac{\pi}{4}x\right).$$

28. This can be represented by a sine function of amplitude 3 and period 18. Thus,

$$f(x) = 3\sin\left(\frac{\pi}{9}x\right).$$

29. The graph is a stretched sine function, so it starts at the midline, $y = 0$, when $x = 0$.

The amplitude is k, so the sinusoidal graph oscillates between a minimum of $-k$ and a maximum of k.

Since k is positive, the function begins by increasing. The period is 2π. See Figure 1.94.

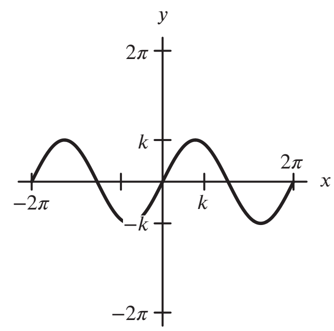


Figure 1.94

30. The graph is a stretched cosine function reflected about the x-axis, so it starts at its minimum when $x = 0$.

The amplitude is k, so the sinusoidal graph oscillates between a minimum of $-k$ and a maximum of k.

The period is 2π. See Figure 1.95.

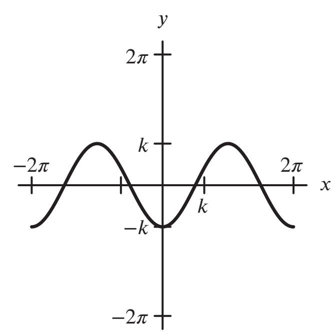


Figure 1.95

31. The graph is a stretched cosine function shifted vertically, so it starts at its maximum when $x = 0$.

We begin by finding the midline, which is k, since the sine function is shifted up by k. The amplitude is k, so the sinusoidal graph oscillates between a minimum of $k - k = 0$ and a maximum of $k + k = 2k$.

The period is 2π. See Figure 1.96.

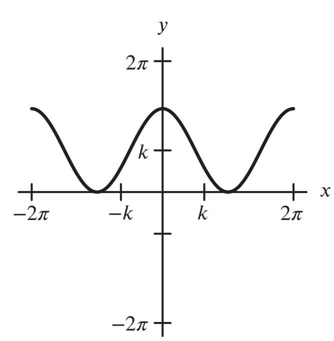


Figure 1.96

32. The graph is a stretched sine function shifted vertically, so it starts at its midline when $x = 0$.

We begin by finding the midline, which is $-k$, since the sine function is shifted up by $-k$ (down by k). The amplitude is k, so the sinusoidal graph oscillates between a minimum of $-k - k = -2k$ and a maximum of $-k + k = 0$.

Since k is positive, the function begins by increasing. The period is 2π. See Figure 1.97.

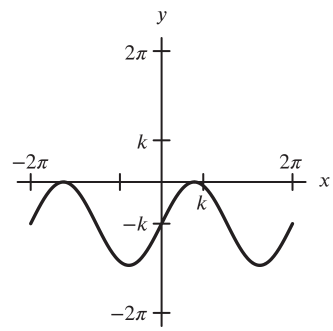


Figure 1.97

33. **(a)** $D =$ the average depth of the water.
(b) $A =$ the amplitude $= 15/2 = 7.5$.
(c) Period = 12.4 hours. Thus $(B)(12.4) = 2\pi$ so $B = 2\pi/12.4 \approx 0.507$.
(d) C is the time of a high tide.

34. The depth function is a vertical shift of a function of the form $A\sin(Bt)$.

The vertical shift is $A = (5.5 + 8.5)/2 = 7$.
The amplitude is $A = 8.5 - 7 = 1.5$.
The period is $6 = 2\pi/B$ so $B = 2\pi/6 = \pi/3$. Thus, we have

$$\text{Depth } = 7 + 1.5\sin\left(\frac{\pi}{3}t\right).$$

35. We use a cosine of the form

$$H = A\cos(Bt) + C$$

and choose B so that the period is 24 hours, so $2\pi/B = 24$ giving $B = \pi/12$.

The temperature oscillates around an average value of 60° F, so $C = 60$. The amplitude of the oscillation is 20° F. To arrange that the temperature be at its lowest when $t = 0$, we take A negative, so $A = -20$. Thus

$$A = 60 - 20\cos\left(\frac{\pi}{12}t\right).$$

36. **(a)**

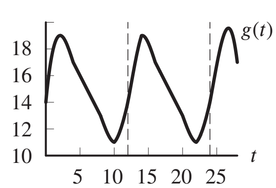


Figure 1.98

From the graph in Figure 1.98, the period appears to be about 12, and the table tells us that $g(0) = 14 = g(12) = g(24)$, which supports this guess.

$$\text{Amplitude} \approx \frac{\max - \min}{2} \approx \frac{19 - 11}{2} = 4$$

.

(b) To estimate $g(34)$, notice that $g(34) = g(24 + 10) = g(10) = 11$ (assuming the function is periodic with period 12). Likewise, $g(60) = g(24 + 12 + 12 + 12) = 14$.

37. **(a)** The monthly mean CO_2 increased about 12 ppm between December 2013 and December 2018. This is because the black curve shows that the December 2013 monthly mean was about 398 ppm, while the December 2018 monthly mean was about 410 ppm. The difference between these two values, $410 - 398 = 12$, gives the overall increase.

(b) The average rate of increase is given by

$$\text{Average monthly increase of monthly mean} = \frac{410 - 398}{60 - 0} = \frac{1}{5} \text{ ppm/month.}$$

This tells us that the slope of a linear equation approximating the black curve is $1/5$. Since the vertical intercept is about 398, a possible equation for the approximately linear black curve is

$$y = \frac{1}{5}t + 398,$$

where t is measured in months since December 2013.

(c) The period of the seasonal CO_2 variation is about 12 months since this is approximately the time it takes for the function given by the blue curve to complete a full cycle. The amplitude is about 3.5 since, looking at the blue curve, the average distance between consecutive maximum and minimum values is about 7 ppm. So a possible sinusoidal function for the seasonal CO_2 cycle is

$$y = 3.5\sin\left(\frac{\pi}{6}t\right).$$

(d) Taking $f(t) = 3.5\sin\left(\frac{\pi}{6}t\right)$ and $g(t) = \frac{1}{5}t + 398$, we have

$$h(t) = 3.5\sin\left(\frac{\pi}{6}t\right) + \frac{1}{5}t + 398.$$

See Figure 1.99.

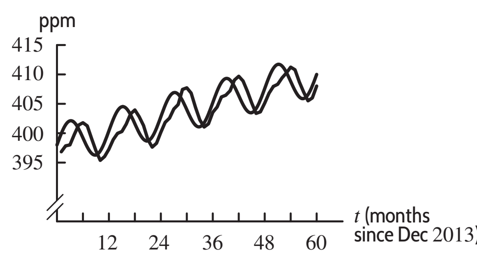


Figure 1.99

Solutions for Chapter 1 Review

1. Since t represents the number of years since 2020, we see that $f(5)$ represents the population of the city in 2025. In 2025, the city's population is 8 million.

2. Dan runs 5 kilometers in 23 minutes.

3. If there are no workers, there is no productivity, so the graph goes through the origin. At first, as the number of workers increases, productivity also increases. As a result, the curve goes up initially. At a certain point the curve reaches its highest level, after which it goes downward; in other words, as the number of workers increases beyond that point, productivity decreases. This might, for example, be due either to the inefficiency inherent in large organizations or simply to workers getting in each other's way as too many are crammed on the same line. Many other reasons are possible.

4. See Figure 1.100.

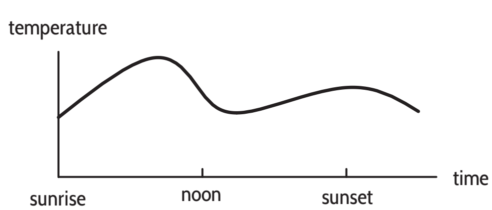


Figure 1.100

5. The contamination is probably greatest right at the tank, at a depth of 6 meters. Contamination probably goes down as the distance from the tank increases. If we assume that the gas spreads both up and down, the graph might look that the one shown in Figure 1.101.

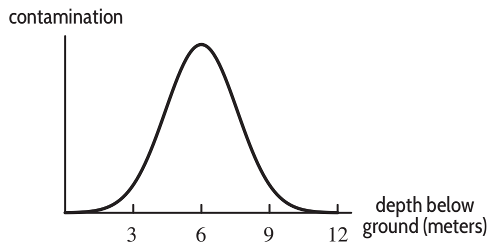


Figure 1.101

6. See Figure 1.102.

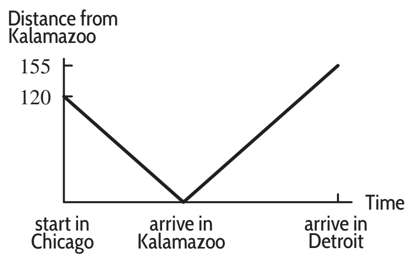


Figure 1.102

7. **(a)** The hottest oven results in the largest final temperature, so the answer is (I).
(b) The initial temperature is represented by the vertical intercept, and the curve with the lowest vertical intercept is (IV).
(c) The initial temperature is represented by the vertical intercept, and the two curves with the same vertical intercepts are (II) and (III).
(d) The loaf represented by curve (III) heats up much faster than the loaf represented by curve (II). One possible reason for this is that the curve shown in (III) is for a much smaller loaf.

