

**2.11** Fill in the details in Example 2.3 and verify the solution.

**Solution:**

The weak form is  $\int_0^1 \left( \frac{d\phi}{dx} \frac{dT}{dx} + W \right) dx - \int_0^1 \phi dx = 0$  Use the Galerkin approximation

$u(x) = a_1 \sin \pi x$ ,  $\phi(x) = \sin \pi x$ ,  $du = a_1 \pi \cos \pi x dx$ ,  $d\phi = \pi \cos \pi x dx$ . Replace in the integrals to get

$\int_0^1 (\pi \cos \pi x) (a_1 \pi \cos \pi x) dx = \int_0^1 \sin \pi x dx$  and using the trigonometric identity

$$\int_0^1 \cos^2 \pi x dx = \int_0^1 \frac{1}{2} (1 + \cos 2\pi x) dx \quad \text{we get} \quad a_1 = \frac{4}{\pi^3}.$$