8. The equation of the line is of the form $y = b + mx$ where m is the slope given by

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{3-(-1)}{2-0} = 2$$

and b is the vertical intercept. Since $y = -1$ when $x = 0$ the vertical intercept is $b = -1$. Therefore, the equation of the line passing through $(0, -1)$ and $(2, 3)$ is

$$y = -1 + 2x.$$

Note that when $x = 0$ this gives $y = -1$ and when $x = 2$ it gives $y = 3$ as required.

9. The equation of the line is of the form $y = b + mx$ where m is the slope given by

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{2-3}{2-(-1)} = -\frac{1}{3}$$

so the line is of the form $y = b - x/3$. For the line to pass through $(-1, 3)$ we need $3 = b + 1/3$ so $b = 8/3$. Therefore, the equation of the line passing through $(-1, 3)$ and $(2, 2)$ is

$$y = \frac{8}{3} - \frac{1}{3}x.$$

Note that when $x = -1$ this gives $y = 3$ and when $x = 2$ it gives $y = 2$ as required.

10. The equation of the line is of the form $y = b + mx$ where m is the slope given by

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{0}{2-0} = 0$$

so the line is $y = 0 \cdot x + b = b$. Since the line passes through the point $(0, 2)$ we have $b = 2$ and the equation of the line is $y = 2$.

11. If the equation of the line were of the form $y = b + mx$ where m is the slope, then the slope would be given by

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{1}{0}$$

so the line cannot be of this form. Instead we have the special case of a vertical line of the form $x = -1$.

12. **(a)** is (V), because slope is positive, vertical intercept is negative
(b) is (IV), because slope is negative, vertical intercept is positive
(c) is (I), because slope is 0, vertical intercept is positive
(d) is (VI), because slope and vertical intercept are both negative
(e) is (II), because slope and vertical intercept are both positive
(f) is (III), because slope is positive, vertical intercept is 0

13. Given that the function is linear, choose any two points, for example (5.2, 27.8) and (5.3, 29.2). Then

$$\text{Slope} = \frac{29.2 - 27.8}{5.3 - 5.2} = \frac{1.4}{0.1} = 14.$$

Using the point-slope formula, with the point $(5.2, 27.8)$, we get the equation

$$y - 27.8 = 14(x - 5.2)$$

which is equivalent to

$$y = 14x - 45.$$

14. **(a)** $\text{Charge per cubic foot} = \dfrac{\Delta\$}{\Delta \text{ cu. ft.}} = \dfrac{102-69}{1600-1000} = \$0.055/\text{cu. ft.}$
Alternatively, if we let c = cost, w = cubic feet of water, b = fixed charge, and m = cost/cubic foot, we obtain $c = b + mw$. Substituting the information given in the problem, we have

$$69 = b + 1000m$$
$$102 = b + 1600m.$$

Subtracting the first equation from the second yields $33 = 600m$, so $m = 0.055$.

(b) The equation is $c = b + 0.055w$, so $69 = b + 0.055(1000)$, which yields $b = 14$. Thus the equation is $c = 14 + 0.055w$.

(c) We need to solve the equation $201 = 14 + 0.055w$, which yields $w = 3400$. It costs \$201 to use 3400 cubic feet of water.

15. **(a)** We find the slope m and intercept b in the linear equation $S = b + mt$. To find the slope m, we use

$$m = \frac{\Delta S}{\Delta t} = \frac{66 - 113}{50 - 0} = -0.94.$$

When $t = 0$, we have $S = 113$, so the intercept b is 113. The linear formula is

$$S = 113 - 0.94t.$$

(b) We use the formula $S = 113 - 0.94t$. When $S = 20$, we have $20 = 113 - 0.94t$ and so $t = 98.9$. If this linear model were correct, the average male sperm count would drop below the fertility level during the year 2038.

16. **(a)** Substituting $x = 1$ gives $f(1) = 3(1) - 5 = 3 - 5 = -2$.

(b) We substitute $x = 5$:

$$y = 3(5) - 5 = 15 - 5 = 10.$$

(c) We substitute $y = 4$ and solve for x:

$$4 = 3x - 5$$
$$9 = 3x$$
$$x = 3.$$

(d) $\text{Average rate of change} = \dfrac{f(4) - f(2)}{4 - 2} = \dfrac{7 - 1}{2} = \dfrac{6}{2} = 3.$

17. The average velocity over a time period is the change in position divided by the change in time. Since the function $s(t)$ gives the position of the particle, we find the values of $s(3) = 12 \cdot 3 - 3^2 = 27$ and $s(1) = 12 \cdot 1 - 1^2 = 11$. Using these values, we find

$$\text{Average velocity} = \frac{\Delta s(t)}{\Delta t} = \frac{s(3) - s(1)}{3 - 1} = \frac{27 - 11}{2} = 8 \text{ mm/sec.}$$

18. The average velocity over a time period is the change in position divided by the change in time. Since the function $s(t)$ gives the position of the particle, we find the values of $s(3) = \ln 3$ and $s(1) = \ln 1$. Using these values, we find

$$\text{Average velocity} = \frac{\Delta s(t)}{\Delta t} = \frac{s(3) - s(1)}{3 - 1} = \frac{\ln 3 - \ln 1}{2} = \frac{\ln 3}{2} = 0.549 \text{ mm/sec.}$$

19. The average velocity over a time period is the change in position divided by the change in time. Since the function $s(t)$ gives the position of the particle, we find the values of $s(3) = 11$ and $s(1) = 3$. Using these values, we find

$$\text{Average velocity} = \frac{\Delta s(t)}{\Delta t} = \frac{s(3) - s(1)}{3 - 1} = \frac{11 - 3}{2} = 4 \text{ mm/sec.}$$

20. The average velocity over a time period is the change in position divided by the change in time. Since the function $s(t)$ gives the position of the particle, we find the values of $s(3) = 4$ and $s(1) = 4$. Using these values, we find

$$\text{Average velocity} = \frac{\Delta s(t)}{\Delta t} = \frac{s(3) - s(1)}{3 - 1} = \frac{4 - 4}{2} = 0 \text{ mm/sec.}$$

Though the particle moves, its average velocity over the interval is zero, since it is at the same position at $t = 1$ and $t = 3$.

21. The average velocity over a time period is the change in position divided by the change in time. Since the function $s(t)$ gives the position of the particle, we find the values on the graph of $s(3) = 2$ and $s(1) = 3$. Using these values, we find

$$\text{Average velocity} = \frac{\Delta s(t)}{\Delta t} = \frac{s(3) - s(1)}{3 - 1} = \frac{2 - 3}{2} = -\frac{1}{2} \text{ mm/sec.}$$

22. The average velocity over a time period is the change in position divided by the change in time. Since the function $s(t)$ gives the position of the particle, we find the values on the graph of $s(3) = 2$ and $s(1) = 2$. Using these values, we find

$$\text{Average velocity} = \frac{\Delta s(t)}{\Delta t} = \frac{s(3) - s(1)}{3 - 1} = \frac{2 - 2}{2} = 0 \text{ mm/sec.}$$

Though the particle moves, its average velocity over the interval is zero, since it is at the same position at $t = 1$ and $t = 3$.

23. **(a)** More fertilizer increases the yield until about 40 lbs.; then it is too much and ruins crops, lowering yield.
(b) The vertical intercept is at $Y = 200$. If there is no fertilizer, then the yield is 200 bushels.
(c) The horizontal intercept is at $a = 80$. If you use 80 lbs. of fertilizer, then you will grow no apples at all.
(d) The range is the set of values of Y attainable over the domain $0 \leq a \leq 80$. Looking at the graph, we can see that Y goes as high as 550 and as low as 0. So the range is $0 \leq Y \leq 550$.
(e) Looking at the graph, we can see that Y is decreasing near $a = 60$.
(f) Looking at the graph, we can see that Y is concave down everywhere, so it is certainly concave down near $a = 40$.

24. **(a)** Advertising is generally cheaper in bulk; spending more money will give better and better marginal results initially, hence concave up. (Spending $5,000 could give you a big newspaper ad reaching 200,000 people; spending $100,000 could give you a series of TV spots reaching 50,000,000 people.) See Figure 1.103. But after a certain point, you may "saturate" the market, and the increase in revenue slows down – hence concave down.
(b) The temperature of a hot object decreases at a rate proportional to the difference between its temperature and the temperature of the air around it. Thus, the temperature of a very hot object decreases more quickly than a cooler object. The graph is decreasing and concave up. See Figure 1.104 (We are assuming that the coffee is all at the same temperature.)

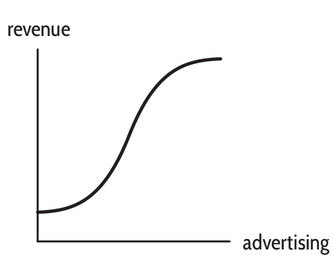


Figure 1.103

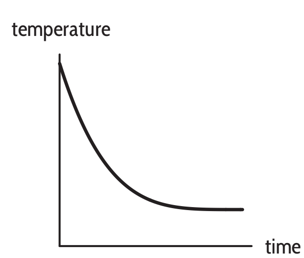


Figure 1.104

25. **(a)** This is the graph of a linear function, which increases at a constant rate, and thus corresponds to $k(t)$, which increases by 0.3 over each interval of 1.

(b) This graph is concave down, so it corresponds to a function whose increases are getting smaller, as is the case with $h(t)$, whose increases are 10, 9, 8, 7, and 6.

(c) This graph is concave up, so it corresponds to a function whose increases are getting bigger, as is the case with $g(t)$, whose increases are 1, 2, 3, 4, and 5.

26. **(a)** See Figure 1.105.

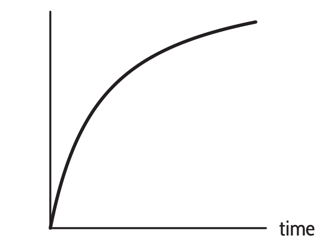


Figure 1.105

(b) "The rate at which new people try it" is the rate of change of the total number of people who have tried the product. Thus, the statement of the problem is telling you that the graph is concave down—the slope is positive but decreasing, as the graph shows.

27. **(a)** The death rate from stomach cancer has decreased fairly steadily. For lung and bronchus cancer the death rate increased fairly steadily until around 1990 and has decreased since. Colorectum cancer death rates increased slightly until about 1945, leveled off until about 1980 and has decreased since. Prostate cancer deaths increased until about 1945 and then stayed level until a small jump between 1990 and 1995. Since 1995, prostate cancer death rates have decreased. Death rates from liver and intrahepatic bile duct decreased slightly until about 1975 and have increased since. Deaths from leukemia increased slightly until around 1980 and then decreased slightly. Deaths from pancreatic cancer increased until around 1980 and then leveled off.

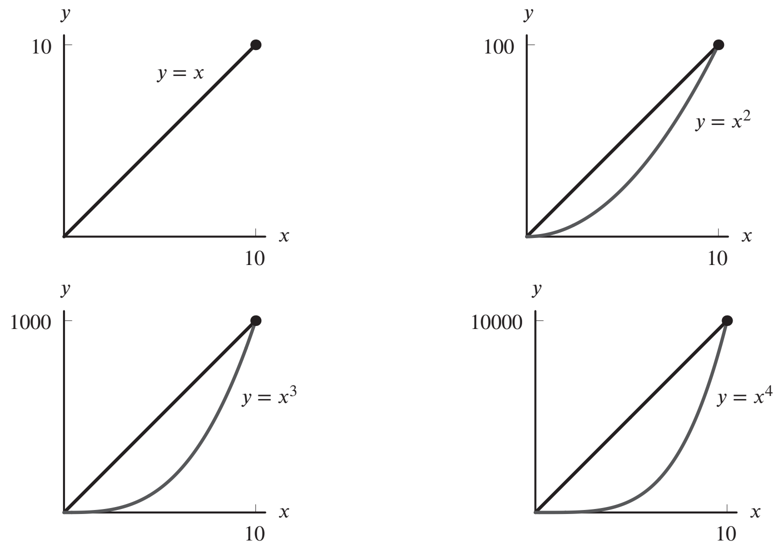
(b) Lung and bronchus cancer deaths have had the greatest average rate of change.

$$\text{Average rate of change} = \frac{55-3}{2012-1930}$$
$$\approx 0.634 \quad \begin{array}{l}\text{additional deaths per 100,000}\\ \text{per year between 1930 and 2012.}\end{array}$$

(c) Stomach cancer deaths have the most negative slope on the graph.

$$\text{Average rate of change} = \frac{3-43}{2012-1930}$$
$$\approx -0.488 \text{ or } 0.488 \quad \begin{array}{l}\text{fewer deaths per 100,000}\\ \text{per year between 1930 and 2012.}\end{array}$$

28. For $y = x$, average rate of change $= \frac{10-0}{10-0} = 1$.
For $y = x^2$, average rate of change $= \frac{100-0}{10-0} = 10$.
For $y = x^3$, average rate of change $= \frac{1000-0}{10-0} = 100$.
For $y = x^4$, average rate of change $= \frac{10000-0}{10-0} = 1000$.
So $y = x^4$ has the largest average rate of change. For $y = x$, the line is the same as the original function.



29. Each of these questions can also be answered by considering the slope of the line joining the two relevant points.

(a) The average rate of change is positive if the volume of water is increasing with time and negative if the volume of water is decreasing.

(i) Since volume is rising from 500 to 1000 from $t = 0$ to $t = 5$, the average rate of change is positive.

(ii) We can see that the volume at $t = 10$ is greater than the volume at $t = 0$. Thus, the average rate of change is positive.

(iii) We can see that the volume at $t = 15$ is lower than the volume at $t = 0$. Thus, the average rate of change is negative.

(iv) We can see that the volume at $t = 20$ is greater than the volume at $t = 0$. Thus, the average rate of change is positive.

(b) (i) The secant line between $t = 0$ and $t = 5$ is steeper than the secant line between $t = 0$ and $t = 10$, so the slope of the secant line is greater on $0 \le t \le 5$. Since average rate of change is represented graphically by the slope of a secant line, the rate of change in the interval $0 \le t \le 5$ is greater than that in the interval $0 \le t \le 10$.

(ii) The slope of the secant line between $t = 0$ and $t = 20$ is greater than the slope of the secant line between $t = 0$ and $t = 10$, so the rate of change is larger for $0 \le t \le 20$.

(c) The average rate of change in the interval $0 \le t \le 10$ is about

$$\frac{750 - 500}{10} = \frac{250}{10} = 25 \quad \text{cubic meters per week}$$

This tells us that for the first ten weeks, the volume of water is growing at an average rate of about 25 cubic meters per week.

30. (a) We know that the fixed cost for this company is the amount of money it takes to produce zero units, or simply the vertical intercept of the graph. Thus, the

$$\text{fixed cost} = \$1000.$$

We know that the marginal cost is the price the company has to pay for each additional unit, or in other words, the slope of the graph. We know that

$$C(0) = 1000$$

and looking at the graph we also see that

$$C(200) = 4000.$$

Thus the slope of the line, or the marginal cost, is

$$\text{marginal cost} = \frac{4000 - 1000}{200 - 0} = \frac{3000}{200} = \$15 \text{ per unit.}$$

(b) $C(q)$ gives the price that the company will have to pay for the production of q units. Thus if

$$C(100) = 2500$$

we know that it will cost the company \$2500 to produce 100 items.

31. We know that the cost function will take on the form

$$C(q) = b + m \cdot q$$

where m is the variable cost and b is the fixed cost. We know that the company has a fixed cost of \$350,000 and that it costs the company \$400 to feed a student. That is, \$400 is the variable cost. Thus

$$C(q) = 350{,}000 + 400q.$$

We know that the revenue function will be of the form

$$R(q) = pq$$

where p is the price that the company charges a student. Since the company intends to charge \$800 per student we have

$$R(q) = 800q.$$

We know that the profit is simply the difference between the revenue and the cost. Thus

$$\pi(q) = 800q - (350{,}000 + 400q) = 800q - 350{,}000 - 400q = 400q - 350{,}000.$$

We are asked to find the number of students that must sign up for the plan in order for the company to make money. That is, we are asked to find the number of students q such that

$$\pi(q) > 0.$$

Solving we get

$$\begin{aligned} 400q - 350{,}000 &> 0 \\ 400q &> 350{,}000 \\ q &> 875. \end{aligned}$$

Thus, if more than 875 students sign up, the company will make a profit.

32. We are looking for a linear function $y = f(x)$ that, given a time x in years, gives a value y in dollars for the value of the refrigerator. We know that when $x = 0$, that is, when the refrigerator is new, $y = 950$, and when $x = 7$, the refrigerator is worthless, so $y = 0$. Thus $(0, 950)$ and $(7, 0)$ are on the line that we are looking for. The slope is then given by

$$m = \frac{950}{-7}$$

It is negative, indicating that the value decreases as time passes. Having found the slope, we can take the point $(7, 0)$ and use the point-slope formula:

$$y - y_1 = m(x - x_1).$$

So,

$$\begin{aligned} y - 0 &= -\frac{950}{7}(x - 7) \\ y &= -\frac{950}{7}x + 950. \end{aligned}$$

33. Generally manufacturers will produce more when prices are higher. Therefore, the first curve is a supply curve. Consumers consume less when prices are higher. Therefore, the second curve is a demand curve.

34. **(a)** We know that the equilibrium point is the point where the supply and demand curves intersect. They appear to intersect at a price of \$250 per unit, so the corresponding quantity is 750 units.

(b) We know that the supply curve climbs upward while the demand curve slopes downward. At the price of \$300 per unit the suppliers will be willing to produce 875 units while the consumers will be ready to buy 625 units. Thus, we see that when the price is above the equilibrium point, more items would be produced than the consumers will be willing to buy. Thus, the producers end up wasting money by producing that which will not be bought, so the producers are better off lowering the price.

(c) Looking at the point on the rising curve where the price is \$200 per unit, we see that the suppliers will be willing to produce 625 units, whereas looking at the point on the downward sloping curve where the price is \$200 per unit, we see that the consumers will be willing to buy 875 units. Thus, we see that when the price is less than the equilibrium price, the consumers are willing to buy more products than the suppliers would make and the suppliers can thus make more money by producing more units and raising the price.

35. This is a line with slope $-3/7$ and y-intercept 3, so a possible formula is

$$y = -\frac{3}{7}x + 3.$$

36. This is a line, so we use $y = mx + b$. The slope is a positive $2/5 = 0.4$ and the y-intercept is 2. The formula is $y = 0.4x + 2$.

37. Starting with the general exponential equation $y = Ae^{kx}$, we first find that for $(0, 1)$ to be on the graph, we must have $A = 1$. Then to make $(3, 4)$ lie on the graph, we require

$$\begin{aligned} 4 &= e^{3k} \\ \ln 4 &= 3k \\ k &= \frac{\ln 4}{3} \approx 0.4621. \end{aligned}$$

Thus the equation is

$$y = e^{0.4621x}.$$

Alternatively, we can use the form $y = a^x$, in which case we find $y = (1.5874)^x$.

38. Since this function has a y-intercept at $(0, 2)$, we expect it to have the form $y = 2e^{kx}$. Again, we find k by forcing the other point to lie on the graph:

$$\begin{aligned} 1 &= 2e^{2k} \\ \frac{1}{2} &= e^{2k} \\ \ln\left(\frac{1}{2}\right) &= 2k \\ k &= \frac{\ln(\frac{1}{2})}{2} \approx -0.34657. \end{aligned}$$

This value is negative, which makes sense since the graph shows exponential decay. The final equation, then, is

$$y = 2e^{-0.34657x}.$$

Alternatively, we can use the form $y = 2a^x$, in which case we find $y = 2(0.707)^x$.

39. This looks like an exponential function. The y-intercept is 3 and we use the form $y = 3e^{kt}$. We substitute the point $(5, 9)$ to solve for k:

$$\begin{aligned} 9 &= 3e^{k5} \\ 3 &= e^{5k} \\ \ln 3 &= 5k \\ k &= 0.2197. \end{aligned}$$

A possible formula is

$$y = 3e^{0.2197t}.$$

Alternatively, we can use the form $y = 3a^t$, in which case we find $y = 3(1.2457)^t$.

40. $z = 1 - \cos\theta$

41. We look first at $f(x)$. As x increases by 1, $f(x)$ decreases by 5, then 6, then 7. The rate of change is not constant so the function is not linear. To see if it is exponential, we check ratios of successive terms:

$$\frac{20}{25} = 0.8, \quad \frac{14}{20} = 0.7, \quad \frac{7}{14} = 0.5.$$

Since the ratios are not constant, this function is not exponential either.

What about $g(x)$? As x increases by 1, $g(x)$ decreases by 3.2 each time, so this function is linear with slope -3.2. The vertical intercept (when $x = 0$) is 30.8 so the formula is

$$g(x) = 30.8 - 3.2x.$$

Now consider $h(x)$. As x increases by 1, $h(x)$ decreases by 6000, then 3600, then 2160. The rate of change is not constant so this function is not linear. To see if it is exponential, we check ratios of successive terms:

$$\frac{9000}{15{,}000} = 0.6, \quad \frac{5400}{9000} = 0.6, \quad \frac{3240}{5400} = 0.6.$$

Since the ratios are constant, this is an exponential function with base 0.6. The initial value (when $x = 0$) is $15,000$ so the formula is

$$h(x) = 15{,}000(0.6)^x.$$

42. **(a)** The slope of the line is

$$\text{Slope} = \frac{\Delta C}{\Delta t} = \frac{36.44 - 35.24}{2019 - 2014} = 0.24 \text{ billion tons per year.}$$

The initial value (when $t = 0$) is 35.24, so the linear formula is

$$C = 35.24 + 0.24t.$$

The annual rate of increase in carbon dioxide emission is 0.24 billion tons per year.

(b) If the function is exponential, we have $C = C_0a^t$. The initial value (when $t = 0$) is 35.24, so we have $C = 35.24a^t$. We use the fact that $C = 36.44$ when $t = 5$ to find a:

$$\begin{aligned} 36.44 &= 35.24a^5 \\ a^5 &= 1.034 \\ a &= (1.034)^{1/5} = 1.0067. \end{aligned}$$

The exponential formula is

$$C = 35.24(1.0067)^t.$$

Carbon dioxide emission is increasing 0.67% per year. (Alternately, if we used the form $C = C_0e^{kt}$, the formula would be $C = 35.24e^{0.0067t}$.)

43. The population has increased by a factor of $48{,}000{,}000/40{,}000{,}000 = 1.2$ in 10 years. Thus we have the formula

$$P = 40{,}000{,}000(1.2)^{t/10},$$

and $t/10$ gives the number of 10-year periods that have passed since 2005.

In 2005, $t/10 = 0$, so we have $P = 40{,}000{,}000$.

In 2015, $t/10 = 1$, so $P = 40{,}000{,}000(1.2) = 48{,}000{,}000$.

In 2020, $t/10 = 1.5$, so $P = 40{,}000{,}000(1.2)^{1.5} \approx 52{,}581{,}000$.

To find the doubling time, solve $80{,}000{,}000 = 40{,}000{,}000(1.2)^{t/10}$, to get $t = 38.02$ years.

44. Taking logs of both sides

$$\begin{aligned} \ln 3^x &= x\ln 3 = \ln 11 \\ x &= \frac{\ln 11}{\ln 3} = 2.2. \end{aligned}$$

45. Isolating the exponential term

$$\begin{aligned} 20 &= 50(1.04)^x \\ \frac{20}{50} &= (1.04)^x. \end{aligned}$$

Taking logs of both sides

$$\begin{aligned} \ln\frac{2}{5} &= \ln(1.04)^x \\ \ln\frac{2}{5} &= x\ln(1.04) \\ x &= \frac{\ln(2/5)}{\ln(1.04)} = -23.4. \end{aligned}$$

46. We use logarithms:

$$\begin{aligned} \ln(e^{5x}) &= \ln(100) \\ 5x &= \ln(100) \\ x &= \frac{\ln(100)}{5} = 0.921. \end{aligned}$$

47. We divide both sides by 25 and then use logarithms:

$$\begin{aligned} e^{3x} &= \frac{10}{25} = 0.4 \\ \ln(e^{3x}) &= \ln(0.4) \\ 3x &= \ln(0.4) \\ x &= \frac{\ln(0.4)}{3} = -0.305. \end{aligned}$$

48. Since $e^{0.08} = 1.0833$, and $e^{-0.3} = 0.741$, we have

$$\begin{aligned} P &= e^{0.08t} = \left(e^{0.08}\right)^t = (1.0833)^t \text{ and} \\ Q &= e^{-0.3t} = \left(e^{-0.3}\right)^t = (0.741)^t. \end{aligned}$$

49. **(a)** The continuous percent growth rate is 15%.

(b) We want $P_0a^t = 10e^{0.15t}$, so we have $P_0 = 10$ and $a = e^{0.15} = 1.162$. The corresponding function is

$$P = 10(1.162)^t.$$

(c) Since the base in the answer to part (b) is 1.162, the annual percent growth rate is 16.2%. This annual rate is equivalent to the continuous growth rate of 15%.

(d) When we sketch the graphs of $P = 10e^{0.15t}$ and $P = 10(1.162)^t$ on the same axes, we only see one graph. These two exponential formulas are two ways of representing the same function, so the graphs are the same. See Figure 1.106.

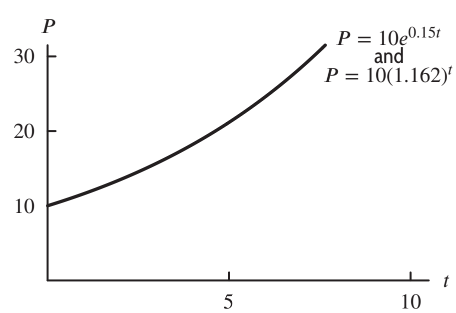


Figure 1.106

50. We know that the formula for the balance after t years in an account which is compounded continuously is

$$P(t) = P_0e^{rt}$$

where P_0 is the initial deposit and r is the interest rate. In our case we are told that the rate is $r = 0.08$, so we have

$$P(t) = P_0e^{0.08t}.$$

We are asked to find the value for P_0 such that after three years the balance would be \$10,000. That is, we are asked for P_0 such that

$$10{,}000 = P_0e^{0.08(3)}.$$

Solving we get

$$\begin{aligned} 10{,}000 &= P_0 e^{0.08(3)} \\ &= P_0 e^{0.24} \\ &\approx P_0(1.27125) \\ P_0 &\approx \frac{10{,}000}{1.27125} \\ &\approx 7866.28. \end{aligned}$$

Thus, \$7866.28 should be deposited into this account so that after three years the balance would be \$10,000.

51. **(a)** We want to find t such that

$$0.15Q_0 = Q_0 e^{-0.000121t},$$

so $0.15 = e^{-0.000121t}$, meaning that $\ln 0.15 = -0.000121t$, or $t = \dfrac{\ln 0.15}{-0.000121} \approx 15{,}678.7$ years.

(b) Let T be the half-life of carbon-14. Then

$$0.5Q_0 = Q_0 e^{-0.000121T},$$

so $0.5 = e^{-0.000121T}$, or $T = \dfrac{\ln 0.5}{-0.000121} = 5728.489 \approx 5730$ years.

52. We use $P = P_0 e^{kt}$. Since 200 grams are present initially, we have $P = 200e^{kt}$. To find k, we use the fact that the half-life is 8 years:

$$\begin{aligned} P &= 200e^{kt} \\ 100 &= 200e^{k(8)} \\ 0.5 &= e^{8k} \\ \ln(0.5) &= 8k \\ k &= \frac{\ln(0.5)}{8} \approx -0.087. \end{aligned}$$

We have

$$P = 200e^{-0.087t}.$$

The amount remaining after 12 years is

$$P = 200e^{-0.087(12)} = 70.4 \text{ grams}.$$

To find the time until 10% remains, we use $P = 0.10P_0 = 0.10(200) = 20$:

$$\begin{aligned} 20 &= 200e^{-0.087t} \\ 0.10 &= e^{-0.087t} \\ \ln(0.10) &= -0.087t \\ t &= \frac{\ln(0.10)}{-0.087} = 26.5 \text{ years}. \end{aligned}$$

53. Given the doubling time of 5 hours, we can solve for the bacteria's growth rate;

$$\begin{aligned} 2P_0 &= P_0 e^{k5} \\ k &= \frac{\ln 2}{5}. \end{aligned}$$

So the growth of the bacteria population is given by:

$$P = P_0 e^{\ln(2)t/5}.$$

We want to find t such that

$$3P_0 = P_0 e^{\ln(2)t/5}.$$

Therefore we divide both sides by P_0 and apply ln. We get

$$t = \frac{5\ln(3)}{\ln(2)} = 7.925 \text{ hours}.$$

54. If the pressure at sea level is P_0, the pressure P at altitude h is given by

$$P = P_0 \left(1 - \frac{0.4}{100}\right)^{h/100},$$

since we want the pressure to be multiplied by a factor of $(1 - 0.4/100) = 0.996$ for each 100 feet we go up to make it decrease by 0.4% over that interval. At Mexico City $h = 7340$, so the pressure is

$$P = P_0(0.996)^{7340/100} \approx 0.745P_0.$$

So the pressure is reduced from P_0 to approximately $0.745P_0$, a decrease of 25.5%.

55. Let A represent the revenue (in billions of dollars) at Apple t years since 2016. Since $A = 216$ when $t = 0$ and we want the continuous growth rate, we write $A = 216e^{kt}$. We use the information from 2020, that $A = 275$ when $t = 4$, to find k:

$$\begin{aligned} 275 &= 216e^{k\cdot 4} \\ 1.27315 &= e^{4k} \\ \ln(1.27315) &= 4k \\ k &= 0.06037. \end{aligned}$$

We have $A = 216e^{0.06037t}$, which represents a continuous growth rate of 6.037% per year.

56. Marine catch, M (in millions), is increasing exponentially, so

$$M = M_0 e^{kt}.$$

If we let t be the number of years since 2007, we have $M_0 = 140.7$ and

$$M = 140.7e^{kt}.$$

Since $M = 158$ when $t = 5$, we can solve for k:

$$\begin{aligned} 158 &= 140.7e^{k(5)} \\ 158/140.7 &= e^{5k} \\ \ln(158/140.7) &= 5k \\ k &= \frac{\ln(158/140.7)}{5} = 0.0232. \end{aligned}$$

Marine catch has increased at a continuous rate of 2.32% per year. Since

$$M = 140.7e^{0.0232t}$$

in the year 2020, we have $t = 13$ and

$$M = 140.7e^{0.0232\cdot 13} = 190.2 \text{ million tons.}$$

57. The following table contains the present value of each of the expenses. Since the total present value of the repairs, \$195.72, is less than the cost of the service contract, you should not buy the service contract.

Present value of repairs

Year	Repairs	Present Value
1	30	$30/(1.05) = 28.57$
2	70	$70/(1.05)^2 = 63.49$
3	120	$120/(1.05)^3 = 103.66$
	Total	195.72

58. **(a)** $g(h(x)) = g(x^3+1) = \sqrt{x^3+1}$.
(b) $h(g(x)) = h(\sqrt{x}) = (\sqrt{x})^3 + 1 = x^{3/2} + 1$.
(c) $h(h(x)) = h(x^3+1) = (x^3+1)^3 + 1$.
(d) $g(x) + 1 = \sqrt{x} + 1$.
(e) $g(x+1) = \sqrt{x+1}$.

59. **(a)** We know that $f(x) = 2x + 3$ and $g(x) = \ln x$. Thus,

$$g(f(x)) = g(2x+3) = \ln(2x+3).$$

(b) We know that $f(x) = 2x + 3$ and $g(x) = \ln x$. Thus,

$$f(g(x)) = f(\ln x) = 2\ln x + 3.$$

(c) We know that $f(x) = 2x + 3$. Thus,

$$f(f(x)) = f(2x+3) = 2(2x+3) + 3 = 4x + 6 + 3 = 4x + 9.$$

60. **(a)** $f(g(x)) = f(x+3) = 2(x+3)^2 = 2(x^2 + 6x + 9) = 2x^2 + 12x + 18$.
(b) $g(f(x)) = g(2x^2) = 2x^2 + 3$.
(c) $f(f(x)) = f(2x^2) = 2(2x^2)^2 = 8x^4$.

61. **(a)** We have $f(g(x)) = f(5x^2) = 2(5x^2) + 3 = 10x^2 + 3$.
(b) We have $g(f(x)) = g(2x+3) = 5(2x+3)^2 = 5(4x^2 + 12x + 9) = 20x^2 + 60x + 45$.
(c) We have $f(f(x)) = f(2x+3) = 2(2x+3) + 3 = 4x + 9$.

62. **(a)** We have $f(g(x)) = f(\ln x) = (\ln x)^2 + 1$.
(b) We have $g(f(x)) = g(x^2+1) = \ln(x^2+1)$.
(c) We have $f(f(x)) = f(x^2+1) = (x^2+1)^2 + 1 = x^4 + 2x^2 + 2$.

63. For $5f(x)$, the values are 5 times as large. See Figure 1.107.

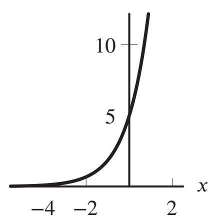


Figure 1.107

64. For $f(x+5)$, the graph is shifted 5 units to the left. See Figure 1.108.

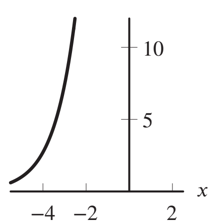


Figure 1.108

65. For $f(x) + 5$, the graph is shifted 5 upward. See Figure 1.109.

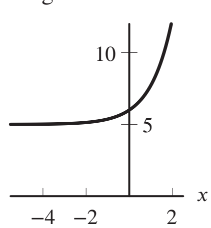
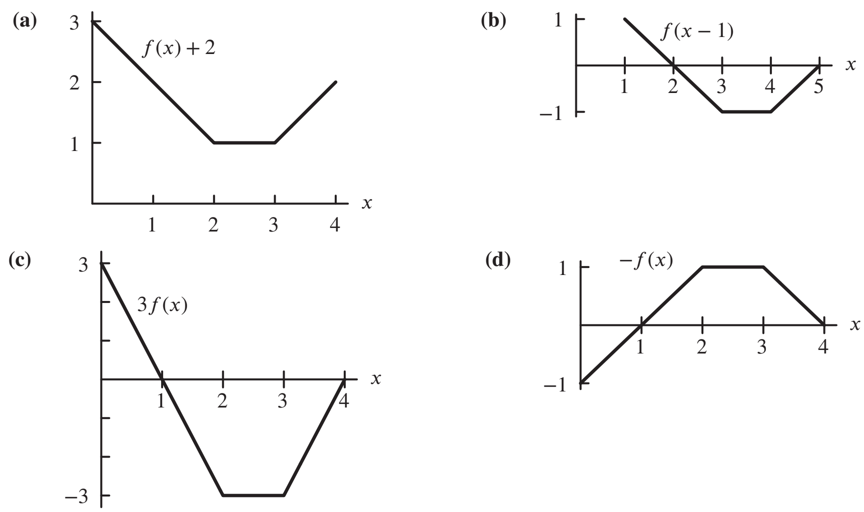
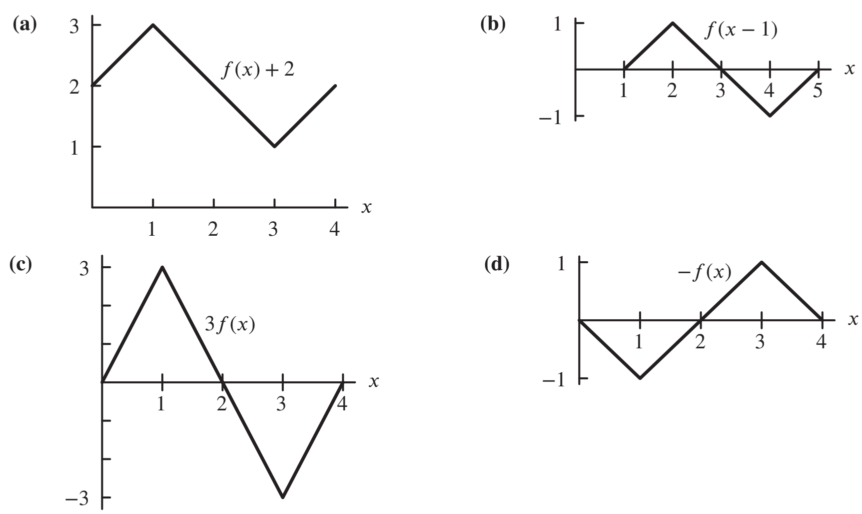


Figure 1.109

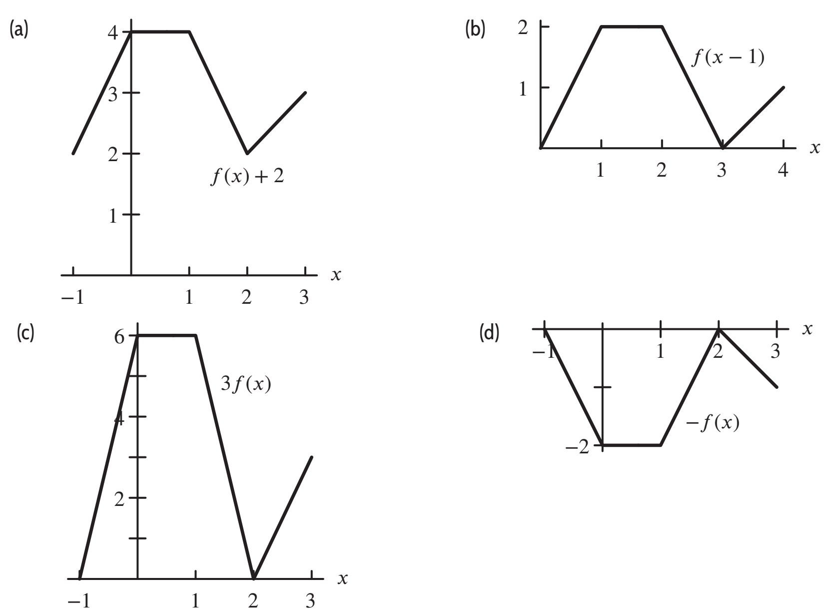
66.



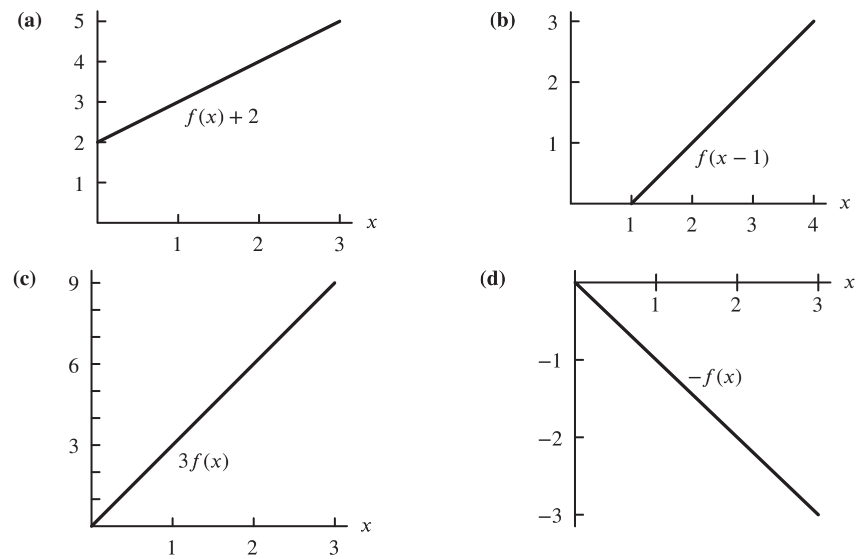
67.



68.



69.



70. This graph is the graph of $m(t)$ shifted upward by two units. See Figure 1.110.

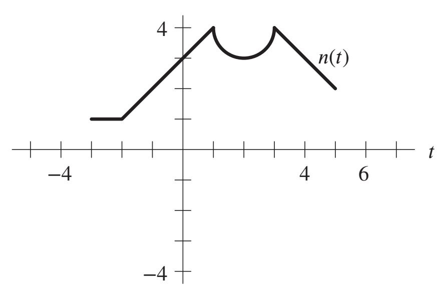


Figure 1.110

71. This graph is the graph of $m(t)$ shifted to the right by one unit. See Figure 1.111.

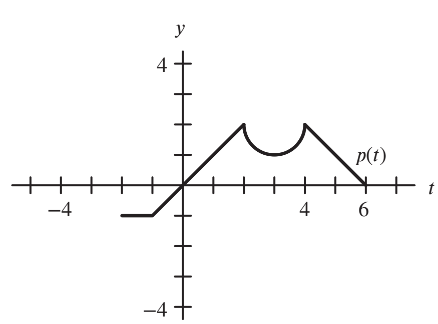


Figure 1.111

72. This graph is the graph of $m(t)$ shifted to the left by 1.5 units. See Figure 1.112.

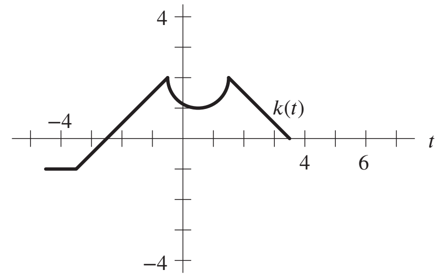


Figure 1.112

73. This graph is the graph of $m(t)$ shifted to the right by 0.5 units and downward by 2.5 units. See Figure 1.113.

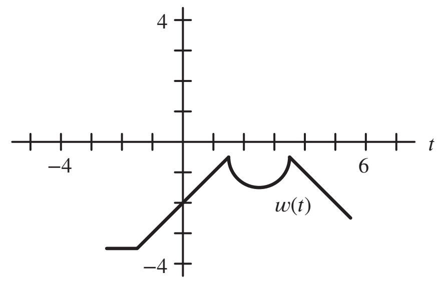


Figure 1.113

74. **(a)** First note that C is negative, because if $C = 0$ then the quantity of pollution $Q = f(t) = A + Be^{Ct}$ does not change over time and if $C > 0$ then $Q = f(t)$ approaches $+\infty$ or $-\infty$ as t approaches ∞, which is not realistic. Since $C < 0$, the quantity $Q = f(t)$ approaches $A + B(0) = A$ as t increases.

Since Q can not be negative or 0, we have $A > 0$. Finally, since $Q = f(t)$ is a decreasing function we have $B > 0$.

(b) Initially we have $Q = f(0) = A + Be^{C(0)} = A + B$.

(c) We saw in the solution to part (a) that $Q = f(t)$ approaches A as t approaches ∞, so the legal limit, which is the goal of the clean-up project, is A.

75. This looks like the graph of $y = -x^2$ shifted up 2 units and to the left 3 units. One possible formula is $y = -(x+3)^2 + 2$. Other answers are possible. You can check your answer by graphing it using a calculator or computer.

76. This looks like the graph of $y = x^2$ shifted down 5 units and to the right 2 units. One possible formula is $y = (x-2)^2 - 5$. Other answers are possible. You can check your answer by graphing it using a calculator or computer.

77. If p is proportional to t, then $p = kt$ for some fixed constant k. From the values $t = 10$, $p = 25$, we have $25 = k(10)$, so $k = 2.5$. To see if p is proportional to t, we must see if $p = 2.5t$ gives all the values in the table. However, when we check the values $t = 20$, $p = 60$, we see that $60 \neq 2.5(20)$. Thus, p is not proportional to t.

78. Substituting $w = 65$ and $h = 160$, we have

(a)

$$s = 0.01(65^{0.25})(160^{0.75}) = 1.3\text{ m}^2.$$

(b) We substitute $s = 1.5$ and $h = 180$ and solve for w:

$$1.5 = 0.01w^{0.25}(180^{0.75}).$$

We have

$$w^{0.25} = \frac{1.5}{0.01(180^{0.75})} = 3.05.$$

Since $w^{0.25} = w^{1/4}$, we take the fourth power of both sides, giving

$$w = 86.8\text{ kg}.$$

(c) We substitute $w = 70$ and solve for h in terms of s:

$$s = 0.01(70^{0.25})h^{0.75},$$

so

$$h^{0.75} = \frac{s}{0.01(70^{0.25})}.$$

Since $h^{0.75} = h^{3/4}$, we take the 4/3 power of each side, giving

$$h = \left(\frac{s}{0.01(70^{0.25})}\right)^{4/3} = \frac{s^{4/3}}{(0.01^{4/3})(70^{1/3})}$$

so

$$h = 112.6s^{4/3}.$$

79. The period is $2\pi/3$, because when t varies from 0 to $2\pi/3$, the quantity $3t$ varies from 0 to 2π. The amplitude is 7, since the value of the function oscillates between -7 and 7.

80. The period is $2\pi/(1/4) = 8\pi$, because when u varies from 0 to 8π, the quantity $u/4$ varies from 0 to 2π. The amplitude is 3, since the function oscillates between 2 and 8.

81. The period is $2\pi/\pi = 2$, since when t increases from 0 to 2, the value of πt increases from 0 to 2π. The amplitude is 0.1, since the function oscillates between 1.9 and 2.1.

82. **(a)** It makes sense that temperature would be a function of time of day. Assuming similar weather for all the days of the experiment, time of day would probably be the over-riding factor on temperature. Thus, the temperature pattern should repeat itself each day.

(b) The maximum seems to occur at 19 hours or 7 pm. It makes sense that the river would be the hottest toward the end of daylight hours since the sun will have been beating down on it all day. The minimum occurs at around 9 am which also makes sense because the sun has not been up long enough to warm the river much by then.

(c) The period is one day. The amplitude is approximately $\dfrac{32-28}{2} = 2°\text{C}$

83. **(a)** The period looks to be about 12 months. This means that the number of mumps cases oscillates and repeats itself approximately once a year.

$$\text{Amplitude} = \frac{\text{max} - \text{min}}{2} = \frac{11{,}000 - 2000}{2} = 4500 \text{ cases}$$

This means that the minimum and maximum number of cases of mumps are within $9000 (= 4500 \cdot 2)$ cases of each other.

(b) Assuming cyclical behavior, the number of cases in 30 months will be the same as the number of cases in 6 months which is about 2000 cases. (30 months equals 2 years and six months). The number of cases in 45 months equals the number of cases in 3 years and 9 months which assuming cyclical behavior is the same as the number of cases in 9 months. This is about 2000 as well.

84.

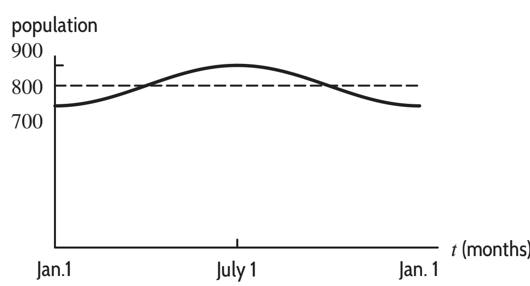


Figure 1.114

85. This graph is the same as in Problem 20 of Section 1.10 but shifted up by 2, so it is given by $f(x) = 2\sin\left(\frac{x}{4}\right) + 2$.

86. The graph is an inverted sine curve with amplitude 1 and period 2π, shifted up by 2, so it is given by $f(x) = 2 - \sin x$.

STRENGTHEN YOUR UNDERSTANDING

1. False, the domain is the set of inputs of a function.

2. False, $f(10)$ is the value of the car when it is 10 years old.

3. True, the point $(2, 4)$ is on the graph since $f(2) = 2^2 = 4$.

4. False, since we include the points 3 and 4, the set of numbers is written $[3, 4]$.

5. True. Plugging in $r = 0$ to find the intercept on the D axis gives $D = 10$.

6. False, a function could be given by a graph or table, or it could be described in words.

7. True, plugging in $x = 3$, we have $f(3) = 3^2 + 2(3) + 1 = 9 + 6 + 1 = 16$.

8. True, consider the function $f(x) = x^2 - 1$. This has horizontal intercepts at $x = \pm 1$.

9. False, if a graph has two vertical intercepts, then it cannot correspond to a function, since the function would have two values at zero.

10. True. The vertical intercept is the value of C when $q = 0$, or the cost to produce no items.

11. True, the slope of a linear function is given by rise over run, where rise is the difference in the function values and run is the difference in the corresponding inputs.

12. False, the linear function $m(x)$ has slope 3.

13. True, any constant function is a linear function with slope zero.

14. True. We first check that the slope is -1, and then make sure that the point $(2, 5)$ is on the graph. Since $-(2) + 7 = 5$, the line $y = -x + 7$ passes through the point $(2, 5)$.

15. True, for every increase in s by 2, the function $h(s)$ decreases by 4, so $h(s)$ could correspond to a linear function with slope -2.

16. False, any two linear functions which have the same slope and different y-intercepts will not intersect, since they are parallel.

17. False, consider the linear function $y = x - 1$. The slope of this function is 1, and the y-intercept is -1.

18. False, the slope of a linear function $f(x)$ is given by $\frac{f(x_2)-f(x_1)}{x_2-x_1}$. Plugging in 4 for x_1 and 9 for x_2, we see that the slope is $\frac{3-2}{9-4} = \frac{1}{5}$.

19. False, the units of the slope are acres per meter.

20. True, since a slope of 3 for a linear function $y = f(x)$ means increasing the x coordinate by 1 causes the y coordinate to increase by 3.

21. False. Since the slope of this linear function is negative, the function is decreasing.

22. True, this is just the definition of the average rate of change.

23. False, the average rate of change is given by $\frac{C(b)-C(a)}{b-a}$. The units of the numerator are dollars, and the units of the denominator are students, so the units of the average rate of change is in dollars per student.

24. True, the average rate of change is given by the formula $\frac{s(b)-s(a)}{b-a}$. Substituting $b = 2$ and $a = -1$, we get $\frac{2^2-(-1)^2}{2-(-1)} = \frac{4-1}{2+1} = \frac{3}{3} = 1 > 0$.

25. True, consider $f(x) = \sqrt{x}$.

26. False, if $Q(r)$ was concave up, then the average rate of change would be increasing. However, the average rate of change from $r = 1$ to $r = 2$ is $\frac{15-10}{2-1} = 5$, and the average rate of change from $r = 2$ to $r = 3$ is $\frac{17-15}{3-2} = 2$.

27. False, the velocity includes the direction of the particle, whereas the speed does not.

28. True, the rate of change formula is the same as the formula for the slope of the secant line.

29. True, for any values $t_1 < t_2$, $\frac{s(t_2)-s(t_1)}{t_2-t_1} = \frac{3t_2+2-(3t_1+2)}{t_2-t_1} = \frac{3(t_2-t_1)}{t_2-t_1} = 3$.

30. False, consider the function $f(x) = 1/x$ for $x > 0$. $f(x)$ is concave up, but it is a decreasing function.

31. True. This is the definition of relative change.

32. False. Percent change does not have units.

33. True. When quantity increases by 100 from 500 to 600, the cost increases by 15%, which is $0.15 \cdot 1000 = 150$ dollars. Cost goes from \$1000 to \$1150.

34. False. Relative change can be positive (if the quantity is increasing) or negative (if the quantity is decreasing.)

35. False. Relative rate of change is given as a percent per hour.

36. False, profit is the difference between revenue and cost.

37. True, this is the definition of revenue.

38. False, although the cost function depends on the quantity produced q, there can also be fixed costs of production which are present even when nothing is being produced.

39. False, as q increases $C(q)$ should increase, since producing more units will cost more than producing less.

40. True, since the more units sold, the more revenue is produced.

41. False. It is possible that supply is greater than demand.

42. True.

43. True, both marginal cost and marginal revenue have units dollars per item.

44. True, since the additional profit in selling one more item is the additional revenue from that item minus the additional cost to produce the item.

45. False. Imposing a sales tax shifts the curves which can change where they intersect.

46. True, this is a function of the form P_0a^t which is the definition of an exponential function.

47. True.

48. False, an exponential function can be increasing or decreasing.

49. False, the percent growth rate is 3%.

50. True, substituting $x = 2$ and $x = 1$, we have $\frac{Q(2)}{Q(1)} = \frac{35(1/3)^2}{35(1/3)} = 1/3$.

51. True, substituting $s = 0$, we have $R(0) = 16$.

52. True, the first two decimal digits of e are 2.7.

53. True.

54. True.

55. False, $Q(r)$ has constant differences in successive values so it is linear. It does not have constant ratios of successive values so it is not exponential.

56. False, $\ln(x)$ is only defined for $x > 0$.

57. True, since $e^0 = 1$.

58. False, for example, $\ln(2) = \ln(1 + 1) \neq \ln(1) + \ln(1) = 0 + 0 = 0$.

59. True, since $\ln x = c$ means $e^c = x$.

60. True, since as x increases, the exponent e needs to be raised to in order to equal x also increases.

61. True. We take ln of both sides and bring the exponent t down.

62. True, an exponential growth function is one which has the form $P = P_0e^{kt}$ for k a positive constant.

63. True, when $b > 0$, $e^{\ln b} = b$ and $\ln(e^b) = b$.

64. False, since if $B = 1$, then $\ln(2B) = \ln(2) \neq 2\ln(B) = 2\ln(1) = 0$.

65. True, by the properties of logarithms.

66. True, the doubling time depends only on the rate of growth, which in this case is the same.

67. False. It will take two half-lives, or 12 years, for the quantity to decay to 25 mg.

68. False, the half-life only depends on the rate of decay, which is the same in this case ($k = -0.6$). So, they have the same half-life.

69. False, if an initial investment Q is compounded annually with interest rate r for t years, then the value of the investment is $Q(1 + r)^t$. So, we should have $1000(1.03)^t$ as the value of the account.

70. False, if the rate of growth is larger, the doubling time will be shorter.

71. True, the doubling time is the value of t which gives double the initial value for P. Since the initial value is 5, we find the value of t when $P = 10$.

72. True, since a deposit made today (present value) will always grow with interest over time (future value).

73. True, since we have a continuous rate of growth, we use the formula P_0e^{kt}. In our case, $P_0 = 1000$, $k = .03$, and $t = 5$, giving $1000e^{.15}$.

74. True, since the longer the initial payment is invested, the more interest will be accumulated.

75. False, since less money needs to be invested today to grow to \$1000 ten years from now than to grow to \$1000 five years from now.

76. False, since $f(x+5)$ shifts the graph of f to the left by 5, while $f(x)+5$ shifts the graph of f up by 5.

77. True, since $f(x+k)$ is just a shift of the graph of $f(x)$ k units to the left, this shift will not affect the increasing nature of the function f. So, $f(x+k)$ is increasing.

78. True, since $-2g(t)$ flips and stretches the concave up graph of $g(t)$.

79. True, crossing the x-axis at $x = 1$ means that $f(1) = 0$. If we multiply both sides by 5, we see that $5f(1) = 0$, which means that $5f(x)$ crosses the x-axis at $x = 1$.

80. True, since the graph of $f(x+1)$ is the graph of $f(x)$ shifted to the left by 1.

81. False. We have $g(3+h) = (3+h)^2 = 9+6h+h^2$ so $g(3+h) \neq 9+h^2$ for many values of h.

82. False, $f(g(t)) = (t+1)^2 = t^2+2t+1$ and $g(f(t)) = t^2+1$ which are clearly not equal.

83. True, we have $f(g(x)) = f(\ln x) = (\ln x)^3 - 5$.

84. True, we can check that $g(u(x)) = g(3x^2+2) = (3x^2+2)^3 = h(x)$.

85. False, since $f(x+h) = (x+h)^2 - 1 = x^2+2xh+h^2-1$, we see that $f(x+h) - f(x) = 2xh + h^2$ which is not equal to h^2.

86. True.

87. False, if A is inversely proportional to B, then $A = k/B$ for some non-zero constant k.

88. True, a power function has the form $g(x) = k \cdot x^p$.

89. False, this is an exponential function.

90. False. It can be written as the power function $h(x) = 3x^{-1/2}$.

91. False. It can be written as a power function in the form $g(x) = (3/2)x^{-2}$

92. True, since $f(x) = (3/2)x^{1/2} = 1.5x^{1/2}$.

93. True.

94. True.

95. True. If p is proportional to q, then $p = kq$ for nonzero constant k, so $p/q = k$, a constant.

96. False, the amplitude is the coefficient of the trig function, which in this case is 3.

97. True, for a trig function of the form $\cos(kx)+b$, the period is given by $2\pi/k$. Since $k = 1$ in this case, the period is 2π.

98. False, if we substitute $t = 0$, then the first expression is not defined, whereas the second expression is $3/5$. Thus, they cannot be equal.

99. True. It is shifted $\pi/2$ to the left.

100. False, for a trig function of the form $A\cos(kx)+b$, the period is given by $2\pi/k$. Since $k = 5$ in this case, the period is $2\pi/5$.

101. False, since the period of $y = \sin(2t)$ is π and the period of $\sin(t)$ is 2π, the period is half.

102. True, since both have amplitude of 5. The second function has a vertical shift of 8.

103. False, one way to see that the graphs are different is that $(\sin x)^2$ is never negative (as it is a square). On the other hand, $y = \sin(x^2)$ is both positive and negative.

104. False, the function $\sin(x)$ oscillates between -1 and 1.

105. True.

PROJECTS FOR CHAPTER ONE

1. (a) Compounding daily (continuously),

$$\begin{aligned} P &= P_0 e^{rt} \\ &= \$450{,}000 e^{(0.06)(213)} \\ &\approx \$1.5977 \cdot 10^{11}. \end{aligned}$$

This amounts to approximately \$160 billion.

(b) Compounding yearly,

$$\begin{aligned} A &= \$450{,}000\,(1+0.06)^{213} \\ &= \$450{,}000(1.06)^{213} \approx \$450{,}000(245{,}555.29) \\ &\approx \$1.10499882 \cdot 10^{11}. \end{aligned}$$

This is only about \$110.5 billion.

(c) We first wish to find the interest that will accrue during 1990. For 1990, the principal is $\$1.105 \cdot 10^{11}$. At 6% annual interest, during 1990 the money will earn

$$0.06 \cdot \$1.105 \cdot 10^{11} = \$6.63 \cdot 10^{9}.$$

The number of seconds in a year is

$$\left(365\frac{\text{days}}{\text{year}}\right)\left(24\frac{\text{hours}}{\text{day}}\right)\left(60\frac{\text{mins}}{\text{hour}}\right)\left(60\frac{\text{secs}}{\text{min}}\right) = 31{,}536{,}000 \text{ sec.}$$

Thus, over 1990, interest is accumulating at the rate of

$$\frac{\$6.63 \cdot 10^{9}}{31{,}536{,}000 \text{ sec}} \approx \$210.24 \text{ /sec.}$$

2. **(a)** Since the population center is moving west at 40 miles per 10 years, or 4 miles per year, if we start in 2020, when the center is in Hartville, its distance d west of Hartville t years after 2020 is given by

$$d = 4t.$$

(b) It moved over 1000 miles over the 230 years from 1790 to 2020, so its average speed was greater than $1000/230 = 4.35$ miles/year, somewhat bigger than its present rate.

(c) According to the function in (a), after 500 years, the population center would be 2000 miles west of Hartville, in the Pacific Ocean, which is impossible.

3. Assuming that T_r decays exponentially, we have

$$T_r = Ae^{-kt}.$$

We find the values of A and k from the data in the table in the problem. We know that:

$$\begin{aligned} Ae^{-k\cdot 4} &= 37 && \text{since } T_r = 37 \text{ at } t = 4 \\ Ae^{-k\cdot 19.5} &= 13 && \text{since } T_r = 13 \text{ at } t = 19.5 \\ \text{so} \quad \frac{Ae^{-k\cdot 4}}{Ae^{-k\cdot 19.5}} &= \frac{37}{13} && \text{dividing} \\ e^{15.5k} &= \frac{37}{13} \\ k &= \frac{1}{15.5}\ln\left(\frac{37}{13}\right) = 0.06748. \end{aligned}$$

Having found k, we can now find A:

$$\begin{aligned} Ae^{-0.06748(4)} &= 37 && \text{since } T_r = 37 \text{ at } t = 4 \\ A &= 48.5 \text{ ng/ml.} \end{aligned}$$

Thus, the concentration of tryptase t hours after surgery is modeled by the exponential decay function

$$T_r = 48.5e^{-0.06748t} \text{ ng/ml.}$$

The greatest concentration is 48.5 ng/ml, which occurs at $t = 0$ when the patient leaves surgery. We can diagnose anaphylaxis since this peak level is above 45 ng/ml.